CS 286r Homework 2 – due Wed, Oct 1 in class

September 24, 2008

1. (Fudenberg and Tirole 3.3 – 20 pts) Player 1, "the government," wishes to influence the choice of player 2. Player 2 chooses an action $a_2 \in A_2 = \{0, 1\}$ and receives a transfer $t = T = \{0, 1\}$ from the government, which observes a_2. Player 2's objective is to maximize the expected value of his transfer, minus the cost of his action, which is 0 for $a_2 = 0$ and $\frac{1}{2}$ for $a_2 = 1$. Player 1's objective is to minimize the sum $2(a_2 - 1)^2 + t$. Before player 2 chooses his action, the government can announce a transfer rule $t(a_2)$. *(a)* Draw the extensive form for the case where the government's announcement is not binding and has no effect on payoffs. *(b)* Draw the extensive form for the case where the government is constrained to implement the transfer rule it announced. *(c)* Give the strategic forms to both games. *(d)* Characterize the subgame-perfect equilibria of the two games.

2. (Strategic Voting – 20 pts) Three legislators $(1, 2, 3)$ are voting on whether to give themselves a pay raise. The raise is worth b but each legislator who votes for the raise incurs a cost of voter resentment equal to $c < b$. The outcome is decided by majority vote. *(a)* Draw a game tree for the problem, assuming the legislators vote sequentially and publicly (i.e., 2 sees 1's vote and 3 sees both 1 and 2's votes). *(b)* Find a Nash equilibrium for this game using backward induction. Show that it is best to go first. *(c)* Show that there are two Nash equilibrium in which 3 votes "no." Clearly specify these Nash equilibria. Why can't these equilibria be found by backward induction? Is there something strange about these equilibria?

3. (The Centipede game – 15 pts) The Centipede game is played by two players, A and B, as follows: Both players start out with \$2 each, and they alternate rounds. On the first round, A can defect (D) by stealing \$2 from B, and the game is over. Otherwise, A cooperates (C) by not stealing, and mother nature gives A \$1 for being nice. Then B can defect and steal \$2 from A or cooperate and get nature's \$1 gift. This continues until one or the other defects, or each player has \$100. Show that the only Nash equilibrium compatible with backward induction has A defecting on the first round. How would you play this game?

4. (Chess – 20 points) Harold Kuhn in 1953 proved that all finite games of perfect information have a Nash equilibrium in pure strategies. Also, there is a theorem stating that in all two-person zero-sum games, all Nash equilibria give the same expected payoffs to both players—so players are indifferent between equilibria. What does this tell you about the game of chess? Do you think chess is a trivial game, based on this observation? (*Hint: Show, based on Kuhn's observation and the zero-sum theorem, that there is a strategy for "white" that guarantees him a victory, no matter what "black" does, or a similar strategy for "black," or a strategy for either player that guarantees him/her a tie. Also keep in mind that repeating a series of moves in chess for more than three times forces a draw.*)

5. (Stackelberg leadership – 20 pts) Look at the following game matrix. A Stackelberg leader is a player who can precommit to following a certain action, so other players consider him as effectively "going first," and they predicate their actions on the preferred choices of the leader. Stackelberg leadership is a form of power due to this capacity to precommit. Write an extensive form game for which the normal form is as below and find all Nash equilibria. Then, assume that *row* chooses first and *column*

follows, having seen *row*'s choice. Write the new extensive form game and find the Nash equilibria. Do you think that being a Stackelberg leader is always to your benefit? Why / why not?

	t_1	t_2
s_1	0,2	3,0
s_2	2,1	1,3

6. (The Rotten Kid – 25 points) A certain family consists of a mother and a son, with increasing, concave utility functions $u(y)$ for the mother and $v(z)$ for the son. The son can affect both his income and that of the mother by choosing a level of familial work commitment a, so $y = y(a)$ and $z = z(a)$. The mother, however, feels a degree of altruism $\beta > 0$ toward the son, so given y and z, she transfers an amount t to the son to maximize the objective function

$$u(y - t) + \beta v(z + t)$$

The son, however, is perfectly selfish ("rotten") and chooses a to maximize his own utility $v(z(a) + t)$ However, he knows that his mother's transfer t depends on y and z, and hence on a. Use backward induction to show that the son chooses a to maximize total family income $y(a) + z(a)$. *Hint: Assume that "maximization" is equivalent to a simple first-order condition.*

Total points: 100

Student submissions will be graded according to the correctness, sophistication and clarity of their answers.

Late policy: Each student is allowed 2 late days for the entire semester. A "day" is defined as a 24-hour period after the submission deadline, i.e., 1 PM. No grades will be deducted within the alloted late days. Grade penalties will be imposed on all submissions exceeding this limit and up to 3 days. Any submissions more than 3 days late will not be accepted.