Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords

by
Benjamin Edelman, Michael Ostrovsky, and Michael Schwarz
(EOS)

presented by
Scott Brinker and Jerry Kung

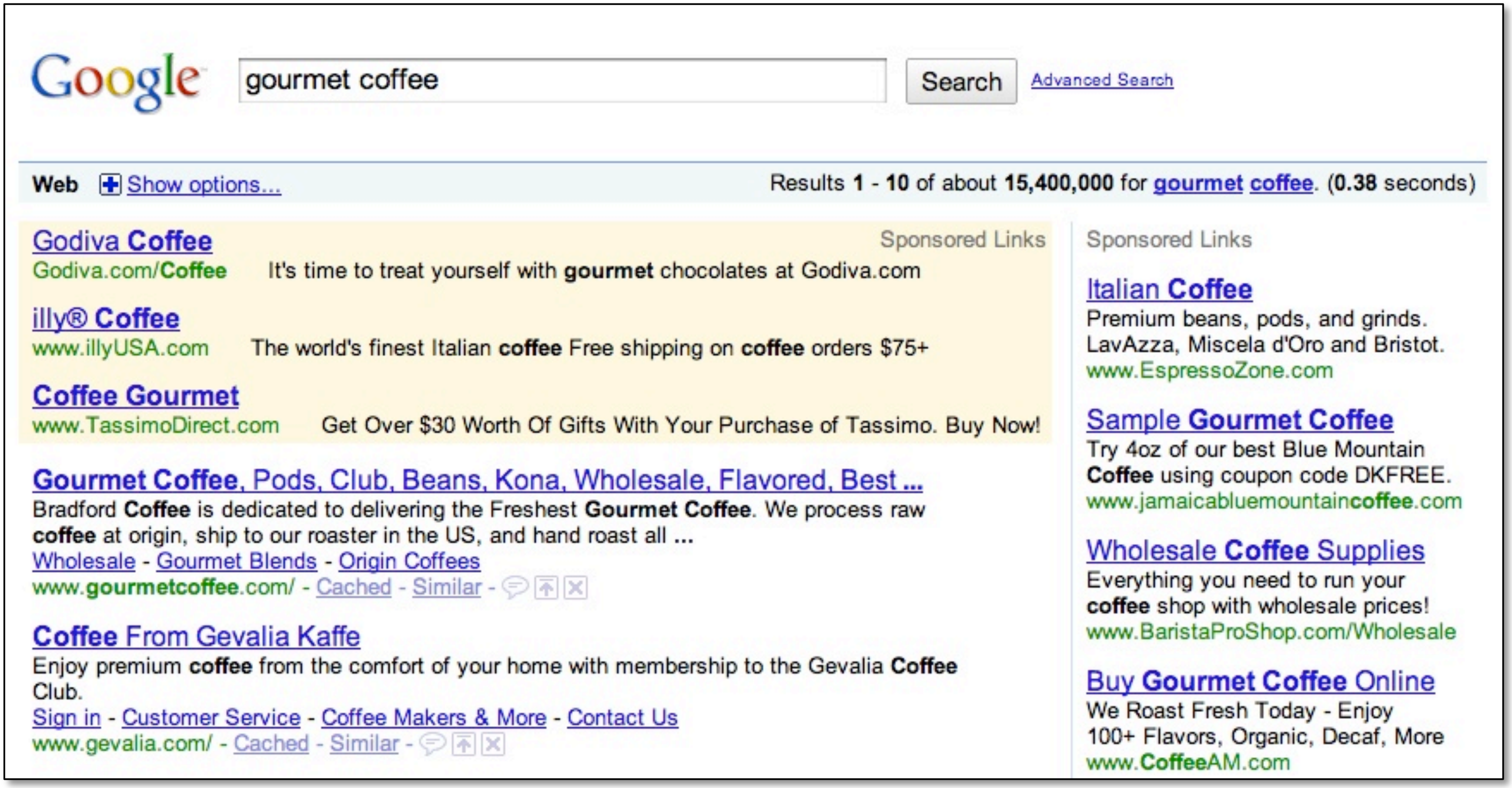
Motivation

> $20 billion

Motivation

- Free for users:
  - Google Search
  - Gmail
  - Google Maps
  - Google Scholar
  - Google Groups
  - Google Images
  - Google Books

Sponsored search auctions



Notable features of the market

- Every search on a keyword is a new auction
    - Well, in our highly stylized theory it is
- Submit single bids that can be changed at any time
- Pay-per-click (PPC) as the “unit” being purchased
- Advertisements ranked according to bid *
- Assumption: click-through rate depends only on position (and perhaps quality of advertiser)

* And “quality score” with Google
** Externalities imposed by relative placement of other ads; see Immorlica (2009)

Evolution of market institutions

- Early Internet advertising: “impressions” (1994)
  - CPM (cost-per-thousand)
- Generalized first-price auctions by Overture (1997)
  - Shift to PPC model
  - But GFP encouraged frequent bid changes
  - No pure strategy equilibrium

A problem with first-price auctions

Example: 3 advertisers with click values of $10, $4, and $2

2 ad slots receiving 200 and 100 clicks-per-hour

Inspires an infinite loop bidding war…

Advertiser 1:	$2.01	$2.03	$2.03	…	$2.99	$2.99	$2.02	$2.02
Advertiser 2:	$2.02	$2.02	$2.04	…	$2.98	$2.01	$2.01	$2.03

The cycle resets when the profit on slot 2 for advertiser 2 — ~100 x ($4 − $2) — is around the same as the profit from slot 1 — ~200 x ($4 − $3).

Evolution of market institutions

- Early Internet advertising: "impressions" (1994)
  - CPM (cost-per-thousand)
- Generalized first-price auctions by Overture (1997)
  - Shift to PPC model
  - But GFP encouraged frequent bid changes
  - No pure strategy equilibrium
- Generalized second-price auctions by Google (2002)
  - Yahoo!, Microsoft both adopted this model too
  - Looks kind of like VCG at first glance…

GSP vs. VCG — bid vs. externality

Example: 3 advertisers with click values of $10, $4, and $2
2 ad slots receiving 200 and 100 clicks-per-hour
If advertisers were to bid truthfully…

GSP

- Advertiser 1 gets slot 1
  - Payment: 200 x $4 = $800
  - Payoff: $1,200
- Advertiser 2 gets slot 2
  - Payment: 100 x $2 = $200
  - Payoff: $200

Total revenue: **$1,000**

VCG

- Advertiser 1 gets slot 1
  - Payment: (100 x $4) + (100 x $2) = $600
  - Payoff: $1,400
- Advertiser 2 gets slot 2
  - Payment: 100 x $2 = $200
  - Payoff: $200

Total revenue: **$800**

The Rules of GSP

N: slots for ads

K: bidders (advertisers)

α_i: clicks per period received in slot i

s_k: value per click to advertiser k

b_k: advertiser k's bid

$b^{(j)}$ and $g(j)$: bid and identity of the j-th highest advertiser

Allocation: $g(1)$ is highest bidder, $g(2)$ is 2nd highest, etc.

Payment: $g(i)$ pays $p^{(i)} = \alpha_i b^{(i+1)}$ for $i \in \{1, \ldots, \min\{N, K\}\}$ * **

Payoff: $g(i)$ receives payoff of $\alpha_i (s_{g(i)} - b^{(i+1)})$

* In practice, advertiser i is charged ($b^{(i+1)}$ + $0.01) per click

** If ($N \geq K$) then $p^{(K)} = 0$ in theory; in practice, search engines charge a reserve price

GSP compared to VCG

- Allocation rule remains the same
- Payment under GSP:

$$p^{(i)} = \alpha_i b^{(i+1)}$$

- Payment under VCG:

$$p^{V,(i)} = (\alpha_i - \alpha_{(i+1)}) b^{(i+1)} + p^{V,(i+1)}$$

- Payment of last advertiser allocated a spot is the same
- If all advertisers bid same amount under both mechanisms:

$$p^{(i)} \geq p^{V,(i)}$$

Truth-telling: a dominant strategy?

- Under **VCG**, yes
- Under **GSP**, no

Example:

3 advertisers with click values \$10, \$4, and \$2

2 ad slots receiving 200 and 199 clicks-per-hour

If all advertisers bid truthfully, advertiser 1's payoff:

(\$10 − \$4) x 200 = \$1,200

If advertiser 1 shades his bid to \$3, his payoff is:

(\$10 - \$2) x 199 = \$1,592

Why not change to VCG?

- VCG may be hard to explain to ad buyers
- Switching to VCG has enormous transition costs
  - Lower revenue for the same bids ($p^{(i)} \geq p^{V,(i)}$)
  - Ad buyers may be slow to stop shading bids
- Importance of strategy-proofness?
- Under GSP, payment is still independent of bid, but may not get outcome that maximizes utility so not DSIC

More assumptions

- All values are common knowledge
- Stable bids are best responses to each other
- Bids form an equilibrium in simultaneous-move, one-shot complete-information game
- Simple strategies to increase payoff?

Locally envy-free Nash equilibria

- Locally envy-free equilibrium: no player can improve her payoff by exchanging positions with the bid above
- Locally: only compare to immediately preceding position

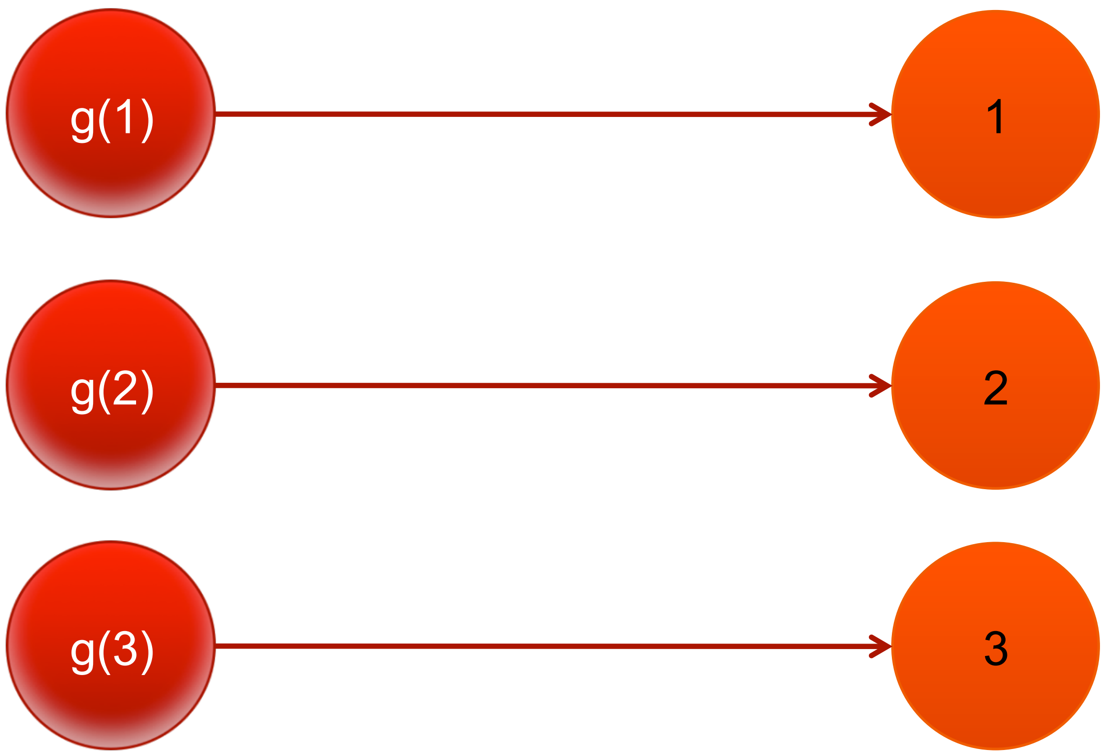
For any $i \leq \min[N + 1, K]$:

$$\alpha_i s_{g(i)} - p^{(i)} \geq \alpha_{i\text{-}1} s_{g(i)} - p^{(i\text{-}1)}$$

- Motivated by a notion of spitefulness
  - Not explicit in payoff function

Connection to matching

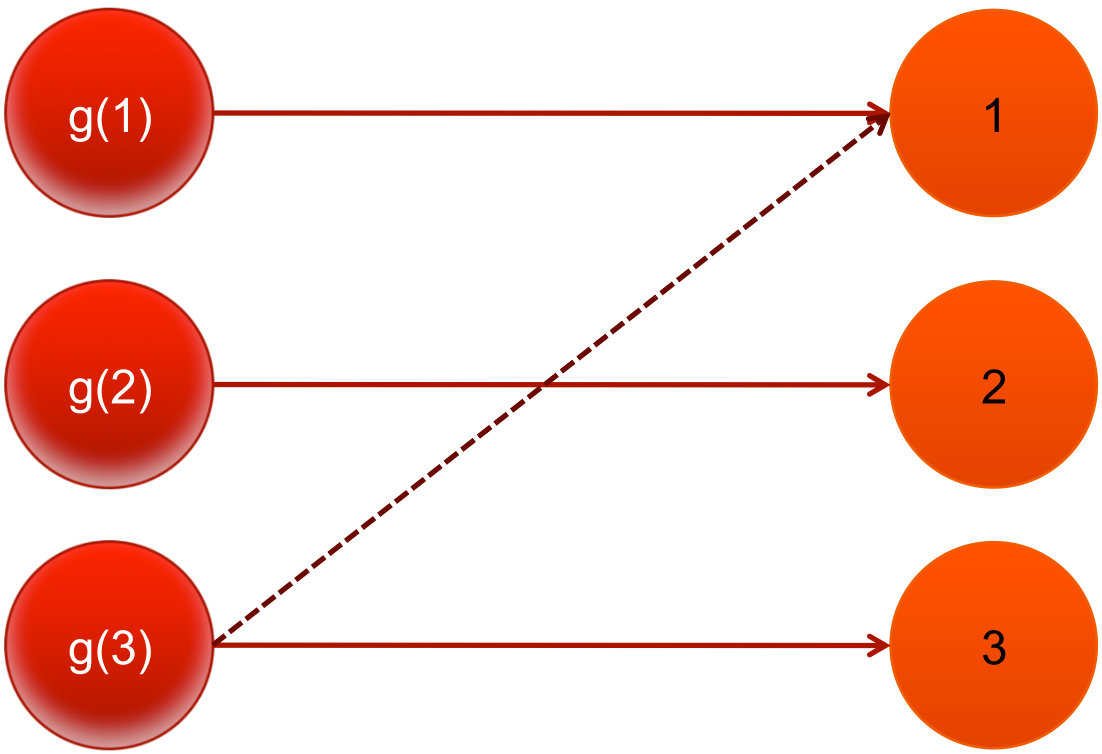
Set of **locally** envy-free equilibria maps to stable two-sided matching:



Connection to matching

Set of **locally** envy-free equilibria maps to stable two-sided matching

But can there still be a blocking pair?



One locally envy-free equilibrium

- Strategy profile *B**, locally envy-free equilibrium
- Position and payment equal to VCG dominant strategy
- The best locally envy-free equilibrium for advertisers

Same example: 3 advertisers with click values of $10, $4, and $2
2 ad slots receiving 200 and 100 clicks-per-hour

$b_1^* = \$10$	$p_1 = \$600$	$\alpha_1 s_1 = \$2{,}000$	payoff = $1,200
$b_2^* = \$600/200 = \$3$	$p_2 = \$200$	$\alpha_2 s_2 = \$400$	payoff = $200
$b_3^* = \$200/100 = \$2$	$p_3 = \$0$	$\alpha_3 s_3 = \$0$	payoff = $0

Note that advertisers 2 and 3 are indifferent between remaining in their existing positions and swapping with the advertiser one position above.

Advertiser-specific factors

- Advertiser CTR factor β_k independent of position
- Different impact on equilibria for Google vs. Yahoo! *

Yahoo!: $\alpha_i \beta_{g(i)} (s_{g(i)} - b^{(i+1)}) \geq \alpha_j \beta_{g(i)} (s_{g(i)} - b^{(j+1)})$

Divide both sides by $\beta_{g(i)}$, no impact on equilibria

Google: γ_k "quality score" (mix of β_k and other factors)

k's rank = $\gamma_k b_k$ determines ordering

$\alpha_i \beta_{g(i)} (s_{g(i)} - \gamma_{g(i+1)} b_{(i+1)}/\gamma_{g(i)}) \geq \alpha_j \beta_{g(i)} (s_{g(i)} - \gamma_{g(j+1)} b_{(i+1)}/\gamma_{g(i)})$

* Yahoo! and Microsoft/Bing now use their own quality score factors too

Interesting questions

- Can advertisers “learn” each other’s values?
- Is there opportunity for collusion?
    - What about third-party agencies?
- Are the simplifying click-model assumptions too simple?
- Are there key strategic dimensions that are missing?
    - Offer, creative, “broad match,” etc.
    - Are advertisers really “risk neutral?”
- If Google charges for API usage, would GFP be better?

Thank you!