Optimizing Scrip Systems

or: mo' money, mo' problems? or: paying with Kash

paper by: Kash, Friedman, Halpern, 2009
presented 10/5/09

What good is money?

why might we want to introduce a scrip system?

- dictate initial distribution / control policy
- “double coincidence of wants” - ease transactions
- control flows within a system

The Great Capitol Hill Baby Sitting Co-op

proof? of an optimum quantity of money?

- 150 couples institute coupon system
  precautionary savings ensue
  more coupons injected
  market crashes

- with strong assumptions, (Hens et al) experiments suggest optimum quantity

- too many hoarders? Or, not a closed model?

Major results from Kash et al

- Theoretical:

  1. monotonicity of best reply: if all others play threshold, you should play threshold

  2. concentration phenomenon: an entropy-maximizing distribution as n->infinity

  (n is agents of a given type)

- Simulation:

  1. collusion doesn't hurt

  2. hoarders and altruists can cause crashes

The model (Complete)

- There are *N* agents
- A *type set T*
- Frequencies of types f_t
- A type $t = (\alpha_t, \beta_t, \gamma_t, \delta_t, \rho_t, \chi_t)$
- A game is described by *G(T, f, h, n,m)* where
  - *h = base number*
  - *n = number of replicas for each base and type*
  - *m= average money M/N*

Decoding types

Cost of providing a service

Utility **discount**

Request **rate**

$$t = (\alpha_t, \beta_t, \gamma_t, \delta_t, \rho_t, \chi_t)$$

Probability of being capable to **fulfill** a request

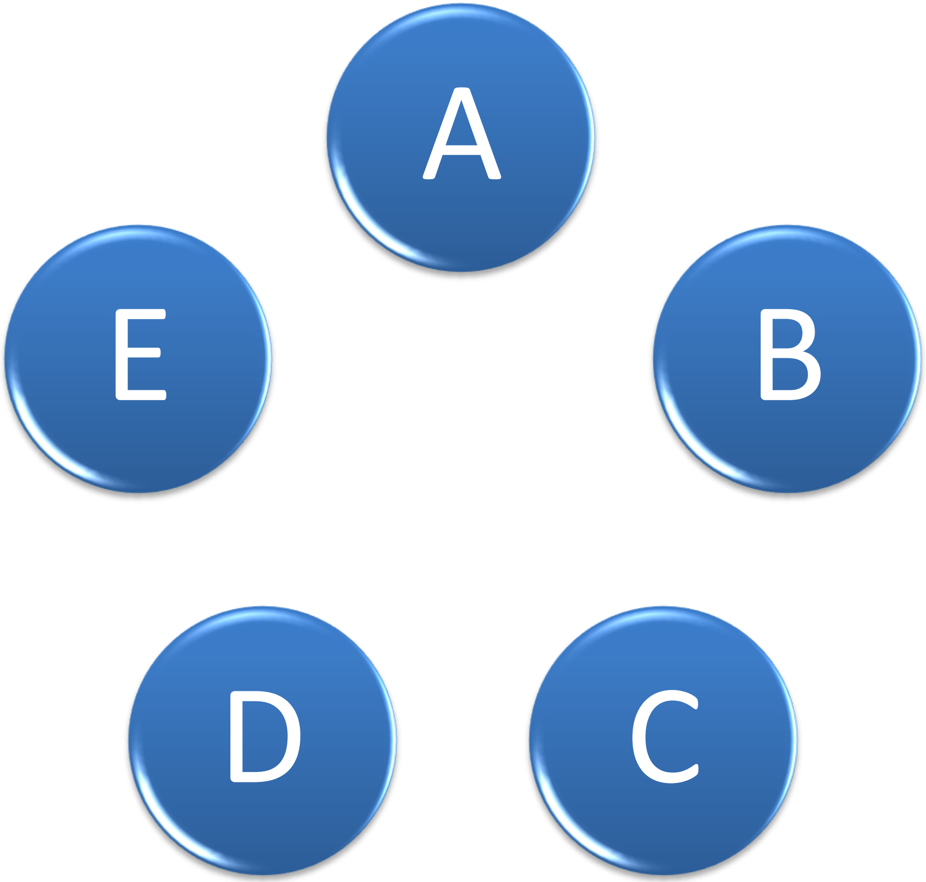
Utility for having a request **satisfied**

Relative probability of being **selected** when volunteered

The Model (Simplified)

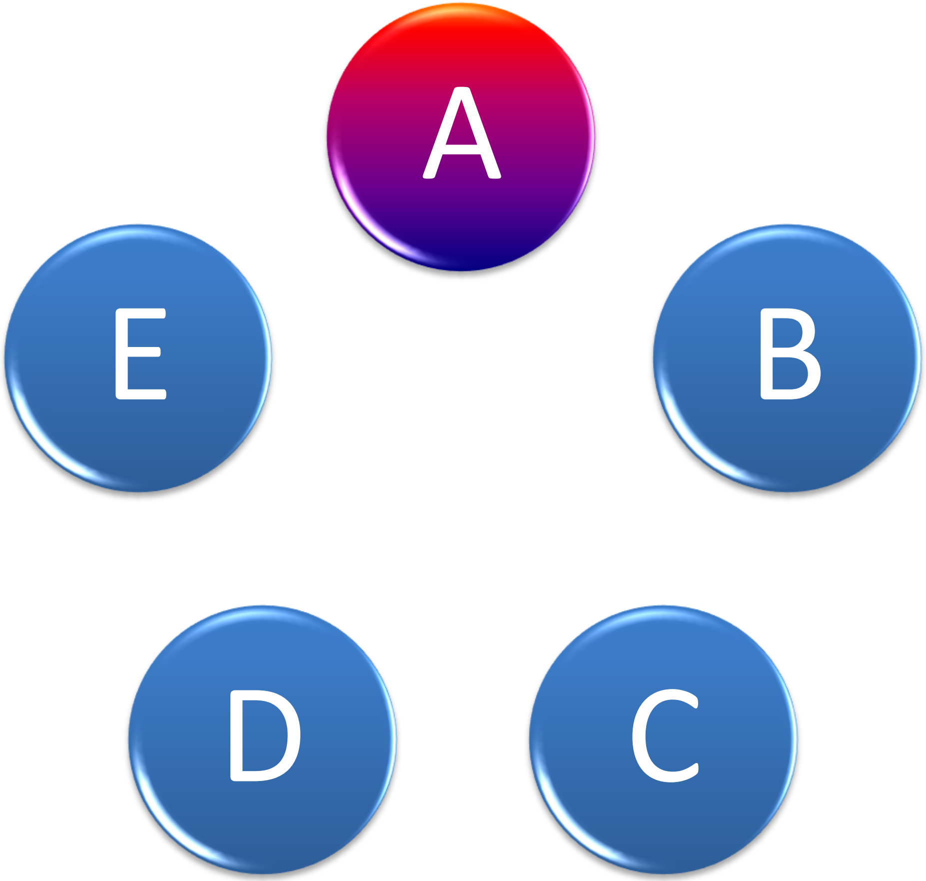
- Just consider only one type:
  - ***N*** *agents*
  - Randomly choose agent ***P*** to *request* service
  - Probability of being **able** to satisfy request, ***β***
  - Choose randomly among volunteers agent ***V***
    - Payoff of *V*, -***α***
    - Payoff of *P*, ***+1***
  - Total utility of a player: $\sum_{t=0}^{\infty} \delta^t u_t$

Sample Run



5 agents in the system

Sample Run

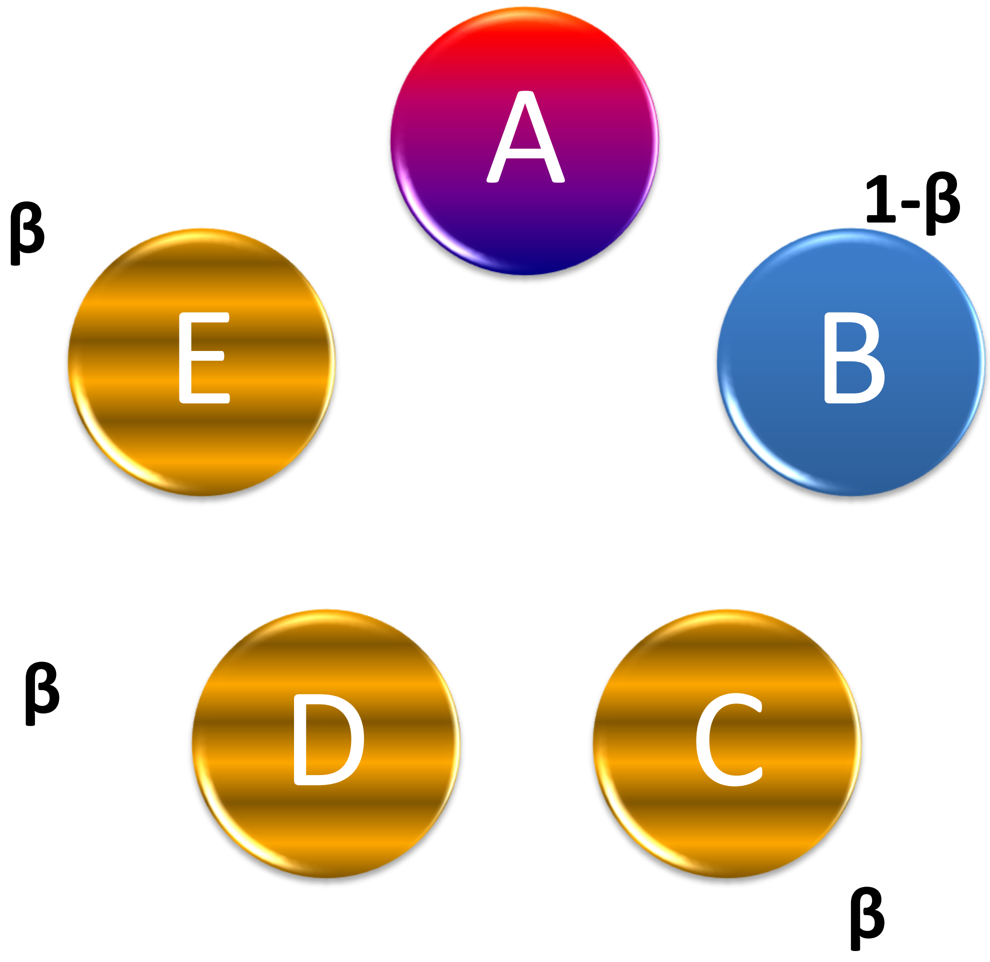


Agent A is chosen to request for a service.

Now we will form a set of volunteers for satisfying this request.

Every other agent has probability *β* of being able to satisfy the request.

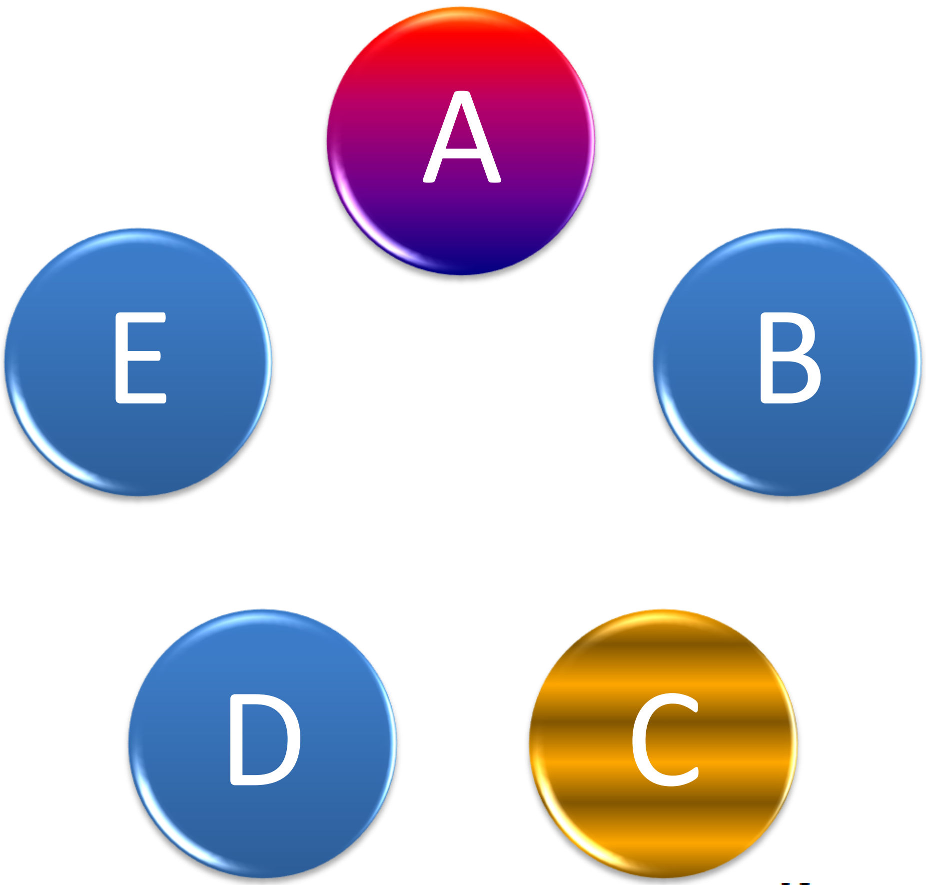
Sample Run



Agents **E, C** and **D** are selected as **capable** of serving a request

Now agents will have to decide if they want to volunter

Sample Run

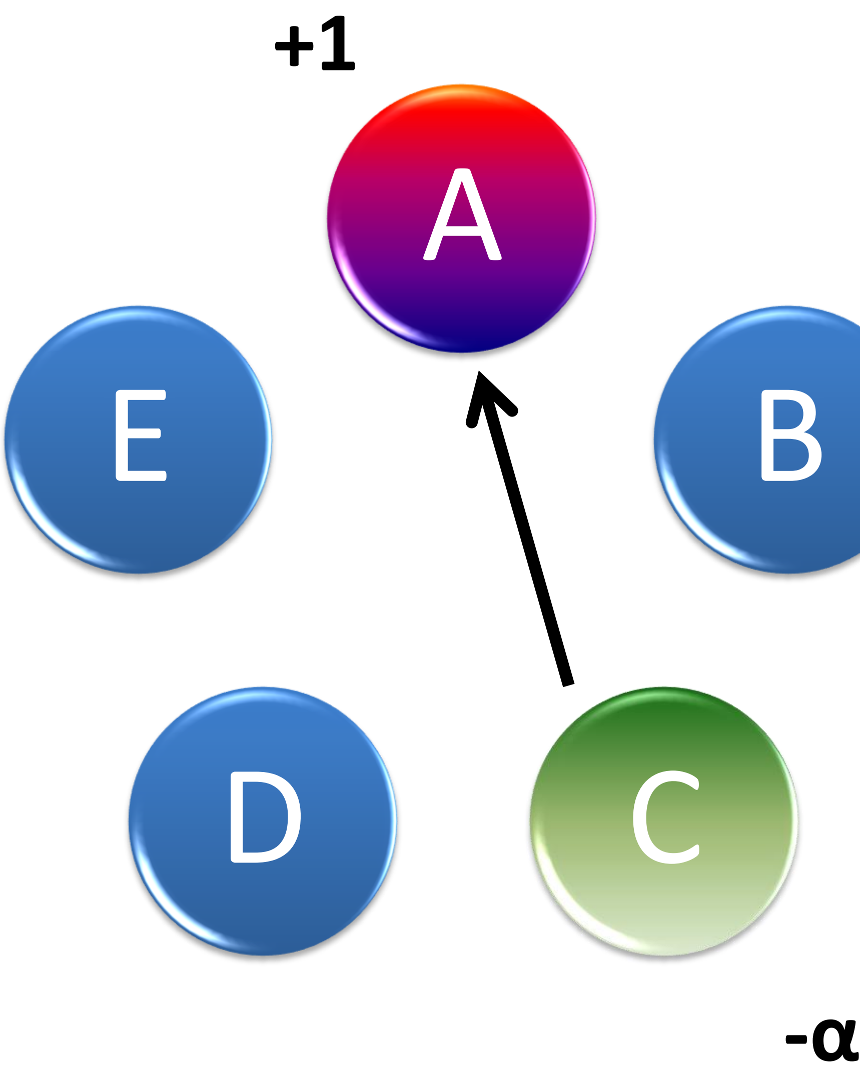


Agent **C** decides to be a volunteer. This decision is based on his particular **strategy**.

The transaction is completed in the subsequent phase

$$S_c^r = \{Volunteer\}$$

Sample Run



Agent **C** was selected to satisfy the request uniformly from the set.

Agent **A** gets **1$** for having his request satisfied and **B** gets ***–α***

Strategies?

- Consider an agent *j*
  - Money : $\boldsymbol{x_j}$ dollars
  - Round : $\boldsymbol{r}$
- How to decide if to be a volunteer ?
- *Threshold strategies* $\boldsymbol{S_k}$ *(k-comfort level)*
  - *IF* $x_j < k$ *then volunteer*
    - S_0 *= never volunteer*
- Others?

The two arguments

- Existence of approximate equilibrium in the model
  - *Existence of ε-best replies*
- *Concentration phenomenon* of wealth distribution
  - *The distribution of money converges (quickly) to a specific distribution, given a **big** enough agent set, and a **long** enough process*
  - *When playing threshold strategies*

The distribution *d**

- Wealth per agent type converges to

$$d^*(t,i) = \frac{f_t \cdot \lambda_i \cdot q(t,i)}{\sum_{j=0}^{k_t} \lambda_j \cdot q(t,j)}$$

- *d(t,i) =* The fraction of agents of type *t* that have *i* dollars

The Volunteer's Dilemma

- If *no*, his money does not change
- If *yes*, agent agrees to
  - Pay an amount a_t
  - Receive a discounted γ_t in the future
- The decision is based on the estimation on how long will it take (say J) to finally *spend* the 1$

$$a_t \leq \delta_t^J \cdot \gamma_t$$

Optimal Threshold Policy

$$a_t \leq \delta_t^J \cdot \gamma_t$$

- The maximum *comfort level k* defines the optimal threshold policy
- *J(k)* is the mean time in which an agent is *depleted* of money, if starting with *k* dollars

Equilibrium through an MDP

- The evolution of the model can be described by a Markov Chain
- States are agent money savings
- Agent optimal response can be modeled through an MDP :
  - P_u = *probability of earning a dollar at each round*
  - P_d = *probability of making a request at each round*

ε-best replies

- Given an agent of type t, then for *large enough* agent populations and a large enough *type discount*, the optimal threshold policy is an *ε-best reply* to all others playing *threshold strategies*

Theorem 5.1. *For all games $G = (T, \vec{f}, h, m, n)$, all vectors $\vec{k}$ of thresholds, and all $\varepsilon > 0$, there exist n^*_ε and $\delta^*_{\varepsilon,n}$ such that for all $n > n^*_\varepsilon$, types $t \in T$, and $\delta_t > \delta^*_{\varepsilon,n}$, an optimal threshold policy for $\mathcal{P}_{G,\vec{S}(\vec{k}),t}$ is an ε-best reply to the strategy profile $\vec{S}(\vec{k})_{-i}$ for every agent i of type t.*

Monotonicity

- The *best-reply* function is *non-decreasing* in *k*. When all others increase their *thresholds,* one does not improve by lowering his own threshold

- Last "piece": There *exists* a *threshold vector* such that the best reply is strictly higher than this vector

Lemma 5.3. *For all games $G = (T, \vec{f}, h, m, n)$, there exists a $\delta^* < 1$ such that if $\delta_t > \delta^*$ for all t, there is a vector $\vec{k}$ of thresholds such that $BR_G(\vec{k}) > \vec{k}$.*

Main theorem – Existence of equilibrium

- There exists a non-trivial equilibrium where all agents play threshold strategies

Theorem 5.2. *For all games $G = (T, \vec{f}, h, m, 1)$ and all ϵ, there exist n^*_ϵ and $\delta^*_{\epsilon,n}$ such that, if $n > n^*_\epsilon$ and $\delta_t > \delta^*_{\epsilon,n}$ for all t, then there exists a nontrivial vector $\vec{k}$ of thresholds that is an ϵ-Nash equilibrium. Moreover, there exists a greatest such vector.*

- Other equilibria?

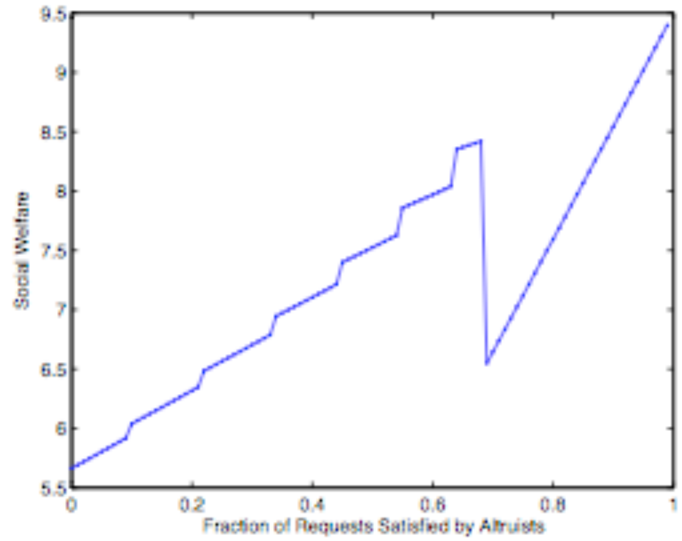
Some equations

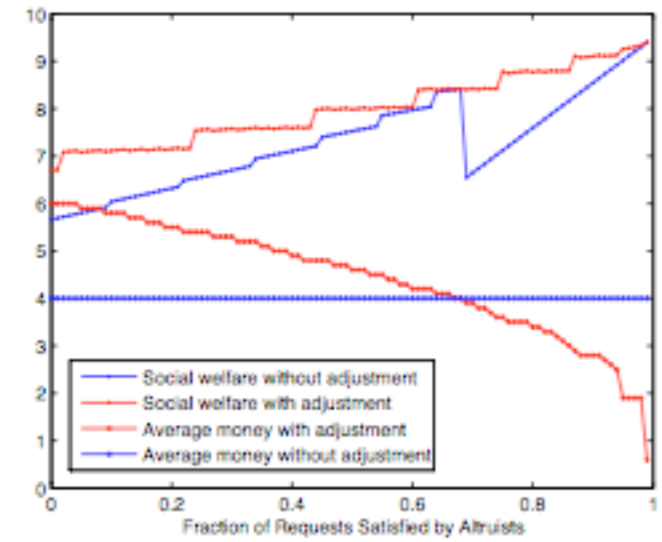
- Total amount of agents = $h*n$
- Agents of type t $= f_t*h*n$
- Total amount of money $M = h*n*m$

Simulations: hoarders

- Hoarders (defined here as non-volunteers) can cause system to break down
- Non-monetary strategy to discourage hoarder? Forced volunteerism?
- Response is to increase m (although babysitters example shows downside, if hoarding strategy is fleeting)

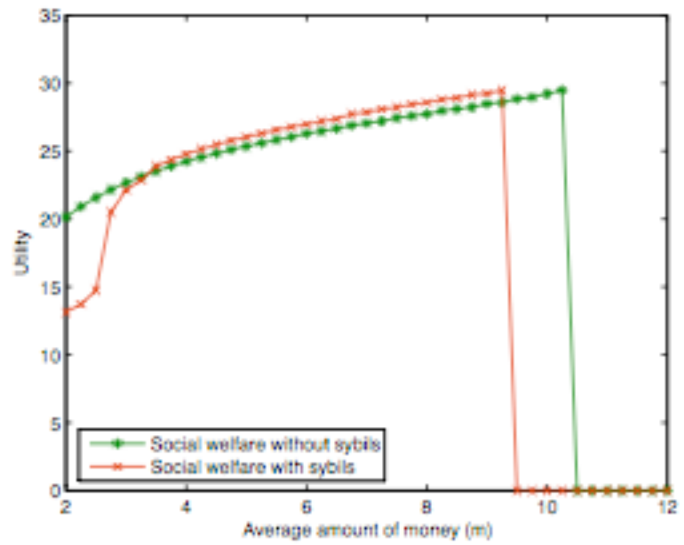
Simulations: altruists

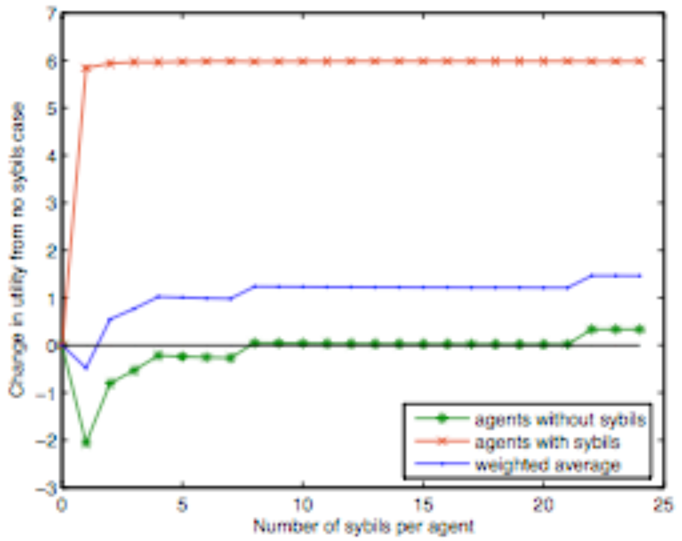




- A little altruism is good; too much can cause a crash

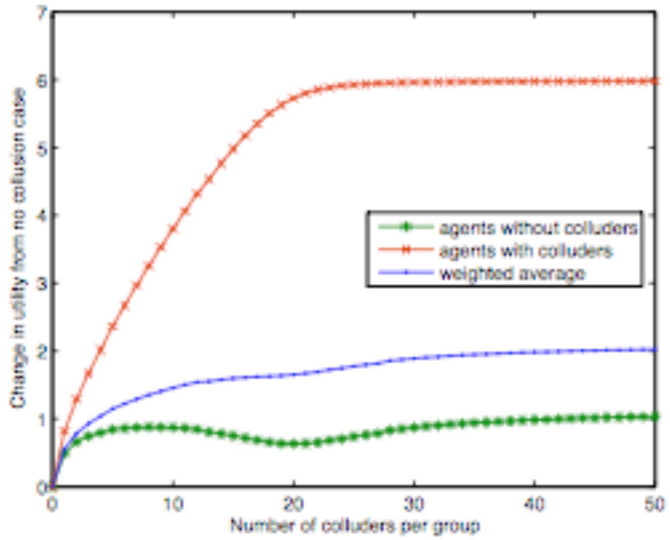
Simulations: sybils





- Only modest gains for sybils if no other agents act as sybils
- However, self-reinforcing process - as number of sybils grows, so does incentive to sybilize - can lead to crash

Simulations: collusion



- Colluders keep money in the system - do not reduce utility - work done by colluding group must = work paid for - net zero
- Implications for loans?