Optimizing Scrip Systems

or: mo’ money, mo’ problems? or: paying with Kash

paper by: Kash, Friedman, Halpern, 2009
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What good is money?

why might we want to introduce a scrip system?

• dictate initial distribution / control policy

• “double coincidence of wants” - ease transactions

• control flows within a system
proof? of an optimum quantity of money?

• 150 couples institute coupon system
  precautionary savings ensue
  more coupons injected
  market crashes

• with strong assumptions, (Hens et al) experiments suggest optimum quantity

• too many hoarders? Or, not a closed model?
Major results from Kash et al

• Theoretical:
  1. monotonicity of best reply: if all others play threshold, you should play threshold
  2. concentration phenomenon: an entropy-maximizing distribution as $n \to \infty$
     $(n$ is agents of a given type)

• Simulation:
  1. collusion doesn’t hurt
  2. hoarders and altruists can cause crashes
The model (Complete)

- There are $N$ agents
- A type set $T$
- Frequencies of types $f_t$
- A type $t = (\alpha_t, \beta_t, \gamma_t, \delta_t, \rho_t, \chi_t)$
- A game is described by $G(T, f, h, n,m)$ where
  - $h =$ base number
  - $n =$ number of replicas for each base and type
  - $m =$ average money $M/N$
Decoding types

\[ t = (\alpha_t, \beta_t, \gamma_t, \delta_t, \rho_t, \chi_t) \]

- Cost of providing a service
- Probability of being capable to fulfill a request
- Utility discount
- Utility for having a request satisfied
- Request rate
- Relative probability of being selected when volunteered
The Model (Simplified)

• Just consider only one type:
  – *N* agents
  – Randomly choose agent *P* to *request* service
  – Probability of being *able* to satisfy request, *θ*
  – Choose randomly among volunteers agent *V*
    • Payoff of *V*, -*α*
    • Payoff of *P*, +1
  – Total utility of a player: \( \sum_{t=0}^{\infty} \delta^t u_t \)
Sample Run

5 agents in the system
Agent A is chosen to request for a service.

Now we will form a set of volunteers for satisfying this request.

Every other agent has probability $\beta$ of being able to satisfy the request.
Agents E, C and D are selected as capable of serving a request.

Now agents will have to decide if they want to volunteer.
Agent C decides to be a volunteer. This decision is based on his particular strategy.

The transaction is completed in the subsequent phase

$$S^r_c = \{\text{Volunteer}\}$$
Sample Run

Agent C was selected to satisfy the request uniformly from the set.

Agent A gets $1$ for having his request satisfied and B gets $-\alpha$.
Strategies?

• Consider an agent $j$
  – Money: $x_j$ dollars
  – Round: $r$

• How to decide if to be a volunteer?

• Threshold strategies $S_k$ ($k$-comfort level)
  – IF $x_j < k$ then volunteer
    • $S_o = never volunteer$

• Others?
The two arguments

• Existence of approximate equilibrium in the model
  – *Existence of* $\varepsilon$-*best replies*

• *Concentration phenomenon* of wealth distribution
  – The distribution of money converges (quickly) to a specific distribution, given a **big** enough agent set, and a **long** enough process
  – When playing threshold strategies
The distribution $d^*$

- Wealth per agent type converges to

\[ d^*(t, i) = \frac{f_t \cdot \lambda_i \cdot q(t, i)}{\sum_{j=0}^{k_t} \lambda_j \cdot q(t, j)} \]

- $d(t, i) = \text{The fraction of agents of type } t \text{ that have } i \text{ dollars} \]
The Volunteer’s Dilemma

• If *no*, his money does not change
• If *yes*, agent agrees to
  – Pay an amount $a_t$
  – Receive a discounted $\gamma_t$ in the future

• The decision is based on the estimation on how long will it take (say $J$) to finally *spend* the 1$.

$$a_t \leq \delta_t^J \cdot \gamma_t$$
Optimal Threshold Policy

\[ a_t \leq \delta^J_t \cdot \gamma_t \]

• The maximum \textit{comfort level} \( k \) defines the optimal threshold policy

• \( J(k) \) is the mean time in which an agent is \textit{depleted} of money, if starting with \( k \) dollars
Equilibrium through an MDP

• The evolution of the model can be described by a Markov Chain

• States are agent money savings

• Agent optimal response can be modeled through an MDP:
  – \( P_u = \text{probability of earning a dollar at each round} \)
  – \( P_d = \text{probability of making a request at each round} \)
**ε-best replies**

- Given an agent of type $t$, then for *large enough* agent populations and a *large enough type discount*, the optimal threshold policy is an *ε-best reply* to all others playing *threshold strategies*.

**Theorem 5.1.** For all games $G = (T, f, h, m, n)$, all vectors $\bar{k}$ of thresholds, and all $\varepsilon > 0$, there exist $n^*_\varepsilon$ and $\delta^*_\varepsilon, n$ such that for all $n > n^*_\varepsilon$, types $t \in T$, and $\delta_t > \delta^*_\varepsilon, n$, an optimal threshold policy for $P_G, \bar{s}(\bar{k}), t$ is an ε-best reply to the strategy profile $\bar{S}(k)_{-i}$ for every agent $i$ of type $t$. 
Monotonicity

• The best-reply function is non-decreasing in $k$. When all others increase their thresholds, one does not improve by lowering his own threshold.

• Last “piece”: There exists a threshold vector such that the best reply is strictly higher than this vector.

Lemma 5.3. For all games $G = (T, f, h, m, n)$, there exists a $\delta^* < 1$ such that if $\delta_t > \delta^*$ for all $t$, there is a vector $\bar{k}$ of thresholds such that $BR_G(\bar{k}) > \bar{k}$. 
Main theorem – Existence of equilibrium

• There exists a non-trivial equilibrium where all agents play threshold strategies

Theorem 5.2. For all games $G = (T, f, h, m, 1)$ and all $\epsilon$, there exist $n_\epsilon^*$ and $\delta_{\epsilon,n}^*$ such that, if $n > n_\epsilon^*$ and $\delta_t > \delta_{\epsilon,n}^*$ for all $t$, then there exists a nontrivial vector $\vec{k}$ of thresholds that is an $\epsilon$-Nash equilibrium. Moreover, there exists a greatest such vector.

• Other equilibria?
Some equations

• Total amount of agents = $h*n$
• Agents of type $t = f_t*h*n$
• Total amount of money $M = h*n*m$
Simulations: hoarders

• Hoarders (defined here as non-volunteers) can cause system to break down
• Non-monetary strategy to discourage hoarder? Forced volunteerism?
• Response is to increase m (although babysitters example shows downside, if hoarding strategy is fleeting)
Simulations: altruists

- A little altruism is good; too much can cause a crash
Simulations: sybils

- Only modest gains for sybils if no other agents act as sybils
- However, self-reinforcing process - as number of sybils grows, so does incentive to sybilize - can lead to crash
Simulations: collusion

- Colluders keep money in the system - do not reduce utility - work done by colluding group must = work paid for - net zero
- Implications for loans?