CS286r Fall 2009: Homework 2

Due: September 23, in class.

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Points will be awarded for clarity, correctness and completeness of the answers. Submissions should be brought to class. **There is one late day, to be used over homework 1 and 2.** *You may work in a pair and submit only one solution. But you must understand your solution.*

Total points: 140

1. **VCG mechanisms**

   (a) (15 pts) Consider the design of a mechanism for a market with m sellers and n buyers, each interested in trading one unit of the same item. Buyers have value b_i ("bids") and sellers value s_i ("asks"). Assume quasi-linear utility.

      i. Specify the Vickrey-Clarke-Groves mechanism for the problem. *Give a concise description of the mechanism in its simplified form for this special case* (don't just give the general formula.)

      ii. Provide a simple example to show that the VCG mechanism runs at a deficit on this problem.

      iii. Consider a mechanism with the same allocation rule, but every buyer and selling trading at the same price p, and this price set to half way between the lowest bid and highest ask that trade. What are the incentive properties of this mechanism? (A couple of sentences, you can be informal.)

   (b) (25 pts) Consider a problem with three advertising slots (slots 1, 2 and 3) and n bidders. The probability of a click is $p_1, \gamma p_1, \gamma^2 p_1$ in slots 1, 2 and 3 for $\gamma \in (0, 1)$ (for all advertisers). Bidders $i \in \{1, \ldots, n\}$ have value $v_i > 0$ for a click, independent of the slot from which a click is received.

      i. Derive a description of the VCG mechanism for this problem (i.e., do not just the generic equations.) You can assume that $n > 3$. Explain how to allocate ads to slots and how to determine payments.

ii. Provide an equation for determining the *per-click* price for an advertiser in slot 1, slot 2 and slot 3 so that the expected payment is equal to the VCG payments.

iii. Compare the per-click price to that in a "Generalized second price auction" in which advertisers are allocated to slots in order of bid price and the price charged is the smallest bid amount for which an advertiser would retain the same slot. (Assume the same set of bids.) What do you notice?

(c) (10 pts) Consider a SCF that is an affine maximizer, $f(v) \in \arg\max_{a \in A}(c_a + \sum_i w_i v_i(a))$ for some fixed agent weights $w_i, \ldots, w_n \geq 0$ and some outcome weights $c_a \in \mathbb{R}$ for every $a \in A$. Show that a Groves mechanism that picks the affine-maximizing alternative and collects payment

$$p_i(v) = h_i(v_{-i}) - \sum_{j \neq i} \frac{w_j}{w_i} v_j(a) - \frac{c_a}{w_i}$$

from every agent i, where h_i is an arbitrary function that does not depend on v_i is strategyproof.

(d) (25 pts) Consider a VCG mechanism applied to a combinatorial auction with two goods $\{A, B\}$ and bids $(AB, \$2), (A, \$2), (B, \$2)$ from three different bidders.

i. What is the outcome of the VCG mechanism in this example?

ii. Use a variation on the example to show that the revenue of the VCG mechanism is not monotonic-increasing in the number of bidders.

iii. Use a variation on the example to show why the VCG mechanism is susceptible to collusion by losers.

iv. Use a variation on the example to show why the VCG mechanism is susceptible to manipulation by false-name bidders (or "sybil-attack"), where one bidder bids under multiple identities.

v. Do any of these problems occur in the single-item Vickrey auction? For each problem, explain why or why not.

2. **Strategyproofness and Core**

(a) (20 pts) Consider a mechanism that is defined in terms of:

(A1) an agent-independent price-function so that for every v_{-i}, every allocation $x_i \in X$ to agent i, and for all v_i, the payment $p_i(v_i, v_{-i}) = p(x_i, v_{-i})$ and depends only on the allocation and the valuations of other agents.

(A2) an allocation rule $x_i(v) \in X$ that selects an allocation $x_i(v) \in \arg\max_{x \in X}\{v_i(x) - p(x_i, v_{-i})\}$, for every agent i.

Assume that X contains a "null" allocation, for which $v_i(x) = 0$ for all possible valuations v_i.

i. Explain intuitively why such a mechanism is strategyproof.

ii. Define the second-price Vickrey auction in these terms (i.e. exhibit an agent-independent price function (A1) and demonstrate that the winner determination rule satisfies (A2).)

iii. (**EXTRA CREDIT**) Define the Vickrey-Clarke-Groves mechanism for a combinatorial auction problem in these terms; i.e., for valuations $v = (v_1, \ldots, v_n)$ from agents, define an agent-independent price function to agent i for every possible bundle of goods and establish that the VCG mechanism allocates every agent the bundle that maximizes its utility at these prices.

(b) (10 pts) Consider a single-parameter domain with known interesting set $W_i \subseteq A$ for each agent i and private value $v_i \in \mathbb{R}$. Fixing v_{-i}, show that the WMON condition implies that if the alternative $f(v_i, v_{-i}) \in W_i$ then $f(v_i', v_{-i}) \in W_i$ for $v_i' \geq v_i$. (Check Chapter 9, the AGT book for the definition of WMON and single-parameter domains).

(c) (35 pts) Allocation without money.

i. Prove that the outcome of the top-trading cycle algorithm (TTCA) is in the core (see p.253-254, chapter 10 AGT book).

ii. Describe a variation on the TTCA that would remain strategyproof and could be used for a problem of allocation rather than reallocation (i.e., the items are not initially owned by agents.)

iii. Describe a strategyproof mechanism for the "course allocation" problem in which there are capacitated courses and each agent has preferences over different sets of courses.

Good luck!