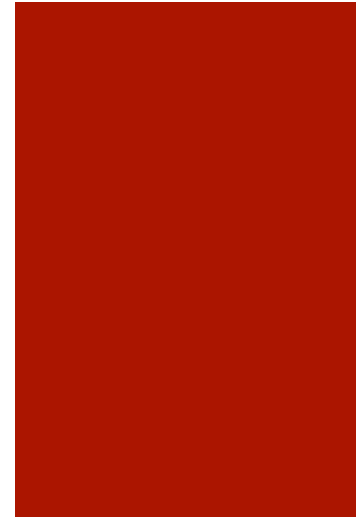


# Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords

by  
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presented by  
Scott Brinker and Jerry Kung



**Motivation**



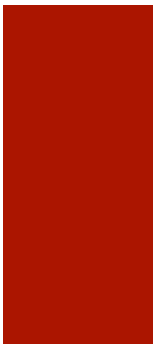
**> \$20 billion**

# Motivation

- Free for users:
  - Google Search
  - Gmail
  - Google Maps
  - Google Scholar
  - Google Groups
  - Google Images
  - Google Books



# Sponsored search auctions



Google  Search [Advanced Search](#)

Web [+ Show options...](#) Results 1 - 10 of about 15,400,000 for [gourmet coffee](#). (0.38 seconds)

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# Notable features of the market



- Every search on a keyword is a new auction
  - Well, in our highly stylized theory it is
- Submit single bids that can be changed at any time
- Pay-per-click (PPC) as the “unit” being purchased
- Advertisements ranked according to bid \*
- Assumption: click-through rate depends only on position (and perhaps quality of advertiser)

\* And “quality score” with Google

\*\* Externalities imposed by relative placement of other ads; see Immorlica (2009)

# Evolution of market institutions



- Early Internet advertising: “impressions” (1994)
  - CPM (cost-per-thousand)
- Generalized first-price auctions by Overture (1997)
  - Shift to PPC model
  - But GFP encouraged frequent bid changes
  - No pure strategy equilibrium

# A problem with first-price auctions

Example: 3 advertisers with click values of \$10, \$4, and \$2  
2 ad slots receiving 200 and 100 clicks-per-hour  
Inspires an infinite loop bidding war...

Advertiser 1:	\$2.01	\$2.03	\$2.03	...	\$2.99	\$2.99	\$2.02	\$2.02
Advertiser 2:	\$2.02	\$2.02	\$2.04	...	\$2.98	\$2.01	\$2.01	\$2.03

The cycle resets when the profit on slot 2 for advertiser 2 —  
~100 x (\$4 - \$2) — is around the same as the profit from slot 1  
— ~200 x (\$4 - \$3).

# Evolution of market institutions



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  - Shift to PPC model
  - But GFP encouraged frequent bid changes
  - No pure strategy equilibrium
- Generalized second-price auctions by Google (2002)
  - Yahoo!, Microsoft both adopted this model too
  - Looks kind of like VCG at first glance...



# GSP vs. VCG — bid vs. externality

Example: 3 advertisers with click values of \$10, \$4, and \$2  
2 ad slots receiving 200 and 100 clicks-per-hour  
If advertisers were to bid truthfully...

## GSP

- Advertiser 1 gets slot 1
  - Payment:  $200 \times \$4 = \$800$
  - Payoff: \$1,200
- Advertiser 2 gets slot 2
  - Payment:  $100 \times \$2 = \$200$
  - Payoff: \$200

Total revenue: **\$1,000**

## VCG

- Advertiser 1 gets slot 1
  - Payment:  $(100 \times \$4) + (100 \times \$2) = \$600$
  - Payoff: \$1,400
- Advertiser 2 gets slot 2
  - Payment:  $100 \times \$2 = \$200$
  - Payoff: \$200

Total revenue: **\$800**

# The Rules of GSP

$N$ : slots for ads

$K$ : bidders (advertisers)

$\alpha_i$ : clicks per period received in slot  $i$

$s_k$ : value per click to advertiser  $k$

$b_k$ : advertiser  $k$ 's bid

$b^{(j)}$  and  $g(j)$ : bid and identity of the  $j$ -th highest advertiser

**Allocation:**  $g(1)$  is highest bidder,  $g(2)$  is 2<sup>nd</sup> highest, etc.

**Payment:**  $g(i)$  pays  $p^{(i)} = \alpha_i b^{(i+1)}$  for  $i \in \{1, \dots, \min\{N, K\}\}$  \* \*\*

**Payoff:**  $g(i)$  receives payoff of  $\alpha_i (s_{g(i)} - b^{(i+1)})$

\* In practice, advertiser  $i$  is charged  $(b^{(i+1)} + \$0.01)$  per click

\*\* If  $(N \geq K)$  then  $p^{(K)} = 0$  in theory; in practice, search engines charge a reserve price

# GSP compared to VCG

- Allocation rule remains the same
- Payment under GSP:

$$p^{(i)} = \alpha_j b^{(i+1)}$$

- Payment under VCG:

$$p^{V,(i)} = (\alpha_j - \alpha_{(j+1)})b^{(i+1)} + p^{V,(i+1)}$$

- Payment of last advertiser allocated a spot is the same
- If all advertisers bid same amount under both mechanisms:

$$p^{(i)} \geq p^{V,(i)}$$

# Truth-telling: a dominant strategy?



- Under **VCG**, yes
- Under **GSP**, no

Example:

3 advertisers with click values \$10, \$4, and \$2

2 ad slots receiving 200 and 199 clicks-per-hour

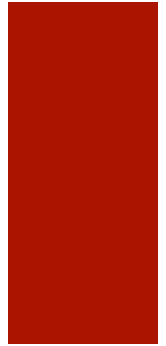
If all advertisers bid truthfully, advertiser 1's payoff:

$$(\$10 - \$4) \times 200 = \$1,200$$

If advertiser 1 shades his bid to \$3, his payoff is:

$$(\$10 - \$2) \times 199 = \$1,592$$

# Why not change to VCG?



- VCG may be hard to explain to ad buyers
- Switching to VCG has enormous transition costs
  - Lower revenue for the same bids ( $p^{(i)} \geq p^{V,(i)}$ )
  - Ad buyers may be slow to stop shading bids
- Importance of strategy-proofness?
- Under GSP, payment is still independent of bid, but may not get outcome that maximizes utility so not DSIC

# More assumptions

- All values are common knowledge
- Stable bids are best responses to each other
- Bids form an equilibrium in simultaneous-move, one-shot complete-information game
- Simple strategies to increase payoff?



# Locally envy-free Nash equilibria

- Locally envy-free equilibrium: no player can improve her payoff by exchanging positions with the bid above
- Locally: only compare to immediately preceding position

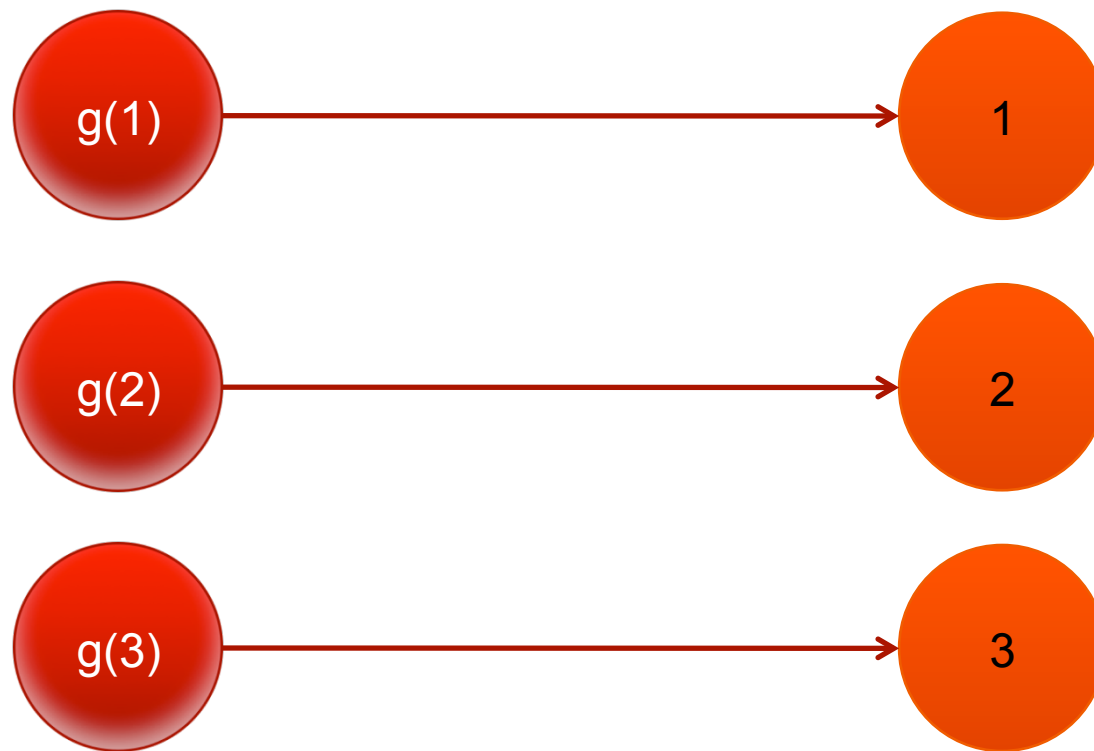
For any  $i \leq \min[N + 1, K]$ :

$$\alpha_i s_{g(i)} - p^{(i)} \geq \alpha_{i-1} s_{g(i)} - p^{(i-1)}$$

- Motivated by a notion of spitefulness
  - Not explicit in payoff function

# Connection to matching

Set of **locally** envy-free equilibria maps to stable two-sided matching:

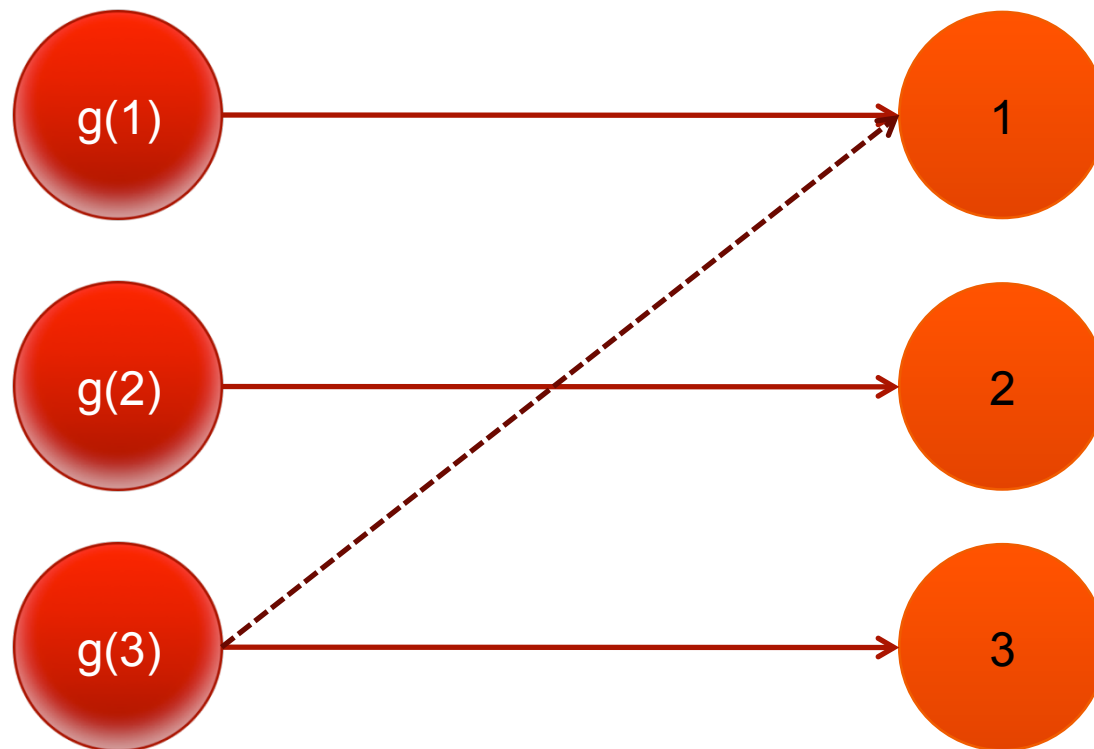




# Connection to matching

Set of **locally** envy-free equilibria maps to stable two-sided matching

But can there still be a blocking pair?



# One locally envy-free equilibrium

- Strategy profile  $B^*$ , locally envy-free equilibrium
- Position and payment equal to VCG dominant strategy
- The best locally envy-free equilibrium for advertisers

Same example: 3 advertisers with click values of \$10, \$4, and \$2  
2 ad slots receiving 200 and 100 clicks-per-hour

$b_1^* = \$10$	$p_1 = \$600$	$\alpha_1 s_1 = \$2,000$	payoff = \$1,200
$b_2^* = \$600/200 = \$3$	$p_2 = \$200$	$\alpha_2 s_2 = \$400$	payoff = \$200
$b_3^* = \$200/100 = \$2$	$p_3 = \$0$	$\alpha_3 s_3 = \$0$	payoff = \$0

Note that advertisers 2 and 3 are indifferent between remaining in their existing positions and swapping with the advertiser one position above.

# Advertiser-specific factors

- Advertiser CTR factor  $\beta_k$  independent of position
- Different impact on equilibria for Google vs. Yahoo! \*

Yahoo!: 
$$\alpha_i \beta_{g(i)} (s_{g(i)} - b^{(i+1)}) \geq \alpha_j \beta_{g(i)} (s_{g(i)} - b^{(j+1)})$$

Divide both sides by  $\beta_{g(i)}$ , no impact on equilibria

Google:  $\gamma_k$  “quality score” (mix of  $\beta_k$  and other factors)

$k$ 's rank =  $\gamma_k b_k$  determines ordering

$$\alpha_i \beta_{g(i)} (s_{g(i)} - \gamma_{g(i+1)} b_{(i+1)} / \gamma_{g(i)}) \geq \alpha_j \beta_{g(i)} (s_{g(i)} - \gamma_{g(j+1)} b_{(j+1)} / \gamma_{g(i)})$$

\* Yahoo! and Microsoft/Bing now use their own quality score factors too

# Interesting questions

- Can advertisers “learn” each other’s values?
- Is there opportunity for collusion?
  - What about third-party agencies?
- Are the simplifying click-model assumptions too simple?
- Are there key strategic dimensions that are missing?
  - Offer, creative, “broad match,” etc.
  - Are advertisers really “risk neutral?”
- If Google charges for API usage, would GFP be better?





**Thank you!**