

CS285 Spring 2009 – Homework 3

(due April 30, in class)

Professor David Parkes

*Points will be awarded for clarity, correctness and completeness of the answers. Typed answers will get a 5% bonus, but typing is not required. Submissions should be electronic or printed in class. Each student will be allotted 3 late days throughout the semester. Delays exceeding 3 days will cause penalties of 5% each to the overall homework grade. Submissions more than 4 days late on any one homework will not be accepted. Students may work **in pairs** and submit only one solution, but they **both** have to understand it. **Any sources can be used in answering these questions, but they should be cited.***

Total points: 220 – extra credit 35 points

1. (30 points) Bidding languages:
 - (a) The majority valuation on m items assigns a value of 1 to any bundle of at least $m/2$ items, and 0 to all other bundles. Prove that representing this valuation using an OR* formula requires size at least $\binom{m}{m/2}$.
 - (b) A downward-sloping symmetric valuation can be represented as $v_i(S) = \sum_{j=1}^{|S|} p_j$ with $p_1 \geq p_2 \geq \dots \geq p_m \geq 0$. Prove that OR-of-XORs can represent any downward sloping symmetric valuation on m items in size m^2 .
2. (40 points) Affine maximization:
 - (a) Describe a deterministic, strategyproof combinatorial auction for general valuations that is not an affine maximizer.
 - (b) Define the Groves mechanism payments for an affine maximizer and prove that the mechanism is strategyproof.
 - (c) Consider a single-parameter domain (i.e., in which each agent's private information is value $v_i \in \mathbb{R}$). Suppose that for each agent there is a *known*

interesting set, $W_i \subseteq A$ so that agent i 's value for an alternative is v_i if $a \in W_i$, and 0 otherwise. Fixing v_{-i} , show that the WMON condition implies that if the alternative $f(v_i, v_{-i}) \in W_i$ then $f(v'_i, v_{-i}) \in W_i$ for $v'_i \geq v_i$.

3. (40 points - 15 extra) A direct, deterministic, mechanism is strategyproof if and only if, for every $i \in N$ and every $v_{-i} \in V_{-i}$:
 - (A1) the payment function $t_i(v)$ can be written as $t_i(v) = p(a, v_{-i})$ and depends only on the alternative selected and the reports of other agents, and
 - (A2) the allocation function $f(v) \in \arg \max_{a \in A} [v_i(a) - p(a, v_{-i})]$.
 - (a) Prove formally that a mechanism that satisfies (A1) and (A2) is strategyproof.
 - (b) Prove formally that a mechanism must satisfy both (A1) and (A2) to be strategyproof.
 - (c) Define the second-price (Vickrey) auction in these terms (i.e., exhibit an agent-independent price function (A1) for every agent, and demonstrate that the winner determination rule satisfies (A2)).
 - (d) Through a very slight generalization to the above characterization, prove that the *random sampling optimal price auction* is strategyproof.
 - (e) (*Extra credit*) Define the VCG mechanism for a combinatorial auction in these terms.

4. (30 points - 20 extra) The Myerson auction:
 - (a) Show that the Myerson auction with symmetric priors is equivalent to a Vickrey auction with a reserve price.
 - (b) Consider the Myerson optimal auction for two agents; agent 1's value is uniform $v_1 \sim U(0, 1)$ and agent 2's value is uniform $v_2 \sim U(0, 2)$. Assume that the seller's value for the item is $v_0 = 0$.
 - i. Define the virtual valuation function for each agent. Are they monotone non-decreasing?
 - ii. Define the auction for this case and provide some intuition for why the asymmetry in the virtual valuation functions seems to be a reasonable way to improve revenue in this auction? (Limit your response to a few sentences.)
 - iii. (*Extra credit*) Explain why it is important that the virtual valuation function is monotone increasing for each agent. What is the idea that Myerson proposes when this is not the case?

5. (35 points) Consider a voting game with 3 agents (e.g., parties) and a quota of $Q = 100$ to pass a vote. Associate integer weights $1 \leq w_i < Q$ with each agent (e.g., the number of representatives). Define the coalitional value function

as $v(S) = 1$ if $\sum_{i \in S} w_i \geq Q$ and 0 otherwise. In the *inverse problem* we must assign weights to achieve a particular vector of Shapley value payoffs, or demonstrate that no such weights exist. Solve the following inverse problems:

- (a) Shapley values = $(1/3, 1/3, 1/3)$.
- (b) Shapley values = $(1/3, 2/3, 1/2)$.
- (c) Shapley values = $(0, 1/3, 2/3)$.
- (d) For the two games to which there is a solution to the inverse problem, check whether or not the core is empty (and if it is not empty, whether the Shapley value payoffs are in the core).

Hint: For thinking about the inverse problem write down the 6 possible agent orderings in determining the Shapley values. In establishing that one of the inverse problems is impossible, exhibit a set of linear constraints that are infeasible.

6. (45 points) Quick Fire (short answers are OK):

- (a) It is possible to have efficiency and exact budget balance in an incentive compatible (not necessarily dominant-strategy IC) double auction? (Point to the possibility/impossibility result.)
- (b) Is it possible to redistribute some of the payment collected from the winner in a second-price Vickrey auction without compromising ex post IR, efficiency, no deficit, or strategyproofness? (Point to an impossibility result, or suggest a mechanism.)
- (c) Consider the following English auction: An ask price is maintained, initialized to \$1. In each round, one or more agents can “shout” a new bid price (at least as high as the current ask price), and become the current winner, while the ask price is set to \$1 greater than this bid price. Ties are broken at random. The auction closes when there are no new bids, with the final winner paying its bid price. The *myopic best-response* strategy is to bid at the new ask price while (i) losing, and (ii) the ask price is less than the agent’s value. Is this MBR strategy a dominant strategy equilibrium of the English auction? Why or why not? (Either provide an intuitive argument to the affirmative, or give a counterexample. If you answer no, please also state whether MBR is an equilibrium under some other equilibrium concept.)

Good luck!