1. Introduction

1.1 Motivation

The public interest in traded securities has continuously grown over the past few years, with an especially strong growth in Germany and other European countries at the end of the 1990s. Consequently, events influencing stock prices, opinions and speculations on such events and their consequences, and even the daily stock quotes, receive much attention and media coverage. A few reasons for this interest are clearly visible in Fig. 1.1 which shows the evolution of the German stock index DAX [1] over the two years from October 1996 to October 1998. Other major stock indices, such as the US Dow Jones Industrial Average, the S&P500, or the French CAC40, etc., behaved in a similar manner in that interval of time. We notice three important features: (i) the continuous rise of the index over the first almost one and a half years which

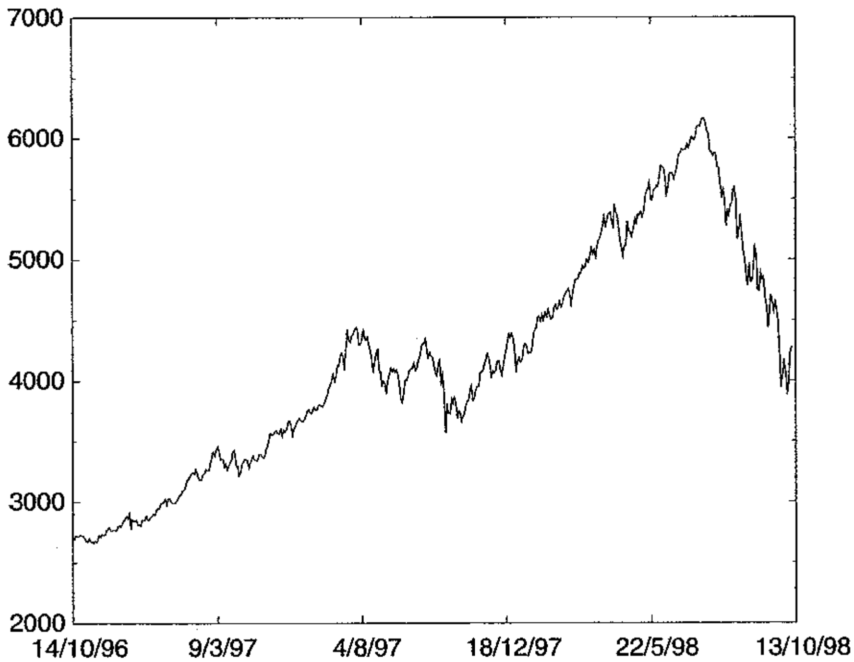

Fig. 1.1. Evolution of the DAX German stock index from October 14, 1996 to October 13, 1998. Data provided by Deutsche Bank Research

was interrupted only for very short periods; (ii) the crash on the "second black Monday", October 27, 1997 (the "Asian crisis", the reaction of stock markets to the collapse of a bank in Japan, preceded by rumors about huge amounts of foul credits and derivative exposures of Japanese banks, and a period of devaluation of Asian currencies). (iii) the very strong drawdown of quotes between July and October 1998 (the "Russian debt crisis", following the announcement by Russia of a moratorium on its debt reimbursements, and a devaluation of the Russian rouble), and the collapse of the Long Term Capital Management hedge fund.

While the long-term rise of the index until 2000 seemed to offer investors attractive, high-return opportunities for making money, enormous fortunes of billions or trillions of dollars were annihilated in very short times, perhaps less than a day, in crashes or periods of extended drawdowns. Such events – the catastrophic crashes perhaps more than the long-term rise – exercise a strong fascination.

To place these events in a broader context, Fig. 1.2 shows the evolution of the DAX index from 1975 to 2005. Several different regimes can be distinguished. In the initial period 1975–1983, the returns on stock investments were extremely low, about 2.6% per year. Returns of 200 DAX points, or 12%, per year were generated in the second period 1983–1996. After 1996, we see a marked acceleration with growth rates of 1200 DAX points, or 33%, per year. We also notice that, during the growth periods of the stock market, the losses incurred in a sudden crash usually persist only over a short

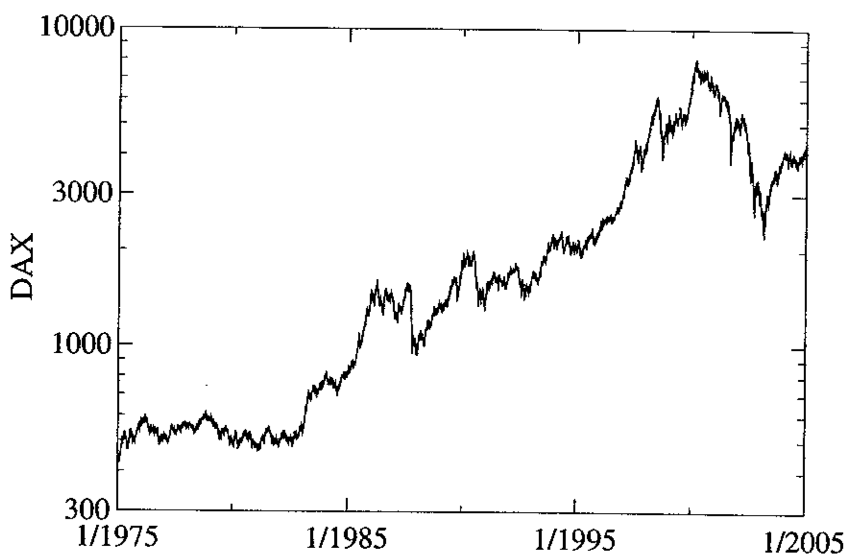

Fig. 1.2. Long-term evolution of the DAX German stock index from January 1, 1975 to January 1, 2005. Data provided by Deutsche Bank Research supplemented by data downloaded from Yahoo, `http://de.finance.yahoo.com`

time, e.g. a few days after the Asian crash [(ii) above], or about a year after the Russian debt crisis [(iii) above]. The long term growth came to an end, around April 2000 when markets started sliding down. The fourth period in Fig. 1.2 from April 2000 to the end of the time series on March 12, 2003, is characterized by a long-term downward trend with losses of approximately 1400 DAX points, or 20% per year. The DAX even fell through its long-term upward trend established since 1983. Despite the overall downward trend of the market in this period, it recovered as quickly from the crash on September 11, 2001, as it did after crashes during upward trending periods. Finally, the index more or less steadily rose from its low at 2203 points on March 12, 2003 to about 4250 points at the end of 2004. Only the future will show if a new growth period has been kicked off.

This immediately leads us to a few questions:

- Is it possible to earn money not only during the long-term upward moves (that appears rather trivial but in fact is not) *but also during the drawdown periods?* These are questions for investors or speculators.
- What are the factors responsible for long- and short-term price changes of financial assets? How do these factors depend on the type of asset, on the investment horizon, on policy, etc.?
- How do the three growth periods of the DAX index, discussed in the preceding paragraph, correlate with economic factors? These are questions for economists, analysts, advisors to politicians, and the research departments of investment banks.
- What statistical laws do the price changes obey? How smooth are the changes? How frequent are jumps? These problems are treated by mathematicians, econometrists, but more recently also by physicists. The answer to this seemingly technical problem is of great relevance, however, also to investors and portfolio managers, as the efficiency of stop-loss or stop-buy orders [2] directly depends on it.
- How big is the risk associated with an investment? Can this be measured, controlled, limited or even eliminated? At what cost? Are reliable strategies available for that purpose? How big is any residual risk? This is of interest to banks, investors, insurance companies, firms, etc.
- How much fortune is at risk with what probability in an investment into a specific security at a given time?
- What price changes does the evolution of a stock price, resp. an index, imply for "financial instruments" (derivatives, to be explained below, cf. Sect. 2.3)? This is important both for investors but also for the writing bank, and for companies using such derivatives either for increasing their returns or for hedging (insurance) purposes.
- Can price changes be predicted? Can crashes be predicted?

1.2 Why Physicists? Why Models of Physics?

This book is about financial markets from a physicist's point of view. Statistical physics describes the complex behavior observed in many physical systems in terms of their simple basic constituents and simple interaction laws. Complexity arises from interaction and disorder, from the cooperation and competition of the basic units. Financial markets certainly are complex systems, judged both by their output (cf., e.g., Fig. 1.1) and their structure. Millions of investors frequent the many different markets organized by exchanges for stocks, bonds, commodities, etc. Investment decisions change the prices of the traded assets, and these price changes influence decisions in turn, while almost every trade is recorded.

When attempting to draw parallels between statistical physics and financial markets, an important source of concern is the complexity of human behavior which is at the origin of the individual trades. Notice, however, that nowadays a significant fraction of the trading on many markets is performed by computer programs, and no longer by human operators. Furthermore, if we make abstraction of the trading volume, an operator only has the possibility to buy or to sell, or to stay out of the market. Parallels to the Ising or Potts models of Statistical Physics resurface!

More specifically, take the example of Fig. 1.1. If we subtract out long-term trends, we are left essentially with some kind of random walk. In other words, the evolution of the DAX index looks like a random walk to which is superposed a slow drift. This idea is also illustrated in the following story taken from the popular book "A Random Walk down Wall Street" by B. G. Malkiel [3], a professor of economics at Princeton. He asked his students to derive a chart from coin tossing.

> "For each successive trading day, the closing price would be determined by the flip of a fair coin. If the toss was a head, the students assumed the stock closed 1/2 point higher than the preceding close. If the flip was a tail, the price was assumed to be down 1/2. ... The chart derived from the random coin tossing looks remarkably like a normal stock price chart and even appears to display cycles. Of course, the pronounced 'cycles' that we seem to observe in coin tossings do not occur at regular intervals as true cycles do, but neither do the ups and downs in the stock market. In other simulated stock charts derived through student coin tossings, there were head-and-shoulders formations, triple tops and bottoms, and other more esoteric chart patterns. One of the charts showed a beautiful upward breakout from an inverted head and shoulders (a very bullish formation). I showed it to a chartist friend of mine who practically jumped out of his skin. "What is this company?" he exclaimed. "We've got to buy immediately. This pattern's a classic. There's no question the stock will be up 15 points next week." He did not respond kindly to me when I told him the chart had been produced by flipping a coin." Reprinted from B. G. Malkiel: *A Random Walk down Wall Street,* ©1999 W. W. Norton

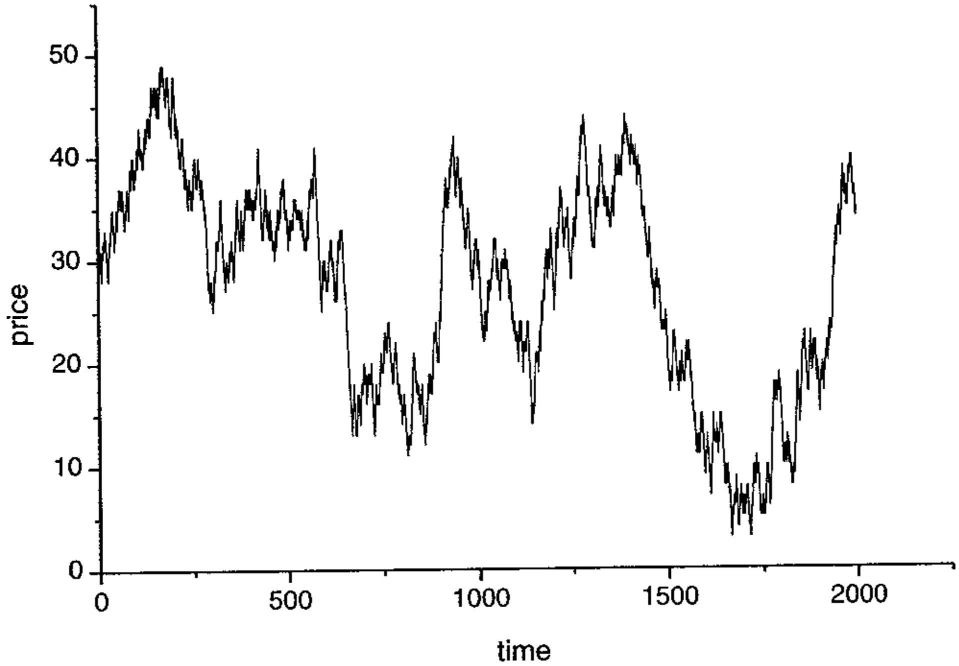

Fig. 1.3. Computer simulation of a stock price chart as a random walk

The result of a computer simulation performed according to this recipe, is shown in Fig. 1.3, and the reader may compare it to the DAX evolution shown in Fig. 1.1. "THE random walk", usually describing Brownian motion, but more generally any kind of stochastic process, is well known in physics; so well known in fact that most people believe that its first mathematical description was achieved in physics, by A. Einstein [4].

It is therefore legitimate to ask if the description of stock prices and other economic time series, and our ideas about the underlying mechanisms, can be improved by

- the understanding of parallels to phenomena in nature, such as, e.g.,
  - diffusion
  - driven systems
  - nonlinear dynamics, chaos
  - formation of avalanches
  - earthquakes
  - phase transitions
  - turbulent flows
  - stochastic systems
  - highly excited nuclei
  - electronic glasses, etc.;
- the associated mathematical methods developed for these problems;
- the modeling of phenomena which is a distinguished quality of physics. This is characterized by

- identification of important factors of causality, important parameters, and estimation of orders of magnitude;
- simplicity of a first qualitative model instead of absolute fidelity to reality;
- study of causal relations between input parameters and variables of a model, and its output, i.e. solutions;
- empirical check using available data;
- progressive approach to reality by successive incorporation of new elements.

These qualities of physicists, in particular theoretical physicists, are being increasingly valued in economics. As a consequence, many physicists with an interest in economic or financial themes have secured interesting, challenging, and well-paid jobs in banks, consulting companies, insurance companies, risk-control divisions of major firms, etc.

Rather naturally, there has been an important movement in physics to apply methods and ideas from statistical physics to research on financial data and markets. Many results of this endeavor are discussed in this book. Notice, however, that there are excellent specialists in all disciplines concerned with economic or financial data, who master the important methods and tools better than a physicist newcomer does. There are examples where physicists have simply rediscovered what has been known in finance for a long time. I will mention those which I am aware of, in the appropriate context. As an example, even computer simulations of "microscopic" interacting-agent models of financial markets have been performed by economists as early as 1964 [5]. There may be many others, however, which are not known to me. I therefore call for modesty (the author included) when physicists enter into new domains of research outside the traditional realm of their discipline. This being said, there is a long line of interaction and cross-fertilization between physics and economy and finance.

1.3 Physics and Finance – Historical

The contact of physicists with finance is as old as both fields. Isaac Newton lost much of his fortune in the bursting of the speculative bubble of the South Sea boom in London, and complained that while he could precisely compute the path of celestial bodies to the minute and the centimeter, he was unable to predict how high or low a crazy crowd could drive the stock quotations.

Carl Friedrich Gauss (1777–1855), who is honored on the German 10 DM bill (Fig. 1.4), has been very successful in financial operations. This is evidenced by his leaving a fortune of 170,000 Taler (contemporary, local currency unit) on his death while his basic salary was 1000 Taler. According to rumors, he derived the normal (Gaussian) distribution of probabilities in

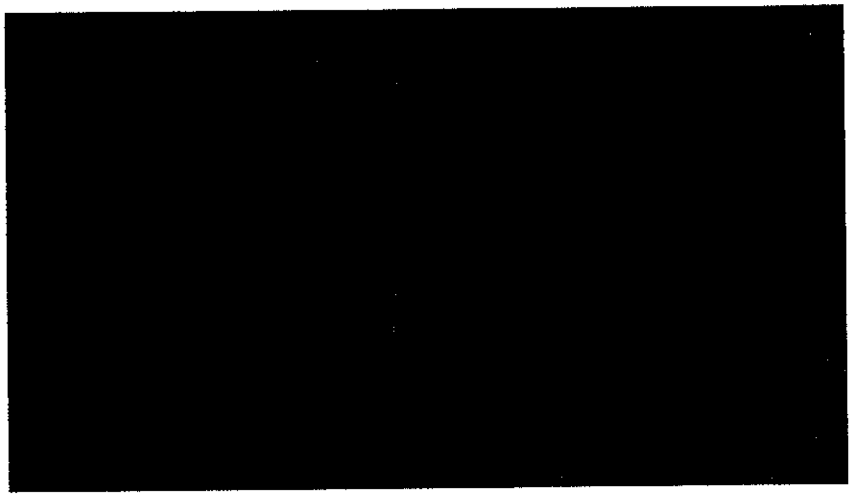

Fig. 1.4. Carl Friedrich Gauss on the German 10 DM bill (detail), courtesy of Deutsche Bundesbank

estimating the default risk when giving credits to his neighbors. However, I have failed to find written documentation of this fact.

His calculation of the pensions for widows of the professors of the University of Göttingen (1845–1851) is a seminal application of probability theory to the related field of insurance. The University of Göttingen, where Gauss was professor, had a fund for the widows of the professors. Its administrators felt threatened by ruin as both the number of widows, as well as the pensions paid, increased during those years. Gauss was asked to evaluate the state of the fund, and to recommend actions to save it. After six years of analysis of mortality tables, historical data, and elaborate calculations, he concluded that the fund was in excellent financial health, that a further increase of the pensions was possible, but that the membership should be restricted. Quite contrary to the present public discussion!

The most important date in the perspective of this book is March 29, 1900 when the French mathematician Louis Bachelier defended his thesis entitled "Théorie de la Spéculation" at the Sorbonne, University of Paris [6]. In his thesis, he developed, essentially correctly and comprehensively, the theory of the random walk – and that five years before Einstein. He constructed a model for exchange quotes, specifically for French government bonds, and estimated the chances of success in speculation with derivatives that are somewhat in between futures and options, on those bonds. He also performed empirical studies to check the validity of his theory. His contribution had been forgotten for at least 60 years, and was rediscovered independently in the financial community in the late 1950s [7, 8]. Physics is becoming aware of Bachelier's important work only now through the interface of statistical physics and quantitative finance.

More modern examples of physicists venturing into finance include M. F. M. Osborne who rediscovered the Brownian motion of stock markets in 1959 [7, 8], and Fisher Black who, together with Myron Scholes, reduced an option pricing problem to a diffusion equation. Osborne's seminal work was first presented in the Solid State Physics seminar of the US Naval Research Laboratory before its publication. Black's work will be discussed in detail in Chap. 4.

1.4 Aims of this Book

This book is based on courses on models of physics for financial markets ("Physikalische Modelle in der Finanzwirtschaft") which I have given at the Universities of Bayreuth, Freiburg, and Ulm, and at Academia Sinica, Taipei. It largely keeps the structure of the course, and the subject choice reflects both my taste and that of my students.

I will discuss models of physics which have become established in finance, or which have been developed there even before (!) being introduced in physics, cf. Chap. 3. In doing so, I will present both the physical phenomena and problems, as well as the financial issues. As the majority of attendees of the courses were physicists, the emphasis will be more on the second, the financial aspects. Here, I will present with approximately equal weight established theories as well as new, speculative ideas. The latter often have not received critical evaluation yet, in some cases are not even officially published and are taken from preprint servers [9]. Readers should be aware of the speculative character of such papers.

Models for financial markets often employ strong simplifications, i.e. treat idealized markets. This is what makes the models possible, in the first instance. On the other hand, there is no simple way to achieve above-average profits in such idealized markets ("there is no free lunch"). The aim of the course therefore is NOT to give recipes for quick or easy profits in financial markets. On the same token, we do not discuss investment strategies, if such should exist. Keeping in line with the course, I will attempt an overview only of the *most basic aspects* of financial markets and financial instruments. There is excellent literature in finance going much further, though away from statistical physics [10]–[16]. Hopefully, I can stimulate the reader's interest in some of these questions, and in further study of these books.

The following is a list of important issues which I will discuss in the book:

- Statistical properties of financial data. Distribution functions for fluctuations of stock quotes, etc. (stocks, bonds, currencies, derivatives).
- Correlations in financial data.
- Pricing of derivatives (options, futures, forwards).
- Risk evaluation for market positions, risk control using derivatives (hedging).

- Hedging strategies.
- Can financial data be used to obtain information on the markets?
- Is it possible to predict (perhaps in probabilistic terms) the future market evolution? Can we formulate equations of motion?
- Description of stock exchange crashes. Are predictions possible? Are there typical precursor signals?
- Is the origin of the price fluctuations exogenous or endogenous (i.e. reaction to external events or caused by the trading activity itself)?
- Is it possible to perform "controlled experiments" through computer simulation of microscopic market models?
- To what extent do operators in financial markets behave rationally?
- Can game-theoretic approaches contribute to the understanding of market mechanisms?
- Do speculative bubbles (uncontrolled deviations of prices away from "fundamental data", ending typically in a collapse) exist?
- The definition and measurment of risk.
- Basic considerations and tools in risk management.
- Economic capital requirements for banks, and the capital determination framework applied by banking supervisors.

The organization of this book is as follows. The next chapter introduces basic terminology for the novice, defines and describes the three simplest and most important derivatives (forwards, futures, options) to be discussed in more detail throughout this book. It also introduces the three types of market actors (speculators, hedgers, arbitrageurs), and explains the mechanisms of price formation at an organized exchange.

Chapter 3 discusses in some detail Bachelier's derivation of the random walk from a financial perspective. Though no longer state of the art, many aspects of Bachelier's work are still at the basis of the theories of financial markets, and they will be introduced here. We contrast Bachelier's work with Einstein's theory of Brownian motion, and give some empirical evidence for Brownian motion in stock markets and in nature.

Chapter 4 discusses the pricing of derivatives. We determine prices of forward and futures contracts and limits on the prices of simple call and put options. More accurate option prices require a model for the price variations of the underlying stock. The standard model is provided by geometric Brownian motion where the logarithm of a stock price executes a random walk. Within this model, we derive the seminal option pricing formula of Black, Merton, and Scholes which has been instrumental for the explosive growth of organized option trading. We also measures of the sensitivity of option prices with respect to the basic variables of the model ("The Greeks"), options with early-exercise features, and volatility indices for financial markets.

Chapter 5 discusses the empirical evidence for or against the assumptions of geometric Brownian motion: price changes of financial assets are uncorrelated in time and are drawn from a normal distribution. While the first assumption is rather well satisfied, deviations from a normal distribution will lead us to consider in more depth another class of stochastic process, stable Lévy processes, and variants thereof, whose probability distribution functions possess fat tails and which describe financial data much better than a normal distribution. Here, we also discuss the implications of these fat-tailed distributions both for our understanding of capital markets, and for practical investments and risk management. Correlations are shown to be an important feature of financial markets. We describe temporal correlations of financial time series, asset–asset correlations in financial markets, and simple models for markets with correlated assets.

An interesting analogy has been drawn recently between hydrodynamic turbulence and the dynamics of foreign exchange markets. This will be discussed in more depth in Chap. 6. We give a very elementary introduction to turbulence, and then work out the parallels to financial time series. This line of work is still controversial today. Multifractal random walks provide a closely related framework, and are discussed.

Once the significant differences between the standard model – geometric Brownian motion – and real financial time series have been described, we can carry on to develop improved methods for pricing and hedging derivatives. This is described in Chap. refchap:risk. An important step is the passage from the differential Black–Scholes world to an integral representation of the life scenarios of an option. Consequently, aside numerical procedures, path integrals which are well-known in physics, are shown to be important tools for option valuation in more realistic situations.

Chapter 8 gives a brief overview of computer simulations of microscopic models for organized markets and exchanges. Such models are of particular importance because, unlike physics, controlled experiments establishing cause–effect relationships are not possible on financial markets. On the other hand, there is evidence that the basic hypotheses underlying standard financial theory may be questionable. One way to check such hypotheses is to formulate a model of interacting agents, operating on a given market under a given set of rules. The model is then "solved" by computer simulations. A criterion for a "good" model is the overlap of the results, e.g., on price changes, correlations, etc., with the equivalent data of real markets. Changing the rules, or some other parameters, allows one to correlate the results with the input and may result in an improved understanding of the real market action.

In Chap. 9 we review work on the description of stock market crashes. We emphasize parallels with natural phenomena such as earthquakes, material failure, or phase transitions, and discuss evidence for and against the hyptothesis that such crashes are outliers from the statistics of "normal" price fluctuations in the stock market. If true, it is worth searching for characteristic patterns preceding market crashes. Such patterns have apparently been found in historical crashes and, most remarkably, have allowed the predicition of the Asian crisis crash of October 27, 1997, but also of milder events such as a reversal of the downward trend of the Japanese Nikkei stock index, in early 1999. On the other hand, bearish trend reversals predicted in many major stock indices for the year 2004 have failed to materialize. We discuss the controversial status of crash predictions but also the improved understanding of what may happen before and after major financial crashes.

Chapters 10 and 11 leave the focus of statistical physics and turn towards banking practice. This appears important because many job opportunities requiring strong quantitative qualifications have been (and continue to be) created in banks. On the other hand, both the basic practices and the hot topics of banking, regrettably, are left out of most presentation for physics audiences. Chapter 10 is concerned with risk management. We define risk and discuss various measures of risk. We classify various types of risk and discuss the basic tools of risk management.

Chapter 11 finally discusses capital requirements for banks. Capital is taken as a cushion against losses which a bank may suffer in the markets, and therefore is an important quantity to manage risk and performance. The first part of the chapter discusses economic capital, i.e. what a bank has to do under purely economic considerations. Regulatory authorities apply a different framework to the banks they supervise. This is explained in the second part of Chap. 11. The new Basel Capital Accord (Basel II) takes a significant fraction of space. On the one hand, it will set the regulatory capital and risk management standards for the decades to come, in many countries of the world. On the other hand, it is responsible for many of the employment opportunities which may be open to the readers.

There are excellent introductions to this field with somewhat different or more specialized emphasis. Bouchaud and Potters have published a book which emphasizes derivative pricing [17]. The book by Mantegna and Stanley describes the scaling properties of and correlations in financial data [18]. Roehner has written a book with emphasis on empirical investigations which include financial markets but cover a significantly vaster field of economics [19]. Another book presents computer simulation of "microscopic" market models [20]. The analysis of financial crashes has been reviewed in a book by one of its main protagonists [21]. Mandelbrot also published a volume summarizing his contributions to fractal and scaling behavior in financial time series [22]. The important work of Olsen & Associates, a Zurich-based company working on trading models and prediction of financial time series, is summarized in *High Frequency Finance* [23]. The application of stochastic processes and path integrals, respectively, to problems of finance is briefly discussed in two physics books [24, 25] whose emphasis, though, is on phyiscal methods and applications. Finally, there has been a series of conferences and workshops whose proceedings give an overview of the state of this rapidly evolving field of research at the time of the event [26]. More sources of information are listed in the Appendix.

2. Basic Information on Capital Markets

2.1 Risk

Risk and profit are the important drivers of financial markets. Briefly, risk is defined as deviation of the actual outcome of an investment from its expected outcome when this deviation is negative. An alternative definition would view risk as the negative changes of a future position with respect to the present position. The difference does not matter much until we define quantitative risk measures in Chap. 10.3. Taking risk, reducing risk, and managing risk are important motivations for many operations in financial markets.

An investor taking risk will expect a certain return as compensation, the more so the higher the risk. Risky assets therefore also possess, at least on the average, high expected growth rates. Investments in risky stocks should be rewarded by a high rate of growth of their price. Investments in risky bonds should be rewarded by a high interest coupon.

Almost all investments are risky. There are very few instances which, to a good approximation, can be considered riskless. An investment in US treasury notes and bonds is considered a riskless investement because there is no doubt that the US treasury will honor its payment obligation. The same applies to bonds emitted by a number of other states and a few corporations (the so-called "AAA-rated" states and corporations). The interest rate paid on these bonds is called the riskless interest rate r, and will play an important role in many theoretical arguments in our later discussion. Interest rates change with time, though, both nominally and effectively. The rate r paid on two otherwise identical bonds emitted at different dates may be different. And the effective return of a traded bond bought or sold at times between emission and maturity fluctuates as a result of trading. In line with neglecting this interest rate risk, we will assume the risk-free interest rate r to be constant over the time scale considered.

2.2 Assets

What are the objects we are concerned with in this book? Let us start by looking into the portfolio of assets of a bank, or into the financial pages of a major newspaper. The bank portfolio may contain stocks, bonds, currencies, commodities, (private) equity, real estate, loans, mutual funds, hedge funds, etc., and derivatives, such as futures, options, or warrants.

The financial pages of the major newspapers contain the quotations of the most important traded assets of this portfolio. In addition, they contain quotations of market indices. Indices measure the composite performance of national markets, industries, or market segments. Examples include (i) for stock markets the Dow Jones Industrial Average, S&P500, DAX, DAX 100, CAC 40, etc., for blue chip stocks in the US, Germany, and France, respectively, (ii) the NASDAQ or TECDAX indices measuring the US and German high-technology markets, (iii) the Dow Jones Stoxx 50 index measuring the performance of European blue chip stocks irrespective of countries, or their participation in the European currency system. (iv) Indices are also used for bond markets, e.g., the REX index in Germany, but bond markets are also characterized by the prices and returns of certain benchmark products [11].

There are several ways to classify these assets. Usually, the assets held by a bank are organized in different groups, called "books". A "trading book" contains the assets held for trading purposes, normally for a rather short time. A simple trading book may contain stocks, bonds, currencies, commodities, and derivatives. The "banking book" contains assets held for longer periods of time, and mostly for business motivations. Assets of the banking book often are loans, mortgage backed loans, real estate, private equity, stocks, etc.

Some assets are *securities.* Securities are normally traded on organized markets (in some cases *over the counter,* OTC, i.e. directly between a bank and its client) and include stock, bonds, currencies, and derivatives. Their prices are fixed by demand and supply in the trading process. The following assets in the bank portfolio are not securities: commodities, equity unless it is in stocks, real estate, loans. Prices of traded securities usually are available as time series with a reasonably high frequency. Market indices are not securities although investments products replicating market indices are securities, often with a hidden derivative element. On the statistical side, very good time series are available for market indices, as illustrated by Figs. 1.1 and 1.2, and many to follow. Good price histories are available, too, for commodities.

Mutual funds, hedge funds, etc., are portfolios of securities. A portfolio in an ensemble of securities held by an investor. Their price is fixed by trading their individual components. We shall explicitly consider portfolios of securities in Chap. 10 where we show that the return of such a portfolio can be maximized at given risk by buying the securities is specific quantities which can be calculated.

A special class of securities merits a general name and discussion of its own. A *derivative* (also derivative security, contingent claim) is a financial instrument whose value depends on other, more basic underlying variables [10, 12, 13]. Very often, these variables are the prices of other securities (such as stocks, bonds, currencies, which are then called "underlying securities" or, for short, just "the underlying") with, of course, a series of additional parameters involved in determining the precise dependence. There are also derivatives on commodities (oil, wheat, sugar, pork bellies [!], gold, etc.), on market indices (cf. above), on the volatility of markets and also on phenomena apparently exterior to markets such as weather. As indicated by the examples of commodities and market indices, the emission of a derivative on these assets produces an "artificial" security. Especially in the case of commodities and markets indicies, the existence of derivatives considerably facilitates investment in these assets. Recently, the related transformation of portfolios of loans into tradable securities, known as securitization, has become an important practice in banking.

Derivatives are traded either on organized exchanges, such as Deutsche Terminbörse, DTB, which has evolved into EUREX by fusion with its Swiss counterpart, the Chicago Board of Trade (CBOT), the Chicago Board Options Exchange (CBOE), the Chicago Mercantile Exchange (CME), etc., or *over the counter* (OTC). Derivatives traded on exchanges are standardized products, while over the counter trading is done directly between a financial institution and a customer, often a corporate client or another financial institution, and therefore allows the tailoring of products to the individual needs of the clients.

Here, we mostly focus on stocks, market indices, and currencies, and their respective derivatives. We do this for two main reasons: (i) much of the research, especially by physicists, has concentrated on these assets; (ii) they are conceptually simpler than, e.g., bonds and therefore more suited to explain the basic mechanisms. Bond prices are influenced by interest rates. The interest rates, however, depend on the maturity of the bond, and the time to maturity therefore introduces an additional variable into the problem. Notice, however, that bond markets typically are much bigger than stock markets. Institutional investors such as insurance companies invest large volumes of money on the bond market because there they face less risk than with investments in, e.g., stocks.

2.3 Three Important Derivatives

Here, we briefly discuss the three simplest derivatives on the market: forward and futures contracts, and call and put options. They are sufficient to illustrate the basic principles of operation, pricing, and hedging. Many more instruments have been and continue to be created. Pricing such instruments, and using them for speculative or hedging purposes may present formidable technical challenges. They rely, however, on the same fundamental principles which we discuss in the remainder of this book where we refer to the three basic derivatives described below. Readers interested in those more complex instruments, are referred to the financial literature [10]–[15].

2.3.1 Forward Contracts

A forward contract (or just: forward for short) is a contract between two parties (usually two financial institutions or a financial institution and a corporate client) on the delivery of an asset at a certain time in the future, the maturity of the contract, at a certain price. This delivery price is fixed at the time the contract is entered.

Forward contracts are not usually traded on exchanges but rather over the counter (OTC), i.e. between a financial institution and its counterparty. For both parties, there is an obligation to honor the contract, i.e., to deliver/pay the asset at maturity.

As an example, consider a US company who must pay a bill of 1 million pound sterling three months from now. The amount of dollars the company has to pay obviously depends on the dollar/sterling exchange rate, and its evolution over the next three months therefore presents a risk for the company. The company can now enter a forward over 1 million pounds with maturity three months from now, with its bank. This will fix the exchange rate for the company as soon as the forward contract is entered. This rate may differ from the spot rate (i.e., the present day rate for immediate delivery), and include the opinion of the bank and/or market on its future evolution (e.g., spot 1.6080, 30-day forward 1.6076, 90-day forward 1.6056, 180-day forward 1.6018, quoted from Hull [10] as of May 8, 1995) but will effectively fix the rate for the company three months from now to 1.6056 US$/£.

2.3.2 Futures Contract

A *futures contract* (futures) is rather similar to a forward, involving the delivery of an asset at a fixed time in the future (maturity) at a fixed price. However, it is standardized and traded on exchanges. There are also differences relating to details of the trading procedures which we shall not explore here [10]. For the purpose of our discussion, we shall not distinguish between forward and futures contracts.

The above example, involving popular currencies in standard quantities, is such that it could as well apply to a futures contract. The differences are perhaps more transparent with a hypothetical example of buying a car. If a customer would like to order a BMW car in yellow with pink spots, there might be 6 months delivery time, and the contract will be established in a way that assures delivery and payment of the product at the time of maturity. Normally, there will be no way out if, during the six months, the customer changes his preferences for the car of another company. This corresponds to the forward situation. If instead one orders a black BMW, and changes opinion before delivery, for a Mercedes-Benz, one can try to resell the contract on the market (car dealers might even assist with the sale) because the product is sufficiently standardized so that other people are also interested in, and might enter the contract.

2.3.3 Options

Options may be written on any kind of underlying assets, such as stocks, bonds, commodities, futures, many indices measuring entire markets, etc. Unlike forwards or futures which carry an *obligation* for both parties, options give their holder the *right* to buy or sell an underlying assets in the future at a fixed price. However, they imply an obligation for the writer of the option to deliver or buy the underlying asset.

There are two basic types of options: *call options* (calls) which give the holder the right to buy, and *put options* (puts) which give their holder the right to sell the underlying asset in the future at a specified price, the strike price of the option. Conversely, the writer has the obligation to sell (call) or buy (put) the asset. Options are distinguished as being of European type if the right to buy or sell can only be exercised at their date of maturity, or of American type if they can be exercised at any time from now until their date of maturity. Options are traded regularly on exchanges.

Notice that, for the holder, there is no obligation to exercise the options while the writer has an obligation. As a consequence of this asymmetry, there is an intrinsic cost (similar to an insurance premium) associated with the option which the holder has to pay to the writer. This is different from forwards and futures which carry an obligation for both parties, and where there is no intrinsic cost associated with these contracts.

Options can therefore be considered as insurance contracts. Just consider your car insurance. With some caveats concerning details, your insurance contract can be reinterpreted as a put option you bought from the insurance company. In the case of an accident, you may sell your car to the insurance company at a predetermined price, resp. a price calculated according to a predetermined formula. The actual value of your car after the accident is significantly lower than its value before, and you will address the insurance for compensation. Your contract protects your investment in your car against unexpected losses. Precisely the same is achieved by a put option on a capital market. Reciprocally, a call option protects its owner against unexpected rises of prices. As in our example, with real options on exercise, one often does not deliver the product (which is possible in simple cases but impossible, e.g., in the case of index options), but rather settles the difference in cash.

As another example, consider buying 100 European call options on a stock with a strike price (for exercise) of $X =$ DM 100 when the spot price for the stock is $S_t =$ DM 98. Suppose the time to maturity to be $T - t = 2m$.

- If at maturity T, the spot price $S_T <$ DM 100, the options expire worthless (it makes no sense to buy the stock more expensively through the options than on the spot market).
- If, however, $S_T >$ DM 100, the option should be exercised. Assume $S_T =$ DM 115. The price gain per stock is then DM 15, i.e., DM 1500 for the entire investment. However, the net profit will be diminished by the price

of the call option C. With a price of C = DM 5, the total profit will be DM 1000.

- The option should be exercised also for DM 100 $< S_T <$ DM 105. While there is a net loss from the operation, it will be inferior to the one incurred ($-$ 100 C) if the options had expired.

The profile of profit, for the holder, versus stock price at maturity is given in Fig. 2.1. The solid line corresponds to the call option just discussed, while the dashed line shows the equivalent profile for a put.

When buying a call, one speculates on rising stock prices, resp. insures against rising prices (e.g., when considering future investments), while the holder of a put option speculates on, resp. insures, against falling prices.

For the holder, there is the possibility of unlimited gain, but losses are strictly limited to the price of the option. This asymmetry is the reason for the intrinsic price of the options. Notice, however, that in terms of practical, speculative investments, the limitation of losses to the option price still implies a total loss of the invested capital. It only excludes losses higher than the amount of money invested!

There are many more types of options on the markets. Focusing on the most elementary concepts, we will not discuss them here, and instead refer the readers to the financial literature [10]–[15]. However, it appears that much applied research in finance is concerned with the valuation of, and risk management involving, exotic options.

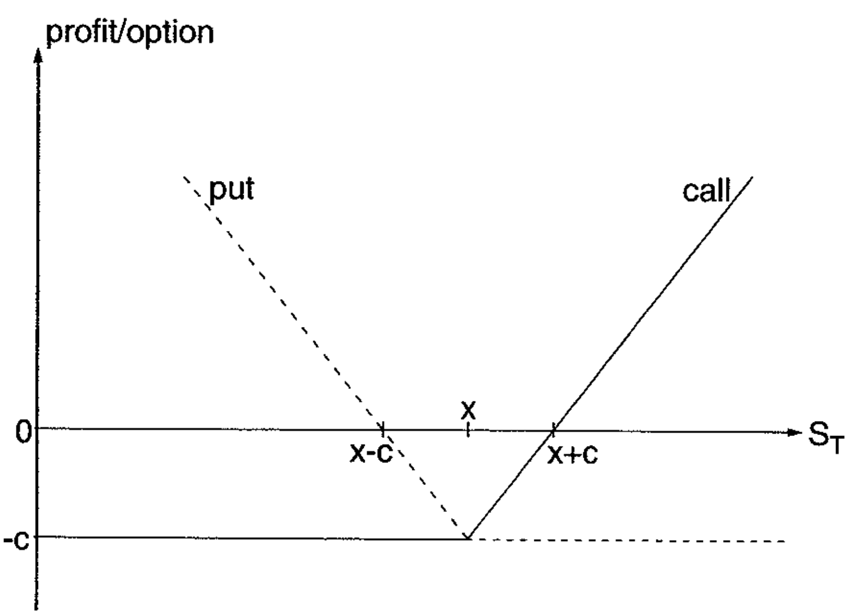

Fig. 2.1. Profit profile of call (*solid line*) and put (*dashed line*) options. S_T is the price of the underlying stock at maturity, X the strike price of the option, and C the price of the call or put

2.4 Derivative Positions

In every contract involving a derivative, one of the parties assumes the *long position,* and agrees to buy the underlying asset at maturity in case of a forward or futures contract, or, as the holder of a call/put option, has the right to buy/sell the underlying asset if the option is exercised. His partner assumes the *short position,* i.e., agrees to deliver the asset at maturity in a forward or futures or if a call option is exercised, resp. agrees to buy the underlying asset if a put option is exercised.

In the example on currency exchange rates in Sect. 2.3.1, the company took the long position in a forward contract on 1 million pounds sterling, while its bank went short. If the acquisition of a new car was considered as a forward or futures contract, the future buyer took the long position and the manufacturer took the short position.

With options, of course, one can go long or short in a call option, and in put options. The discussion of options in Sect. 2.3.3 above always assumed the long position. Observe that the profit profile for the writer of an option, i.e., the partner going short, is the inverse of Fig. 2.1 and is shown in Fig. 2.2. The possibilities for gains are limited while there is an unlimited potential for losses. This means that more money than invested may be lost due to the liabilities accepted on writing the contract.

Short selling designates the sale of assets which are not owned. Often there is no clear distinction from "going short". In practice, short selling is possible quite generally for institutional investors but only in very limited circumstances for individuals. The securities or derivatives sold short are

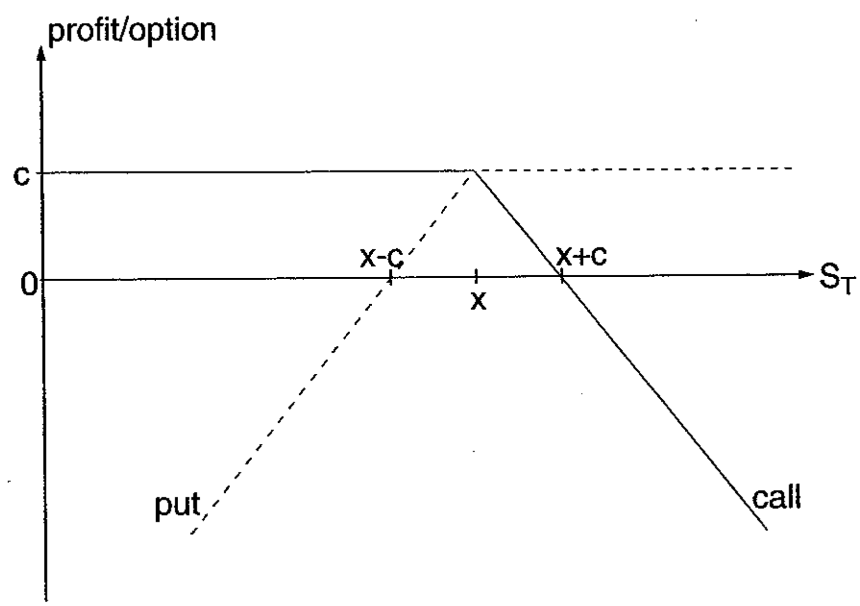

Fig. 2.2. Profit profile of call (*solid line*) and put (*dashed line*) options for the writer of the option (short position)

taken "on credit" from a broker. The hope is, of course, that their quotes will rise in the near future by an appreciable amount. We shall use short selling mainly for theoretical arguments.

Closing out an open position is done by entering a contract with a third party that exactly cancels the effect of the first contract. In the case of publicly traded securities, it can also mean selling (buying) a derivative or security one previously owned (sold short).

2.5 Market Actors

We distinguish three basic types of actors on financial markets.

- *Speculators* take risks to make money. Basically, they bet that markets will make certain moves. Derivatives can give extra leverage to speculation with respect to an investment in the underlying security. Reconsider the example of Sect. 2.3.3, involving 100 call options with $X =$ DM 100 and $S_t =$ DM 98. If indeed, after two months, $S_T =$ DM 115, the profit of DM 1000 was realized with an investment of $100 \times C =$ DM 500, i.e., amounts to a return of 200% in two months. Working with the underlying security, one would realize a profit of $100 \times (S_T - S_t) =$ DM 1700 but on an investment of DM 9,800, i.e., achieve a return of "only" 17.34%. On the other hand, the risk of losses on derivatives is considerably higher than on stocks or bonds (imagine the stock price to stay at $S_T =$ DM 98 at maturity). Moreover, even with simple derivatives, a speculator places a bet not only on the direction of a market move, but also that this move will occur before the maturity of the instruments he used for his investment.
- *Hedgers,* on the other hand, invest into derivatives in order to eliminate risk. This is basically what the company in the example of Sect. 2.3.1 did when entering a forward over 1 million pounds sterling. By this action, all risk associated with changes of the dollar/sterling exchange rate was eliminated. Using a forward contract, on the other hand, the company also eliminated all opportunities of profit from a favorable evolution of the exchange rate during three months to maturity of the forward. As an alternative, it could have considered using options to satisfy its hedging needs. This would have allowed it to profit from a rising dollar but, at the same time, would have required to pay upfront the price of the options. Notice that hedging does not usually increase profits in financial transactions but rather makes them more controllable, i.e., eliminates risk.
- *Arbitrageurs* attempt to make riskless profits by performing simultaneous transactions on two or more markets. This is possible when prices on two different markets become inconsistent. As an example, consider a stock which is quoted on Wall Street at \$172, while the London quote is £100. Assume that the exchange rate is 1.75 \$/£. One can therefore make a riskless profit by simultaneously buying N stocks in New York and selling

the same amount, or go short in N stocks, in London. The profit is \$$3N$. Such arbitrage opportunities cannot last for long. The very action of this arbitrageur will make the price move up in New York and down in London, so that the profit from a subsequent transaction will be significantly lower. With today's computerized trading, arbitrage opportunities of this kind only last very briefly, while triangular arbitrage, involving, e.g., the European, American, and Asian markets, may be possible on time scales of 15 minutes, or so.

Arbitrage is also possible on two national markets, involving, e.g., a futures market and the stock market, or options and stocks. Arbitrage therefore makes different markets mutually consistent. It ensures "market efficiency", which means that all available information is accounted for in the current price of a security, up to inconsistencies smaller than applicable transaction costs.

The absence of arbitrage opportunities is also an important theoretical tool which we will use repeatedly in subsequent chapters. It will allow a consistent calculation of prices of derivatives based on the prices of the underlying securities. Notice, however, that while satisfied in practice on liquid markets in standard circumstances, it is, in the first place, an assumption which should be checked when modeling, e.g., illiquid markets or exceptional situations such as crashes.

2.6 Price Formation at Organized Exchanges

Prices at an exchange are determined by supply and demand. The procedures differ slightly according to whether we consider an auction or continuous trading, and whether we consider a computerized exchange, or traders in a pit.

Throughout this book, we assume a single price for assets, except when stated otherwise explicitly. This is a simplification. For assets traded at an exchange, prices are quoted as bid and ask prices. The bid price is the price at which a trader is willing to buy; the ask price in turn is the price at which he is willing to sell. Depending on the liquidity of the market, the bid–ask spread may be negligible or sizable.

2.6.1 Order Types

Besides the volume of a specific stock, buy and sell orders may contain additional restrictions, the most basic of which we now explain. They allow the investor to specify the particular circumstances under which his or her order must be executed.

A *market order* does not carry additional specifications. The asset is bought or sold at the market price, and is executed once a matching order arrives. However, market prices may move in the time between the decision of the investor and the order execution at the exchange. A market order does not contain any protection against price movements, and therefore is also called an unlimited order.

Limit orders are executed only when the market price is above or below a certain threshold set by the investor. For a buy (sell) order to limit S_L, the order is executed only when the market price is such that the order can be excecuted at $S \leq S_L$ ($S \geq S_L$). Otherwise, the order is kept in the order book of the exchange until such an opportunity arises, or until expiry. A sell order with limit S_L guarantees the investor a minimum price S_L in the sale of his assets. A limited buy order, vice versa, guarantees a maximal price for the purchase of the assets.

Stop orders are unlimited orders triggered by the market price reaching a predetermined threshold. A stop-loss (stop-buy) order issues an unlimited sell (buy) order to the exchange once the asset price falls below S_L. Stop orders are used as a protection against unwanted losses (when owning a stock, say), or against unexpected rises (when planning to buy stock). Notice, however, that there is no guarantee that the price at which the order is executed is close to the limit S_L set, a fact to be considered when seeking protection against crashes, cf. Chap. 5.

2.6.2 Price Formation by Auction

In an auction, every trader gives buy and sell orders with a specific volume and limit (market orders are taken to have limit zero for sell and infinity for buy orders). The orders are now ordered in descending (ascending) order of the limits for the buy (sell) orders, i.e., $S_{L,1} > S_{L,2} > \ldots > S_{L,m}$ for buy orders, and $S_{L,1} < S_{L,2} < \ldots < S_{L,n}$ for the sell orders. Let $V_b(S_i)$ and $V_s(S_i)$ be the volumes of the buy and sell orders, respectively, at limit S_i. We now form the cumulative demand and offer functions $D(S_k)$ and $O(S_k)$ as

$$D(S_k) = \sum_{i=1}^{k} V_b(S_i) , \quad k = 1, \ldots, m \tag{2.1}$$

$$O(S_k) = \sum_{i=1}^{k} V_s(S_i) , \quad k = 1, \ldots, n . \tag{2.2}$$

The market price of the asset determined in the auction then is that price which allows one to execute a maximal volume of orders with a minimal residual of unexecuted order volume, consistent with the order limits. If the order volumes do not match precisely, orders may be partly executed.

We illustrate this by an example. Table 2.1 gives part of a hypothetical order book at a stock exchange. One starts executing orders from top to bottom on both sides, until prices or cumulative order volumes become inconsistent. In the first two lines, the buy limit is above the sell limit so

Table 2.1. Order book at a stock exchange containing limit orders only. Orders with volume in boldface are executed at a price of 162. With a total transaction volume of 900, the buy order of 300 shares at 162 is executed only partly

Buy			Sell		
Volume	Limit	Cumulative	Volume	Limit	Cumulative
200	164	200	**400**	160	400
500	163	700	**400**	161	800
300	162	1000	**100**	162	900
200	161	1200	300	163	1200
300	160	1500	300	164	1500
$V_b(S_i)$	S_i	$D(S_i)$	$V_s(S_i)$	S_i	$O(S_i)$

that the orders can be executed at any price $163 \geq S \geq 161$. In the third line, only 900 (cumulated) shares are available up to 162 compared to a cumulative demand of 1000. A transaction is possible at 162, and 162 is fixed as the transaction price for the stock because it generates the maximal volume of executed orders. However, while the sell order of 100 stocks at 162 is executed completely, the buy order of 300 stocks is exectued only partly (volume 200). Depending on possible additional instructions, the remainder of the order (100 stocks) is either cancelled or kept in the order book.

The problem can also be solved graphically. The cumulative offer and demand functions are plotted against the order limits in Fig. 2.3. The solid line is the demand, and the dash-dotted line is the offer function. They intersect at a price of 162.20. The auction price is fixed as that neighboring allowed price (we restricted ourselves to integers) where the order volume on the lower of both curves is maximal. This happens at 162 with a cumulative volume of 900 (compare to a volume of 750 at 163).

The dotted line in Fig. 2.3 shows the cumulative buy functions if an additional market order for 300 stocks is entered into the order book. The demand function of the previous example is shifted upward by 300 stocks, and the new price is 163. All buy orders with limit 163 and above are executed completely, including the market order (total volume 1000). Sell orders with limit below 163 are executed completely (total volume 900), and the order with limit 163 can sell only 100 shares, instead of 300. The corresponding order book is shown in Table 2.2.

2.6.3 Continuous Trading: The XETRA Computer Trading System

Elaborate rules for price formation and priority of orders are necessary in the computerized trading systems such as the XETRA (EXchange Electronic

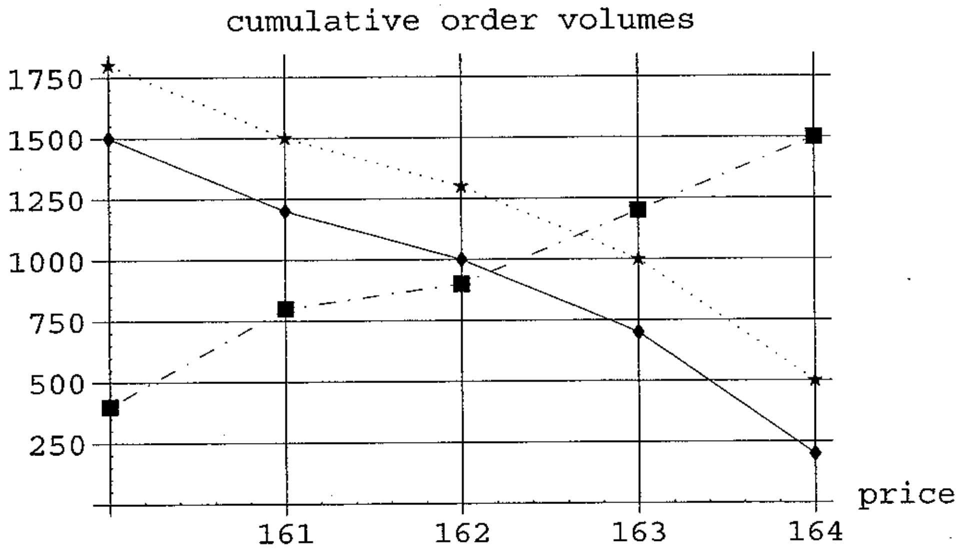

Fig. 2.3. Offer and demand functions in an auction at a stock exchange. The solid line is the demand function with limit orders only, and the dotted line includes a market order of 300 shares. The dash-dotted line is the offer function

Table 2.2. Order book including a market buy order. Orders with volume in boldface are executed at a price of 163. With a total transaction volume of 1000, the sell order of 300 shares at 163 is executed only partly

Buy			Sell		
Volume	Limit	Cumulative	Volume	Limit	Cumulative
300	market	300	**400**	160	400
200	164	500	**400**	161	800
500	163	1000	**100**	162	900
300	162	1300	**300**	163	1200
200	161	1500	300	164	1500
300	160	1800			
$V_b(S_i)$	S_i	$D(S_i)$	$V_s(S_i)$	S_i	$O(S_i)$

Trading) system introduced by the German Stock Exchange in late 1997 [27]. Here, we just describe the basic principles.

Trading takes place in three main phases. In the pretrading phase, the operators can enter, change, or delete orders in the order book. The traders cannot access any information on the order book.

The matching (i.e., continuous trading) phase starts with an opening auction. The purpose is to avoid a crossed order book (e.g., sell orders with limits significantly below those of buy orders). Here, the order book is partly closed, but indicative auction prices or best limits entered, are displayed continuously. Stocks are called to auction randomly with all orders left over from the preceding day, entered in the pretrading phase, or entered during the auction until it is stopped randomly. The price is determined according to the rules of the preceding section. It is clear, especially from Fig. 2.3, that in this way a crossed order book is avoided.

In the matching phase, the order book is open and displays both the limits and the cumulative order volumes. Any newly incoming market or limit order is checked immediately against the opposite side of the order book, for execution. This is done according to a set of at least 21 rules. More complete information is available in the documentation provided by, e.g., Deutsche Börse AG [27]. Here, we just mention a few of them, for illustration. (i) If a market or a limit order comes in and faces a set of limit orders in the order book, the price will be the highest limit for a sell order, resp. the lowest limit for a buy order. (ii) If a market buy order meets a market sell order, the order with the smaller volume is executed completely, while the one with the larger volume is executed partly, at the reference price. The reference price remains unchanged. (iii) If a limit sell order meets a market buy order, and the currently quoted price is higher than the lowest sell limit, the trade is concluded at the currently quoted price. If, on the other hand, the quoted price is below the lowest sell limit, the trade is done at the lowest sell limit. (iv) If trades are possible at several different limits with maximal trading volume and minimal residual, other rules will determine the limit depending on the side of the order book, on which the residuals are located.

If the volatility becomes too high, i.e., stock prices leave a predetermined price corridor, matching is interrupted. At a later time, another auction is held, and continuous trading may resume. Finally, the matching phase is terminated by a closing auction, followed by a post-trading period. As in pretrading, the order book is closed but operators can modify their own orders to prepare next day's trading.

On a trading floor where human traders operate, such complicated rules are not necessary. Orders are announced with price and volume. If no matching order is manifested, traders can change the price until they can conclude a trade, or until their limit is reached.