CS 286r Homework 3 - Due Wed, Oct 8, in class

1. (Selten's horse - 35 points) The game in Figure 1 is known as Selten's horse from the name of its author. The game starts with player 1 (upper left corner). Let $\sigma_i(a_i)$ represent the probability that player i players strategy a_i when he's at his information set I_i. ($I_1 = \{\phi\}$, $I_2 = \{\{C\}\}$, and $I_3 = \{\{D\}, \{(C, d)\}\}$.) There are two types of Nash equilibria:

1. $\sigma_1(D) = 1$, $\frac{1}{3} \leq \sigma_2(c) \leq 1$, and $\sigma_3(L) = 1$;
2. $\sigma_1(C) = 1$, $\sigma_2(c) = 1$, and $\frac{3}{4} \leq \sigma_3(R) \leq 1$.

Verify that (a) any Nash Equilibrium of the first type is not a sequential equilibrium; (b) any Nash Equilibrium of the second type is a sequential equilibrium at which player 3's belief assigns probability $\frac{1}{3}$ to history (D) in his information set. Remember that a sequential equilibrium is a strategy-belief pair. In your answer, please clearly specify the beliefs of players at their information sets. (*Hint for (b): To check for consistency, consider the sequence* (σ^ϵ) *of profiles in which* $\sigma_1^\epsilon(C) = 1 - \epsilon$, $\sigma_2^\epsilon(d) = \frac{2\epsilon}{1-\epsilon}$, and $\sigma_3^\epsilon(R) = \sigma_3(R) - \epsilon$.)

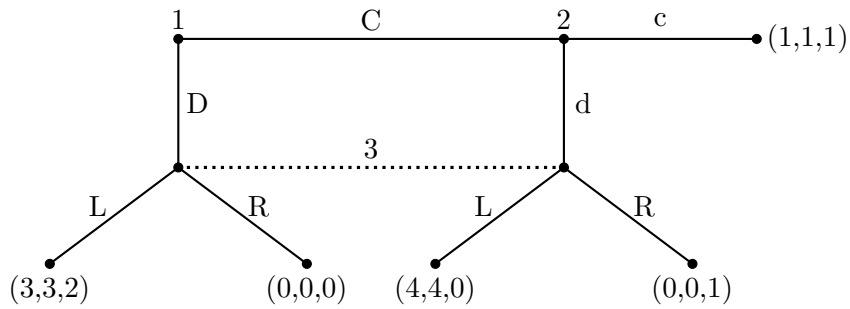

Figure 1: Selten's Horse

2. (MWG 8.E.2 - 35 points) Consider a first-price sealed-bid auction of an object with two bidders. Each bidder has a valuation distributed uniformly

and independently on $[0, \overline{v}]$ for each bidder. (a) Derive a symmetric (pure strategy) Bayesian Nash equilibrium of this auction. *Hint: Look for an equilibrium in which bidder i's bid is a linear function of his valuation.* (b) What if there are I bidders? What happens to each bidder's equilibrium bid function $s(v_i)$ as I increases?

3. (MWG 13.C.5 - 35 points) Assume a single firm and a single consumer. The firm's product may be either high or low quality and is of high quality with probability λ. The consumer cannot observe quality before purchase and is risk neutral. The consumer's valuation of a high-quality product is v_H; her valuation of a low-quality product is v_L. The costs of production for high (H) and low (L) quality are c_H and c_L, respectively. The consumer desires at most one unit of the product. Finally, the firm's price is regulated and is set at p. Assume that $v_H > p > v_L > c_H > c_L$. (a) Given the level of p, under what conditions will the consumer buy the product. (b) Suppose that before the consumer decides whether to buy, the firm (which knows its type) can advertise. Advertising conveys no information directly, but consumers can observe the total amount of money that the firm is spending on advertising, denoted by A. Can there be a separating perfect Bayesian equilibrium, that is, an equilibrium in which the consumer rationally expects firms with different quality levels to pick different levels of advertising?

Total points: 105

Student submissions will be graded according to the correctness, sophistication and clarity of their answers.

Late policy: Each student is allowed 2 late days for the entire semester. A "day" is defined as a 24-hour period after the submission deadline, i.e., 1 PM. No grades will be deducted within the allotted late days. Grade penalties will be imposed on all submissions exceeding this limit and up to 3 days. Any submissions more than 3 days late will not be accepted.