The Great Capitol Hill Baby Sitting Co-op: Anecdote or Evidence for the Optimum Quantity of Money?*

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Abstract

This paper studies a centralized market with idiosyncratic uncertainty and money as a medium of exchange from a theoretical as well as an experimental perspective. In our model, prices are fixed and markets are cleared by rationing. We prove the existence of stationary monetary equilibria and of an optimum quantity of money. The rational solution of our model, which is based on the assumption of individual rationality and rational expectations, is compared with actual behavior in a laboratory experiment. The theoretical results are strongly supported by this experiment.

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1 Introduction

Sweeney and Sweeney's (1977) report on the Great Capitol Hill Baby Sitting Co-op, which has been popularized by Krugman (1999), is without doubt an entertaining anecdote to illustrate the optimum quantity of money. This paper analyzes whether this story is more than a mere anecdote.

The Great Capitol Hill Baby Sitting Co-op was a "cooperative" of about 150 couples, with the goal of sharing baby-sitting fairly amongst themselves by introducing a coupon system. One coupon entitled each member to receive half an hour's worth of baby-sitting. Initially, one coupon of baby-sitting was issued to each couple. Supposing that coupons circulated, each couple would, over time, do as many units of baby-sitting as they received in return. After a short while, however, the system collapsed, because there was not enough demand for baby-sitting. Krugman (1999) attributes this breakdown to precautionary savings. The Co-op solved this problem simply by issuing more coupons. Having found out that each couple was better off with an increase in the number of coupons available, the Co-op continued to issue more coupons, which eventually resulted in a breakdown of the system. The moral of this anecdote is that a market in which prices are fixed only works efficiently for a specific, 'optimal' quantity of money.

The Great Capitol Hill Baby Sitting Co-op is just one example of a trade circle. Trade circles, which have become increasingly common, provide a field for clinical studies on the role of money. Trade circle members can buy or sell services at fixed prices. The supplier of a service receives 'artificial' money, or coupons, which he/she can then use to buy services. Prices are fixed by fairness considerations and credit is limited. Money usually has a positive value in exchange for services, and its introduction leads to a Pareto-improvement over the situation without money.

While the importance of the quantity of money has not been reported from other trade circles, the Capitol Hill Baby Sitting Co-op is a beautiful anecdote to illustrate the role of money in simple markets with idiosyncratic uncertainty and fixed prices. It suggests that individual rationality is typically at odds with collective rationality, and that only a specific amount of money helps to overcome this problem if prices are fixed. From a collective point of view, it is preferable that all participants hold more money if there is an excessive amount of it. If money is scarce, then precautionary savings are contrary to the common interest. The report on the Capitol Hill Baby Sitting Co-op only gives anecdotal evidence for this quite general claim on the existence of an optimum quantity of money. It could still be conjectured that the reasons for these observations are totally different from those put forward above. For example, one might argue that money holdings are determined by myopic expectations on the future resale value, so that the market breaks down because expectations are incorrect in the longterm. Whether the Capitol Hill Baby Sitting Co-op is, in fact, more than an anecdote for the optimum quantity of money can only be decided by extracting the underlying fundamental mechanisms and by studying them in a formal model. This approach will provide us with analytical results, and it will make the above reasoning testable under controlled conditions in a laboratory experiment. A sound theory, in combination with additional experimental observations, can provide more solid evidence for the story told by the Capitol Hill Baby Sitting Co-op.

In pursuit of this goal, we develop a formal model of a monetary economy with a perishable good that can be traded in a centralized market. Competitive behavior is ensured by assuming a continuum of agents, each having stochastic preferences. According to this specification, no agent thinks of him/herself to be in a position to change market averages resulting from the actions of all agents. An agent cannot consume his/her own goods, but needs to receive money in order to finance future consumption. The price is fixed, and trade is a one-to-one exchange of money against goods. Market clearing is ensured by rationing. While this model may not be ideal for studying the long-term effects of monetary policy (because of the assumption of fixed prices), it is arguably well suited for studying short-term effects.

The model shares with Levine (1991) and Kehoe, Levine, and Woodford (1992) in assuming stochastic preferences that generate a need for inter-temporal transfers. But in contrast to these papers, we do not pursue a mechanism design approach. There is no possibility of lump sum subsidies, no making or losing money (i.e. no government and no interest involved), and prices are fixed. In Wallace's (2002) view, our model takes the extreme position of idiosyncratic uncertainty across agents and types, anonymity, the absence of any monitoring and the largest possible market. As a result of these features, the optimum outcome cannot be achieved in our model. But by using the total trading volume in the economy as a welfare measure, we can show the existence of an optimum quantity of money. Money is beneficial in our model of a centralized market, in the sense that more allocations are possible with money than without it.

The model also shares some features with the neo-Keynesian models developed by Clower, Barro and Grossman, Benassy, Malinvaud and Dréze.[1] In those

[1] For a comprehensive account of this theory, see, e.g., Malinvaud (1977) and Benassy (1982).

models, prices are fixed, and demand and supply is coordinated by quantity rationing. However, while the use of money in the standard neo-Keynesian model is justified by a temporary equilibrium approach, the microeconomic foundation of money that our model provides is based on complete rationality under rational expectations.

Last, but not least, our model has some features in common with the recent literature on the micro-foundation of money, originating in the seminal papers of Kiyotaki and Wright (1989, 1991, 1993). In this strand of research, markets are no longer considered to be well organized, but traders meet randomly in pairs, see e.g. Boldrin, Kiyotaki, and Wright (1993), Trejos and Wright (1995). Our paper suggests a micro-foundation of money that is complementary to the standard search model. On the one hand, we assume that markets are well-organized, in the sense that each potential supplier and each potential demander of a service can meet traders of the other side of the market in each period. On the other hand, we assume that the market participants have stochastic preferences: one day a trader would prefer to supply a service, while on another day he/she prefers to demand it. In this view, the role of money is to enable the traders to transfer income from supply days to demand days.[2] From a more general perspective, our model is similar to the Kiyotaki–Wright model because, in both approaches, the incentive to hold money arises from the uncertainty of whether one will be a demander or a supplier of a service in the future. This uncertainty arises from the stochastic preferences in our model, and from the possibility of being matched with a trading partner who does not have "coincident wants" in the Kiyotaki–Wright model. Indeed, in the formal analysis of our model (relying on the study of Bellman equations) we reach a

[2] There are random-matching models with centralized markets and bilateral trade, e.g., Lagos and Wright (2003, 2005), Rocheteau and Wright (2005). In these models, however, money is not needed to trade in the centralized market, as goods are exchanged against goods.

similar conclusion to Berentsen (2000, Lemma 1)—using the Kiyotaki–Wright framework. From this viewpoint, the essential difference of our model is the use of rationing to clear the market.

Since our model will be based on a rigorous idealization—the notion of stationary competitive equilibria with rational expectations in a stochastic game—it is important to contrast the theoretical predictions with the actual behavior of only a few, possibly bounded rational participants in a laboratory experiment with a finite horizon. The experimental design benefits from the previous laboratory test of the Kiyotaki–Wright model by Duffy and Ochs (1999) and McCabe (1989). Duffy and Ochs (1999) conclude that implementing a model with a continuum of players, an infinite time horizon and discounting as a game between finitely many players and a finite time horizon, has only a minor influence on the results. McCabe (1989) finds that, even if the number of time periods is fixed and small (six periods in his paper), only experienced participants show behavior inducing a non-monetary equilibrium. Our main interest is in the behavior of experienced participants. All participants therefore play a sequence of increasingly demanding games that constitute a 'learning' phase that lasts about four hours. Afterwards, a strategy game is conducted with the aim of eliciting the strategies from experienced participants. The games comprise individual decision making experiments and market experiments with interaction. This will allow to determine whether a participant's behavior is shaped by individual optimization or market interaction. This design allows for a rather detailed test of the predictions derived from the theory.

The laboratory experiment culminated in games in which groups of six participants formed a market to repeatedly buy and sell services in exchange for money, called coupons. Each participant's potential payoff in a period (representing an instantaneous utility) was drawn randomly across agents and time from a common probability distribution. While this distribution was common knowledge, the realized, individual payoff values were not revealed to other participants. All contract positions (buy, sell, stay idle) were available to a participant, except for 'buy' if he/she had no money.

Even though each group consisted of only six participants, their behavior conformed with the best response to market averages. This observation shows that the participants' behavior coincided exactly with the theoretical results, based on a rational solution concept: agents behave competitively and act according to the optimal solution of the individual maximization problem. We attribute this result to the fact that, as we prove analytically, stationary monetary equilibria can be implemented using three simple heuristics: (1) holding no money implies offering baby-sitting services; (2) a high payoff-value results in seeking baby-sitting services, provided money holdings are positive; and (3) a low payoff-value leads to offering baby-sitting services, as long as money holdings are below a certain quantity $\overline{m}$ (specified later), and to seeking baby-sitting services if money holdings are above the quantity $\overline{m}$.

Moreover, the median maximum demand for money corresponded closely with the model's prediction. It turned out that there was also an optimum quantity of money, which depended on the average potential payoff from buying. This optimum quantity, and the maximum money holdings, were slightly higher in a setting with lower average time value, a case where there is a higher likelihood of demand shortage. By and large, the experiment provides evidence that the rational solution concept applied in our model has its descriptive merits.

The essence of our paper is that the story told by the Capitol Hill Baby Sitting Co-op indeed goes deeper than its anecdotal nature might suggest.

The paper is organized in the following fashion. The next section gives a precise description of the model that provides a sound theoretical foundation for the Capitol Hill Baby Sitting Co-op story. Then, in Section 3, we define an equilibrium concept, show the existence of two types of equilibria (monetary and non-monetary), and prove that there is an optimum quantity of money. Section 4 describes the implementation of the model in a laboratory setting and discusses the experimental results that provide evidence for the claim derived from the anecdote. Section 5 concludes.

2 The Model

Our formal model of a monetary economy with a centralized market and anonymous agents (which rules out credit) has three distinctive features. First, the good, or service, cannot be stored and is indivisible. Agents can either consume or produce one unit in any one period of time, but cannot consume their own good. Production is costless. Second, there is one centralized market in each period, where all potential demanders meet all potential suppliers. Every agent has to decide whether he/she wants to buy or sell before the market opens. As the price of the good is fixed (to one unit of money for convenience) and the good is indivisible, the market needs to coordinate and demand supply by some non-pricing mechanism. Here, the long side of the market is rationed by randomly selecting as many agents on the long side as there are on the short side. Any agent on the long side is thus subject to a probability of not being able to carry out his/her planned transaction.[3] Third, the agent's decision problem is of an intertemporal nature because any future consumption requires the transfer of income across time. This can only be achieved by the acceptance of money in exchange for the good, because any form of private credit is ruled out by the

[3]The one-stage game in any one period of time is such that agents cannot condition their buy/sell decision on the individual outcome of rationing procedure. The ability to do so would unambiguously increase the agents' utility. This, however, constitutes a more general game, with a secondary market after the first has closed. In such a game, the current situation is established again by restricting the agents' strategy space.

agents' anonymity. Money is therefore essential.[4]

Time is discrete, $t = 1, 2, 3, ...$ There is a continuum of agents, $[0, 1]$. Agent i's discount factor is denoted as β^i, $0 < \beta^i < 1$. In period t, his/her money holdings are denoted as $m_t^i \geq 0$, and the instantaneous utility from consumption is $w_t^i \in \{l, h\}$ $(0 < l < h)$. The individual utility w_t^i is determined by a chance move (independently across agents and time) that assigns probability p_h to the high value h and probability $p_l = 1 - p_h$ to the low value l. Then the agent selects one of the three alternatives S (sell), B (buy) or I (stay idle). Anonymity implies that agents can only select alternative B if $m_t^i > 0$. The agent's choice is executed with probability p_S resp. p_B if he/she chooses alternative S resp. B. These success probabilities are determined by rationing and, naturally, depend on the choices of *all* agents. Since at most one market side is rationed, one of the probabilities is always equal to one. Depending on the outcome of the rationing procedure, money holdings change (decrease resp. increase by one if the agent is successful in buying resp. selling) and instantaneous utility is realized (if successful in buying).

We will drop the index i in the remainder of this section for notational ease.

A stationary policy $\psi(m, w) \in \{B, S, I\}$ describes an agent's choice as a function of current money holdings m and instantaneous utility from consumption w (which can be either high, $w = h$, or low, $w = l$, where $0 < l < h$). The Bellman equation relates the current value and the continuation value of the game. $V(m)$ denotes the lifetime utility, i.e. the present value of expected future utility, for an agent with money holdings m. Provided the agent is risk neutral, and the parameters $0 < p_h < 1$, $0 < p_S \leq 1$ and $0 < p_B \leq 1$ are given,

[4]In a non-monetary barter economy (and without anonymity), agents would need access to a complete market of contingent commodities, because the future states of the individual preferences are unknown.

one has

$$
\begin{aligned}
V(0) &= \beta \ \max\Big\{p_S\, V(1) + (1-p_S)\, V(0), V(0)\Big\} \\
V(m) &= \beta \sum_{w=h,l} p_w \ \max\Big\{p_B\ [w/\beta + V(m-1)] + (1-p_B)\, V(m), \\
&\qquad\qquad\qquad p_S\, V(m+1) + (1-p_S)\, V(m), V(m)\Big\}
\end{aligned}
\tag{1}
$$

Existence and uniqueness of a solution to the Bellman equation (1) is straightforward, using Blackwell's sufficient condition for a contraction on the space of positive functions V with domain $\mathbb{N}$ and range $\mathbb{R}_+$. Denote this solution by $V^\star$. Analogously to Berentsen (2000, Lemma 1), one can show that $V^\star$ has the following properties:

Lemma 2.1 *The solution $V^\star$ to (1) is strictly positive, strictly increasing, strictly concave, and bounded by $h/(1-\beta)$.*

There also is a maximum amount of money, $\overline{m}$, that an agent wants to accumulate. It is given by

$$\overline{m} := \min\Big\{m \geq 1 \mid \frac{p_B\, l}{\beta} > p_S\, [V^\star(m+1) - V^\star(m)] + p_B\, [V^\star(m) - V^\star(m-1)]\Big\}$$

This result is straightforward according to the Bellman equation (1). Obviously, if $w = h$, then "buy" is optimal, provided the agent holds money. It thus remains to consider the case $w = l$. An agent (weakly) prefers to sell if, and only if, $p_B\,(l/\beta + V^\star(m-1)) + (1-p_B)\, V^\star(m) \leq p_S\, V^\star(m+1) + (1-p_S)\, V^\star(m)$. ("Buy" is preferred when the inequality is reversed.) This condition is equivalent to $p_B\, l/\beta \leq p_B(V^\star(m) - V^\star(m-1)) + p_S(V^\star(m+1) - V^\star(m))$. Concavity and boundedness of $V^\star$ immediately imply that $\overline{m}$ is well-defined and that $\psi^\star(m, l) = S$ for all $0 < m < \overline{m}$ and $\psi^\star(m, l) = B$ for all $m \geq \overline{m}$.

Summarizing these findings, one can state:

Theorem 2.1 *Fix any values $0 < l < h$, $0 < p_h < 1$, $0 < p_S \leq 1$, $0 < p_B \leq 1$, and $0 < \beta < 1$. The optimal stationary policy $\psi^\star$ is given by*

- *Rule 1: $\psi^\star(0, w) = S$ for $w = h, l$;*
- *Rule 2: $\psi^\star(m, h) = B$ for all $m > 0$; and*
- *Rule 3: $\psi^\star(m, l) = S$ for all $0 < m < \overline{m}$; and $\psi^\star(m, l) = B$ for all $m \geq \overline{m}$.*

The maximum money holdings $\overline{m}$ are defined above.

The intuition behind the optimal policy $\psi^\star$ is as follows. Alternative I, staying idle, is weakly dominated because the other alternatives may increase either utility or money holdings. An agent with no money is certainly better off choosing alternative S (sell), aiming to obtain money for future consumption. As alternative B (buy) is not available to him/her, rule 1 gives the optimal choice. Rule 2 uses the fact that the optimal choice of an agent with high instantaneous utility h (and with money) is alternative B (buy), because this yields the maximal current utility. Keeping the money would reduce his/her present value, as future utility is discounted. Rule 3 states that an agent has an endogenous maximum quantity of money holdings. If his/her money holdings exceed, or are equal to $\overline{m}$, he/she chooses alternative B, even if his/her instantaneous utility is low. The reason is as follows. The current decision of an agent with money holdings $\overline{m}$ is irrelevant for the optimal continuation in the next $\overline{m} - 1$ periods. An agent can use an additional unit of money obtained today only after at least $\overline{m} - 1$ periods. The lower instantaneous utility in the current period l (times the success probability) is preferred to alternative S (sell) if the discounted expected utility in period $\overline{m} - 1$ is higher from now on for the first choice than for the second one. Otherwise an agent should choose to go for an additional unit of money.

A numerical solution of the Bellman equation gives $\overline{m} = 4$ for both success probabilities being equal to 1, with equal chances of high, $h = 10$, and low, $l = 5$, instantaneous utility (i.e. $p_h = p_l = 0.5$), and a continuation probability of $\beta = 95\%$. Increasing the probability for h to $p_h = 0.8$ (and letting $p_l = 0.2$) gives, ceteris paribus, $\overline{m} = 9$.

3 The Market Equilibrium

Agents can be heterogenous with respect to their discount factor. We allow for finitely many different types $i \in I$, with an individual discount factor $0 < \beta^i < 1$. The relative number of agents of type i is denoted by $\lambda^i > 0$, the Lebesgue measure of the respective type. Recall that there is a continuum of agents.

In a stationary market equilibrium, the "consistency" of all individually rational actions has to hold. That is, every agent's policy is the result of the above optimization, given the other factors, that the distribution of money holdings is stationary (across types and quantities), that the success probabilities given by the stationary distribution are those taken for the determination of the policy, and that the quantity of money has to be equal to the average money holdings of agents. The formal definition is as follows:

Definition 3.1 *A* stationary equilibrium *is a tuple* $((\psi_i^\star)_{i\in I}, \mu^\star, p_S^\star, p_B^\star, M^\star)$, *consisting of a stationary policy for every type, a distribution of types over money holdings, success probabilities, and a quantity of money, such that*

(i) given $p_S^\star$ *and* $p_B^\star$, $\psi_i^\star$ *is an optimal stationary policy for each agent of type* i*;*

(ii) given $(\psi_i^\star)_{i\in I}$, $p_S^\star$, *and* $p_B^\star$, $\mu^\star$ *is a stationary probability measure for the corresponding Markov chain on* $I \times \mathbb{N}$ *with* $\sum_m \mu_i^\star(m) = \lambda^i$ *for all* $i \in I$*;*

(iii) given $(\psi_i^\star)_{i\in I}$ and $\mu^\star$, the probabilities $p_S^\star$ and $p_B^\star$ satisfy the rationing given by (2); and

(iv) the average money holdings are equal to the average money supply, i.e. $\sum_{m\in\mathbb{N},i\in I} m\,\mu_i^\star(m) = M^\star$.

The rationing probabilities are given by[5]

$$\begin{aligned} p_S &= \min\left\{1, \frac{\sum_{m\in\mathbb{N},i\in I}\mu_i(m)\sum_{w=h,l}p_w\,\mathbf{1}_{\psi_i(m,w)}(B)}{\sum_{m\in\mathbb{N},i\in I}\mu_i(m)\sum_{w=h,l}p_w\,\mathbf{1}_{\psi_i(m,w)}(S)}\right\} \\ p_B &= \min\left\{1, \frac{\sum_{m\in\mathbb{N},i\in I}\mu_i(m)\sum_{w=h,l}p_w\,\mathbf{1}_{\psi_i(m,w)}(S)}{\sum_{m\in\mathbb{N},i\in I}\mu_i(m)\sum_{w=h,l}p_w\,\mathbf{1}_{\psi_i(m,w)}(B)}\right\} \end{aligned} \tag{2}$$

A stationary equilibrium is called monetary*, if the value function solving (1) is strictly positive on $\mathbb{N}$ for at least one agent type.*

3.1 Monetary equilibria and the optimum quantity of money

The existence of stationary equilibria is easily proved. A non-monetary stationary equilibrium, for instance, exists for any characteristics of agents and any amount of money; in this equilibrium, $p_B^\star$ is zero. In this market, everybody wants to buy and nobody wants to sell. It is also obvious that no agent improves his/her situation by selling, because the additional unit of money received can never be spent.

It requires more work to prove the existence of monetary equilibria for given agent types and success probabilities $p_B > 0$ and $p_S > 0$. For given success probabilities, the optimal policies are determined by solving the Bellman equation for every agent type. Then a stationary distribution over money holdings is constructed. This distribution has to be consistent with the success probabilities taken for the determination of the individually optimal policies.

[5] $\mathbf{1}_{\psi_i(m,w)}(S)$ is the indicator function, which takes on the value 1 if $\psi_i(m,w) = S$ and the value zero otherwise.

Theorem 3.1 *Given any agent types and any prescribed success probabilities $p_B > 0$ and $p_S > 0$ (one of them being equal to one), there exists a quantity of money and a corresponding monetary stationary equilibrium such that $p_B^\star = p_B$, $p_S^\star = p_S$ and each type i agent's optimal policy $\psi_i^\star$ is given by Theorem 2.1.*

The proof of Theorem 3.1 relies on the results of the value function $V^\star$ and the optimal policy. Let $\psi_i^\star$ denote the optimal policy for agents of type i for given success probabilities $p_B > 0$ and $p_S > 0$. Denote the corresponding maximum money holdings by $\overline{m}_i$.

The distribution of time values can be implemented such that (a) time values are iid for each agent and (b) the distribution prevails across types and money holdings. Rationing can be implemented such that (a) each rationed agent faces an iid draw and (b) for each type of agent and money holdings, the actual ratio of rationed agents to all agents on the same side of the market is equal to the rationing probability. Throughout the following procedure, such a mechanism is supposed to operate in the economy.

Fix a type i. Given the policy $\psi_i^\star$ and success probabilities $p_B > 0$ and $p_S > 0$, a Markov chain on the set of money holdings $X^i := \{0, ..., \overline{m}_i\}$ is defined. The transition probabilities $P_i(m, m')$, $m, m' \in X^i$ are as follows. Trade takes place at price one, therefore $P_i(m, m')$ can be strictly positive if and only if $|m - m'| \leq 1$. One has $P_i(0, 1) = p_S$ (because any agent without money prefers to sell), $P_i(\overline{m}_i, \overline{m}_i - 1) = p_B$ (because $\overline{m}_i$ is the maximal money holdings), and, for all $0 < m < \overline{m}_i$, $P_i(m, m-1) = p_B\, p_h$ and $P_i(m, m+1) = p_S\, p_l$. All of these transition probabilities are strictly positive because $p_B > 0$, $p_S > 0$, and $0 < p_h < 1$. These findings imply that the Markov chain on X^i is irreducible. Denote the corresponding unique stationary distribution by $\tilde{\mu}^i$. We can now define a probability measure $\mu^\star$ on $I \times \mathbb{N}$ by $\mu_i^\star(m) := \lambda^i\, \tilde{\mu}^i(m)$, $i \in I$, and define the corresponding quantity of money by $M^\star := \sum_{i\in I, m\in\mathbb{N}} m\, \mu_i^\star(m)$.

It remains to show that $((\psi_i^\star)_{i\in I}, \mu^\star, p_B^\star, p_S^\star, M^\star)$ actually is a stationary equilibrium, where $p_B^\star := p_B$ and $p_S^\star := p_S$. We check each condition in Definition 3.1 in turn. First, the stationary policies $\psi_i^\star$ are optimal by construction. Second, $\mu^\star$ is stationary and the marginal distribution over types is $\sum_m \mu_i^\star(m)$ $= \lambda^i \sum_m \tilde{\mu}^i(m) = \lambda^i$, also by construction. Third, we have to show that the actual success probabilities, cf. (2), are equal to the prescribed ones $p_B^\star$ and $p_S^\star$. Let us consider the case $p_S^\star = 1$ (the case $p_B^\star = 1$ is analogous). Condition (iii) is satisfied if and only if

$$p_B^\star \sum_{m\in\mathbb{N}, i\in I, w=l,h} \mu_i^\star(m)\, p_w\, \mathbf{1}_{\psi_i^\star(m,w)}(B) = \sum_{m\in\mathbb{N}, i\in I, w=l,h} \mu_i^\star(m)\, p_w\, \mathbf{1}_{\psi_i^\star(m,w)}(S) \quad (3)$$

Stationarity of $\tilde{\mu}^i$ implies

$$p_B^\star \left(\tilde{\mu}^i(\bar{m}_i) + \sum_{0<m<\overline{m}_i} \tilde{\mu}^i(m) p_h \right) = \tilde{\mu}^i(0) + \sum_{0<m<\overline{m}_i} \tilde{\mu}^i(m) p_l$$

That is, for each fixed type of agent, the amount of agents whose money holdings decrease by one unit (due to actually buying the good) is equal to the amount of agents whose money holdings increase by one unit(due to actually selling the good). This holds by stationarity of $\tilde{\mu}^i$ for the corresponding Markov chain on X^i. Multiplying both sides of the last equation by λ^i and summing up over $i \in I$, we obtain (3), taking the optimal policy into account.

Finally, condition (iv) holds by our definition of $M^\star$. The equilibrium is monetary. This completes the proof.

This result can also be interpreted as follows. Agents assume certain success probabilities (for example: $p_S = p_B = 1$, i.e. that no rationing will occur) and choose their policies accordingly. The quantity of money and the policies determine a dynamic process over money holdings, which in turn determine the success probabilities in the market. The quantity of money is the parameter that allows to adjust the success probabilities of the process to the success probabilities taken for the determination of the policy.

The general existence result Theorem 3.1 for monetary equilibria, in which the quantity of money can be chosen by an outside institution, raises the question of an optimum quantity of money. A non-monetary equilibrium is obviously Pareto-dominated by every monetary equilibrium. This does not provide a satisfactory answer, however, in the case of a benevolent social planner who maximizes the welfare in the economy. The most simple measure is that of the number of trades, which is maximal if no rationing occurs ($p_B^\star = p_S^\star = 1$). If there is rationing, some agents cannot trade, and thus obtain lower utility than in an equilibrium without rationing. While this criterion measures individual welfare only, and not social welfare, it provides a simple benchmark for the social planner's policy. This observation resembles findings in the random-matching literature, e.g. Berentsen (2002, Proposition 1), where total welfare is maximal if the number of those pair-wise matchings is minimized in which either buyers have no money or sellers are at their maximum money holdings.

We have the following result:

Corollary 3.1 *Given any agent types, there exists some quantity of money $M^\star$ and a corresponding monetary stationary equilibrium in which no rationing occurs ($p_B^\star = p_S^\star = 1$).*

Every quantity of money different to $M^\star$ leads to a monetary stationary equilibrium in which the number of trades is strictly less than with $M^\star$.

This Corollary can be shown as follows. Theorem 3.1 ensures the existence of an equilibrium with $p_B^\star = p_S^\star = 1$. In this equilibrium, there is no rationing. Consequently, the number of trades is maximal.

We have to compare an agent's utility in this equilibrium with the one obtained in any other equilibrium with a different quantity of money M_1. In the second equilibrium, rationing has to occur because, if $p_B^\star = p_S^\star = 1$, the policies of the agents would be the same. This leads to the same Markov chain and the same quantity of money. In an equilibrium with rationing, an agent's utility under his/her optimal policy is lower than when adopting the same policy in an economy (which is not in equilibrium) without rationing and a different quantity of money M_2. This non-equilibrium policy combination is dominated by the equilibrium policy combination with $p_B^\star = p_S^\star = 1$ and the quantity of money $M^\star$, because the equilibrium policy is the solution to the Bellman equations without rationing.

This shows that it is legitimate to call the quantity of money that is determined this way the *optimum quantity of money.*

For instance, if agents are risk neutral, and $p_h = p_l = 0.5$, the optimum quantity of money is equal to half of the average maximum money holdings across types (weighted according to their frequency) because of the symmetry of the stationary distribution of the Markov chain. This finding is related to Berentsen (2002), where instantaneous utility is certain. If the probability $p_h \neq 0.5$, the optimum quantity of money will, in general, *not* have this property. This gives a theoretical foundation for the claim made in the anecdote of the Capitol Hill Baby Sitting Co-op.

4 The Experiment

Since the model analyzed above is based on a rigorous idealization—the notion of stationary competitive equilibria with rational expectations in a stochastic game—it is important to compare the predictions with actual behavior in a laboratory experiment. Our goal in this experiment is to test the main predictions of the model. The two central issues are whether participants' individual behavior coincides with the optimal policy derived in Theorem 2.1, and whether there is indeed an optimum quantity of money as assured by Corollary 3.1. This section presents the experiments and its results. A description of the experiment's design is followed by an outline of its specific implementation. Then, hypotheses are formulated that draw on the theoretical model and its findings. Finally, the results of the experiment are presented in detail.

4.1 Design

The goal of our experiment is to study a game in which a finite number of participants can repeatedly buy or sell an abstract, perishable commodity in a central market. Since the experimental analysis of the interaction of a relatively small number of players in a game with private information, common knowledge about some distributions, endogenous execution probabilities of subjects' contractual positions, and potential non-stationarity is quite an ambitious goal, the experiment is constructed as a sequence of increasingly more difficult treatments.

Participants first tackle individual decision problems, which are followed by two treatments with market interaction. In the first individual decision problem, discounting is implemented, but every demand for the abstract commodity is satisfied, i.e. no rationing (Game N). In the second decision problem, discounting and rationing are implemented (Game R). A chance move determines whether the player is rationed, i.e. whether the contractual buy/sell position is not satisfied. Both market games have discounting and endogenous rationing, but differ in the probability distribution of the participants' time values. In the first market game (Game M.5), the high time value occurred with 50% probability, while, in the second market game (Game M.2), the high time value had a 20% probability. The same group of participants interacts in all market games.

The actual play of these four games constitutes the learning phase. A strategy game is then played to extract the participants' (stationary) strategies. In this game, the participants have to specify in writing their strategies for each game they played in the learning phase. The specified strategies of all members of one group are then implemented and simulated on a computer. The participants' success (in terms of accrued time values) in all games—actual and simulated—is rewarded in Swiss currency. Realized time values have ten times more weight in the strategy game than in the actual game.

The following list summarizes the sequence of the treatments:

1. Actual Play
   (a) Individual Games
      i. Game N (no rationing)
      ii. Game R (rationing)
   (b) Market Games
      i. Game M.5 (high time value with 50% probability)
      ii. Game M.2 (high time value with 20% probability)
2. Strategy Game for the Games N, R, M.5 and M.2

4.2 Implementation

This section explains in detail how the model is operationalized to allow for an experimental test in the laboratory. The number of participants in a market is finite rather than infinite, as postulated in the model, with six subjects interacting in each market game. Potential monetary payoffs correspond to time values. Discounting is implemented as the probability of the continuation of the game after the current period. A detailed description of the two treatment types, individual decision and market interaction, follows. The reader may also consult the instructions given to the participants, see www.schenk-hoppe.net/babysitting.html.

Figure 1 is a schematic presentation of the games. In the individual decision problems, rationing is determined exogenously by a random draw, while it is endogenous in the market games. Rationing in the market games is implemented as a random draw that selects who can trade out of those agents on the longer side of the market.

Individual Games

Two games have been implemented to analyze the individual behavior in a stationary situation. It is instructive to think of these games as trading in a market with perfectly stable conditions, such as never-changing rationing probabilities. The following describes how Figure 1 is to be interpreted in the individual decision games in which the market only has a 'shadow existence.'

Every period follows the same sequence of events and decisions. At the beginning of each period, a participant's current potential payoff w_t is determined by a random draw that is independent and identically distributed (iid). The time values are $l = 5$ and $h = 10$. The probability of the two values is set to $p_h = p_l = 0.5$. After observing the time value, the subject has to choose one of the three alternatives S (sell), B (buy), or I (stay idle). Alternative B is only available if money holdings are positive. A random draw determines whether the choice is executed (and leads to a transaction) or the subject fails to trade. In the first individual game (Game N = no rationing), the success probabilities are set to $p_B = p_S = 1$, i.e. subjects are never rationed and each choice leads to a transaction. The second individual game (Game R = rationing) has success probabilities $p_B = p_S = 0.8$, i.e. rationing takes place and each 'buy' or 'sell' choice is only executed with 80% probability. According to the outcome of the

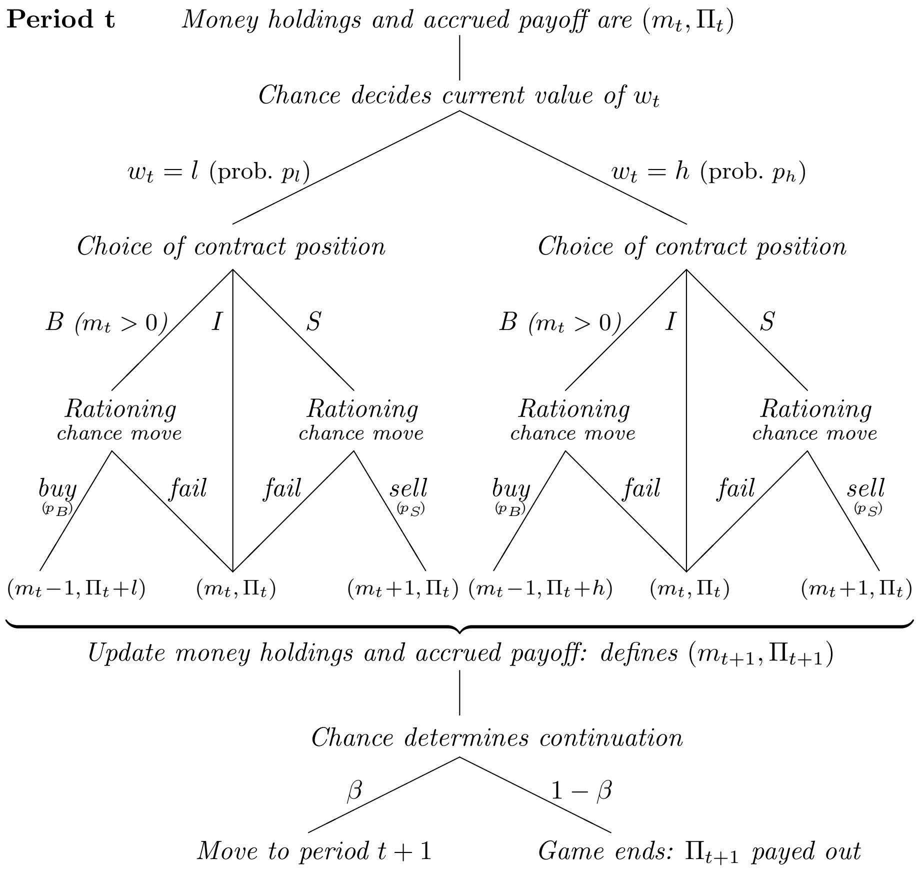

Figure 1: Schematic presentation of the games N, R, M.5 and M.2. Rationing is exogenous in individual decision games N and R, while it is endogenous in the market games M.5 and M.2. Subjects' initial money holdings m_0 coincide with the quantity of money, and are exogenous. The payoff account has a balance of zero at the start of the game, $\Pi_0 = 0$.

rationing process, money holdings and accumulated payoffs are updated. If S is chosen and the subject is successful, money holdings increase by one. The successful choice of B decreases money holdings by one and adds the current time value to the payoff account. Nothing happens if I is chosen, or the subject is singled out not to trade by the rationing mechanism. At the end of the period, a random draw determines whether the game is finished or not. The probability of continuation is set to $\beta = 0.95$. If the game is finished, the accumulated payoffs are paid out as cash, and the current money holdings become worthless.

Market Games

The behavior of participants interacting in a centralized market was analyzed in two games with six subjects per market. The order of events within each period was analogous to the individual games. This is illustrated in Figure 1. The random draw, determining current potential payoffs for each participant, was iid across time and subjects. The high and low time values, and the continuation probability, were the same as in the individual games.

The main difference to the above game is that the success probabilities p_S and p_B depended on the choices of *all* participants. In each period, the longer side of the market was rationed according to an iid random draw, so that only as many participants were selected to trade on the long side as there were subjects on the short side. In the two market games, we considered different probability distributions of time values while retaining the potential outcomes. In the first market game (Game M.5), the probability of the high value was $p_h = 0.5$; and, in the second market game (Game M.2), we set $p_h = .2$.

A central issue in these market games arises from the fact that only six participants interact in a market. This implementation stretches the theory to quite an extreme, as rationing probabilities vary considerably, even if all participants follow stationary strategies. The lack of stable market conditions is attributable to the fact that the average of the assigned time values very often does not match the expected value. For instance, if $p_h = 0.5$, then, in almost 11% of all cases, at least five participants were assigned the low value. This effect is further amplified by the induced variation of the distribution of money holdings. In any finite market, this distribution cannot be strictly stationary.

If the two market experiments are successful despite this 'noise', the support for the model is even stronger. The extent of this distortion for the individually optimal decisions in the market games can be gauged by comparison with the benchmark provided by Game R, in which the rationing probabilities are exogenously fixed.

All treatments of the games followed the same procedure. This structure, and all parameter values, were listed in the instructions and, therefore, were common knowledge to the participants. Each game was played for 20 periods without discounting. Then discounting was implemented. The game was terminated after 100 periods if it did not end before. The continuation probability is $\beta = 0.95$ (i.e. the break–off probability is 5% in each period). It was implemented by throwing a pair of identical and independent 10-sided dice. The game ended if both faces matched each other with 1, 2, 3, 4, or 5. All participants were given identical initial endowments M of coupons at the beginning of each game. For each game, there were three treatments, with $M = 1$, $M = 3$ and $M = 8$, respectively. In Game N and Game R, the time values were determined by flipping a coin, while this was computerized in Game M.5 and Game M.2.

The experiment was conducted with 36 students in the computer laboratory of the Institute for Empirical Research in Economics at the University of Zurich. All participants first played the four games on computer terminals (learning phase). After this learning phase, participants played the strategy games, in which the strategies for each game had to be detailed in writing. A form was supplied for the strategy game, but participants could alternatively use a blank sheet of paper. In the market games, each market comprised six participants who stayed together throughout the experiment. With a total of 36 participants, we had six independent observations of the market games.

Communication between participants was not allowed and computer terminals were separated from one another. Each game was played for about one hour, with instructions given in the first 15 minutes. The experiment lasted about four hours. The payment of each participant was based on individual performance. Each accrued unit of time value in the strategy game was worth 0.05 CHF ($\approx$ \$0.03). Performance in the learning phase was rewarded with 10% of this amount. Participants earned, on average, about 100.00 CHF ($\approx$ \$70).

4.3 Hypotheses

This section details the hypotheses that are tested in the experiment. Predictions derived from the theoretical model fall into two categories; individual behavior in a market and the outcome of the market interaction.

Optimal individual behavior in a market equilibrium is described in Theorem 2.1. The optimal strategy is stationary, depends only on current money holdings and time value (for given parameters), and has a cut-off feature. It can be summarized in three simple statements: (1) if an agent holds no money, he/she chooses action S (sell), regardless of the time value. For positive money holdings, the following two rules apply: (2) if the current time value is equal to the high value, the agent chooses action B (buy); and (3) if the current time value is equal to the low value, the agent chooses action S, as long as the money holdings are below a certain quantity $\overline{m}$. If the agent's money holdings are equal to or larger than $\overline{m}$, he/she chooses action B. The term *cut-off* is justified, as up to the threshold money holdings $\overline{m}$, a high time value implies 'buy' and a low time value implies 'sell' (except for zero money holdings, where there is no real choice). Money holdings beyond the threshold $\overline{m}$ carry too much risk of becoming worthless, and 'buy' is the right action even when the time value is low. The maximum amount of money an agent wants to accumulate is determined by the parameters of the model, cf. Theorem 2.1.

This prediction is summarized as follows:

Hypothesis 1 (Optimal Individual Policy)

Every subject follows cut-off strategies in all games.

The model provides precise information on the values of the maximum money holdings $\overline{m}$ for all games. A risk neutral subject, with a correct perception of the discount factor, has maximum holdings $\overline{m} = 4$ in Game N and $\overline{m} = 3$ in Game R. If rationing is assumed to be absent, then $\overline{m} = 4$ in Game M.5 and $\overline{m} = 2$ in Game M.2. These values are independent of the quantity of money and initial holdings. We used MATLAB scripts to numerically determine these values, and the scripts were also used in all further simulations. They are available on the web at www.schenk-hoppe.net/babysitting.html.

The second hypothesis concerns our assumption that agents play the field and do not engage in strategic reasonings on how to influence market averages. This assumption is central to the equilibrium notion in the model, Definition 3.1. Its implication is that the individual behavior described in the first hypothesis should be observed with or without market interaction. This assumption also underlies the result on the optimum quantity of money, Corollary 3.1.

Hypothesis 2 (Playing the Field)

Subjects take the market conditions as given and do not act strategically to influence rationing or the distribution of money.

This hypothesis would not be rejected if subjects follow cut-off strategies with identical $\overline{m}$ in Games R and M.5, and use a smaller $\overline{m}$ in Game M.2 than in Game M.5. In particular, an increase in the total money supply should not have an effect on the threshold $\overline{m}$.

The last hypothesis is concerned with the optimum quantity of money, which is interpreted as the quantity maximizing the average number of transactions in a market. Corollary 3.1 ensures its existence under the assumptions of our model. As the number of market participants is small, market conditions can vary quite substantially, even if all subjects follow stationary strategies. Subjects may, in addition, have different maximum money holdings, due to individual perceptions of the actual continuation probability β, or varying degrees of risk aversion. Another source of distortion could be the uniform initial distribution of money (i.e. helicopter money). It is, therefore, of interest to determine whether there are any systematic deviations from the theoretical forecast.

Hypothesis 3 (Optimum Quantity of Money)

In each market game, there is a quantity of money $M^\star$ which, if given as an initial endowment to the participants, maximizes the average number of trades.

The benchmark for the actual optimum quantity of money $M^\star$ is provided by our model. Computer simulations of Game M.5, with six agents following cut-off strategies where $\overline{m} = 4$, show that the average number of trades (as well as payoffs) is maximal for $M^* = 2$. One finds an average number of trades of 2.13 and an average payoff of 3.35. In Game M.2, where $\overline{m} = 2$, the optimum quantity of money is $M^* = 1$, with an average number of trades of 1.85 and an average payoff of 2.30. As reported in detail later, most subjects are willing to hold a higher amount of coupons in Game M.2 than is predicted by the model. We therefore simulate this with a maximum amount $\overline{m} = 3$, and find an optimum quantity of money of $M = 2$, an average number of trades of 2.08 and an average payoff of 2.57. Both averages are higher than with $\overline{m} = 2$ and $M^* = 1$. This deviation from full individual rationality leads to a Pareto improvement which benefits all members in the market.

4.4 Results

The experimental outcomes of the strategy game are analyzed to discuss the validity of the three hypotheses. The strategy game captured the behavior of experienced subjects who had completed the learning phase described above. In all treatments in this game, it was made explicitly clear that the game lasts at least 20 periods and at most 100 periods. The strategies specified by subjects were labeled with the periods of their validity. Almost all participants chose to supply strategies for three phases, corresponding to the beginning, middle and end part of the treatment (roughly equal on average to periods 1-20, 21-94, and 95-100). The following analysis focuses only on the (stationary) behavior during the middle phase.

Discussion of Hypothesis 1 (Optimal Individual Policy)

This hypothesis is fully backed up by the experiment. All 36 subjects in all of the games followed a cut-off strategy[6], though the maximum money holdings $\overline{m}$ varied substantially across participants within each game. Figures 2–5 provide a full description of the results of the individual decision games N and R and of the market games M.5 and M.2. We can conclude that our first prediction is not rejected by the data.

The maximum number of coupons $\overline{m}$ held by participants can be studied in more detail. The figures report the median of $\overline{m}$ across each group and show the theoretical benchmark. The median $\overline{m}$ over *all* participants is given by $\overline{m} = 4$ in the Games N, R, and M.5 for all initial endowments of coupons ($M = 1, 3, 8$), except for Game M.5 with $M = 1$, in which the median is $\overline{m} = 3.5$. In Game M.2, one has a median of $\overline{m} = 2.75$ for the initial endowment $M = 1$, and a

[6] The hypothesis that half of the population from which participants were drawn would pursue a different strategy was rejected in a one-sided binomial test on a significance level of 0.1%.

median of $\overline{m} = 3$ for the initial endowments $M = 3, 8$. These medians are in good overall agreement with the theoretical predictions, which are based on the assumption of risk-neutrality and correct perception of all probabilities. The considerable variation of the maximum amount $\overline{m}$ across individuals can, for instance, be due to a misperception of the continuation probability β. If a subject perceives the discount factor in Games N and M.5 as $.957 < \beta < .971$, then $\overline{m} = 5$; $.931 < \beta < .957$ yields $\overline{m} = 4$; and $.868 < \beta < .931$ gives $\overline{m} = 3$.

A remarkable, systematic deviation in the subjects' behavior was observed when rationing was quite common, which is the case in Game R and Game M.2. There was a clear tendency to hold more coupons at the maximum than predicted. Another interesting feature was the change of maximum holdings as a function of the endowment. While most subjects chose the same $\overline{m}$ in all treatments within a game, a few had increasing maximum holdings. One can anticipate that these properties have an effect on the optimum quantity of money. These points will be discussed in detail later.

Summarizing these findings, we can state that the experimental results on the maximum amount of money holdings are in good agreement with the theoretical model.

Discussion of Hypothesis 2 (Playing the Field)

The hypothesis that subjects do not engage in strategic reasonings, how to influence market averages and thus 'play the field,' is central to our model. Its validity can be studied by comparing the change in participants' behavior between games.

The first comparison has to be between the individual decision games and the market games. Only in the latter part can the participants influence aggregates. Table 1 summarizes the changes in maximum money holdings $\overline{m}$, with a perspective on the median behavior within groups. We have six independent observations, one for each group. The individual changes are tabulated in Figures 2–5.

Median of change of $\overline{m}$	N → R (initial M)			N → M.5 (initial M)			R → M.5 (initial M)			M.5 → M.2 (initial M)		
	1	3	8	1	3	8	1	3	8	1	3	8
positive	0	0	0	0	0	1	0	0	2	0	0	0
zero	5	6	6	4	5	3	4	4	3	1	0	0
negative	1	0	0	2	1	2	2	2	1	5	6	6

Table 1: Evaluation of the change in subjects' behavior by comparison of maximum money holdings across different strategy games (with the same initial money endowments). The table indicates the number of groups in which the median (taken within a group) of the changes of individual maximum money holdings $\overline{m}$ is positive, zero or negative.

Table 1 looks at the change of subjects' maximum holdings $\overline{m}$, and summarizes this information by reporting the direction of change of the maximum holdings' median within each group. It gives the number of groups in which the median of changes in individual's $\overline{m}$ is positive, zero or negative. There is obviously no clear-cut pattern in the observations, except for the comparison between Games M.5 and M.2.

The theoretical prediction states that there should be no change in the medians when moving from Game N to Game M.5 while holding the quantity of money fixed. In both cases, the median of actual maximum holdings and the theoretical prediction are $\overline{m} = 4$. Table 1 shows that the median of individual changes is zero in most groups. There is a tendency, however, for a decrease in the median within groups. If subjects were to act strategically, there should be a (strong) increase in the money holdings, in particular for $M = 8$.

Another important aspect is related to the change in behavior between the two market games, Games M.5 and M.2. If subjects were to try to positively influence averages, the maximum money holdings should not decrease, but rather increase, as this means that more of the benefits of the infrequent high time value can be reaped. Table 1 shows that the change in $\overline{m}$ from Game M.5 to Game M.2 is significant and negative. In all groups but one (and this only for $M = 8$), $\overline{m}$ decreases.[7]

Further evidence for hypothesis 2 can be drawn from a comparison of different money endowments within the same game. If subjects act in a strategic fashion, both market games M.5 and M.2 should see a strong increase in $\overline{m}$ as M increases from 1 to 8. Indeed, given the uniform initial distribution of coupons in order to have any trade, $\overline{m}$ must be higher than the endowment. Table 2 shows that nothing of this sort happens. In 44 out of 48 cases, the median of the changes is zero. The only exception is Game M.5, in which an increase in the quantity of money from 1 to 3 induces some subjects to increase $\overline{m}$. A further increase to $M = 8$ however, does not lead to a significant increase in maximum holdings.

The conclusion is that our second hypothesis is supported by the data.

Median of change of $\overline{m}$	Game N		Game R		Game M.5		Game M.2	
	$1 \rightarrow 3$	$3 \rightarrow 8$	$1 \rightarrow 3$	$3 \rightarrow 8$	$1 \rightarrow 3$	$3 \rightarrow 8$	$1 \rightarrow 3$	$3 \rightarrow 8$
positive	0	0	0	0	3	0	1	0
zero	6	6	6	6	3	6	5	6
negative	0	0	0	0	0	0	0	0

Table 2: Evaluation of the change in subjects' behavior by comparing changes in behavior for different money holdings within the same game. The table indicates the number of groups in which the median (taken within a group) of the changes of individual maximum money holdings $\overline{m}$ is positive, zero or negative.

Discussion of Hypothesis 3 (Optimum Quantity of Money)

[7]The hypothesis that half of the population would hold more money in Game M.2 than in Game M.5 is rejected in a one-sided binomial test at a significance level of 2% for the changes of the medians and at the 5% level for the medians of the changes.

The analysis of the third hypothesis is based on the strategies supplied by the participants for the market games M.5 and M.2 with $M = 1, 3, 8$, see Figures 4 and 5. To extract the average number of trades and realized payoffs for each of the six groups, numerical simulations were carried out for different quantities of money. The outcome of this simulation of the strategy game also determined the participants' monetary payoffs. In order to obtain comprehensive information on the effect of different quantities of money, we linearly interpolated individuals' maximum holdings $\overline{m}$ over non-retrieved initial money holdings by using the available data for $M = 1, 3, 8$.

Starting from a uniform initial distribution of money holdings M to participants (M is also the quantity of money), we simulated the market interaction for $50,000$ iterations, ignoring the break-off probability. From this data, we calculated the average number of trades in a single period, as well as the average time values realized by the participants. Figures 6 and 7 give the results.

In both games, increasing the quantity of money had a similar effect. The average number of transactions was positive for $M = 1$ and it increased when M was raised to 2. Then the frequency of transactions leveled out and decreased rapidly as M was raised further. Trade broke down completely for all groups in Game M.5 (M.2) if the quantity of money $M \geq 8$ ($M \geq 5$). The hypothesis that the number of transactions does not depend on the initial endowment is rejected at a 2% significance level in a binomial test for all markets. A comparison of the averages taken across all groups showed that the optimum quantity of money was $M^*_{experiment} = 2$ in both games. In Game M.2, the average number of transaction was actually slightly larger with $M = 3$, but the average payoffs proved to be about 10% higher with $M = 2$. The median payoff was also higher with $M = 2$.

In Game M.5, with $M = 2$, the strategy game gave rise to a realized trade frequency of 2.11, and a realized average payoff of 3.30. The theoretical benchmark, which assumes $\overline{m} = 4$ and $M^* = 2$ in Game M.5, was given by an average number of trades of 2.13 and an average payoff of 3.35. A comparison of these numbers shows that the monetary economy, determined by the strategy game, works surprisingly well. It almost achieves the benchmark. From a statistical viewpoint, 4 out of 6 observations conform with $M = 2$ being the optimum quantity of money in Game M.5. The hypothesis that the optimum quantity is not in the interval $[2, 3]$ can be rejected at a significance level of 5% in a binomial test. In group 1, the behavior of participant 3 stood out because of a very high maximum holding $\overline{m}$. This behavior stimulated trade within the respective group for high quantities of money.

The discussion of the optimum quantity of money in Game M.2 is slightly more complicated, due to a systematic deviation from the theoretical prediction on the individually optimal maximum money holdings. The outcome of the strategy game for Game M.2, with $M = 2$, gave rise to a frequency of trade of 1.58 and a realized average payoff of 2.06. The median of the maximum money holdings is calculated as $\overline{m} = 3$ (with linear interpolation).

The theoretical benchmark for Game M.2 is the stationary equilibrium with $\overline{m} = 2$ and an optimum quantity of money $M^* = 1$. The average number of trades is 1.85 and the average payoff is 2.31. As the median of actual maximum holdings for $M = 2$ is one unit higher than in this benchmark, we simulate the modified economy with $\overline{m} = 3$. In Game M.2, with homogenous $\overline{m} = 3$, the optimum quantity of money is $M = 2$. The corresponding average number of trades is 2.08 and the average payoff is 2.57. Both averages are higher than in Game M.2 with $\overline{m} = 2$ and $M^* = 1$. These numbers imply that the systematic deviation from full individual rationality leads to a Pareto improvement, which benefits all subjects within a group.

Compared to these benchmarks, both the actual frequency of trade and the realized payoffs fall short of the optimum. The actual frequency of trade 1.58 is 15% less than in the stationary equilibrium. The gap for the potential number of trades ($\overline{m} = 3$ and $M = 2$) is even larger, and subjects fall short by 24%. This gap is not surprising because of the heterogeneity in the subjects' maximum money holdings. On the one hand, this behavior gives rise to some inefficiency for the optimum quantity of money, but, on the other hand, it can stimulate trade within the respective group for high quantities of money. For instance, participants 31 and 35 in group 6 had very high maximum holdings for $M = 8$. Their holdings could compensate for the reluctance of others to hold money; though trade ceased after finitely many transactions for $M = 8$, since the sum of the maximum holdings was less than $49 (= 6 \times 8 + 1)$, the minimum permitting trade at $M = 8$.

Overall, there is a reasonable level of agreement between the prediction on the optimum quantity of money and the actual outcome in the laboratory experiment. The third hypothesis is well supported.

Summary of Discussion

Summarizing the experimental results, one can state that the appropriateness of our theoretical model is confirmed and the evidence is quite strong. The individual strategies of the participants coincide with the optimal behavior of rational agents, as predicted by the model. The concept of a stationary monetary market equilibrium, which assumes that agents "play the field," was also confirmed by the experiments. Even though each market consisted of only six participants, the median of their behavior conformed with the best response to market averages. Only in Game M.2, the market game with uneven distribution of potential payoffs and frequent rationing, did participants show a systematic tendency to hold one unit of money more than predicted. Money circulated well in the different markets and the average number of trades and realized time values were surprisingly high. In each market game, an optimum quantity of money could be determined.

5 Conclusion

Our paper provides a combined theoretical and empirical investigation of the optimum quantity of money. The backdrop to this analysis was provided by the anecdote about the Great Capitol Hill Baby Sitting Co-op. This co-op was modeled as a centralized market with idiosyncratic uncertainty, and money as a medium of exchange. Prices were fixed, and markets were cleared by rationing. We first showed the existence of stationary monetary equilibria and proved that there is an optimum quantity of money. The rational solution of our model, which is based on individual rationality, competitive behavior and rational expectations, was then compared with actual behavior in a laboratory experiment. The experimental results strongly supported the hypotheses on optimal individual behavior, competitive behavior and the optimum quantity of money, which were derived from the model. In essence, we showed that the Great Capitol Hill Baby Sitting Co-op is more than a mere anecdote for the optimum quantity of money. It can be modeled very exactly using advanced dynamic equilibrium concepts, and its predictions can be verified in controlled laboratory experiments.

One important feature of the Great Capitol Hill Baby Sitting Co-op is the fixed prices. This is a characteristic trait of trade circles in which fairness considerations play a role. The next natural step would be to consider markets with flexible prices. One might venture to claim that, in a model in which prices are flexible but sticky downwards, a reduction in the quantity of money will result in an efficiency loss (in the very short run). One application might be labor markets in which wages are sticky downwards (e.g. in Continental Europe). We hope that our paper stimulates further research in this direction.

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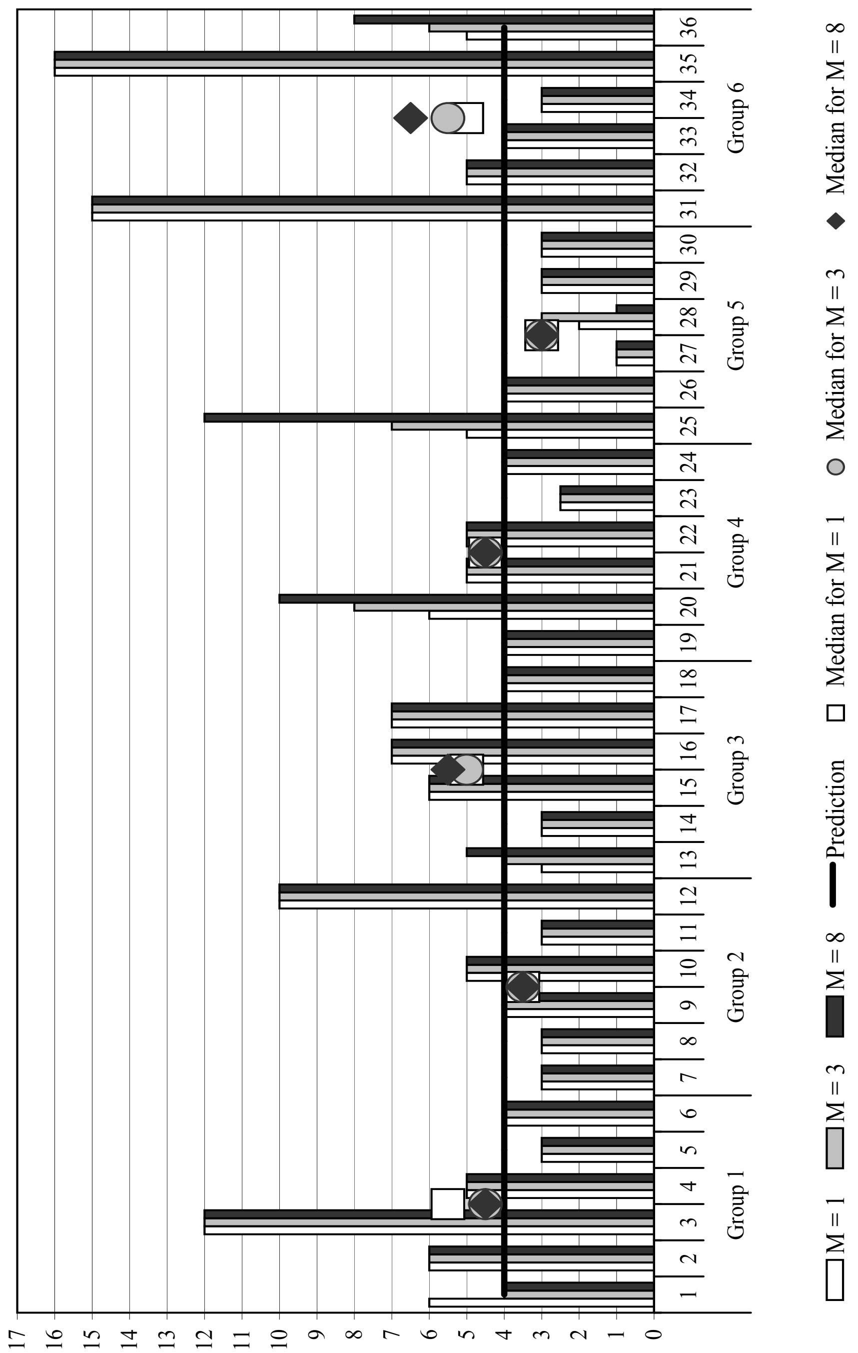

Figure 2: Game N (individual decision game). The figure reports individual maximum money holdings and medians across groups. These groups later formed markets, each with six subjects. All subjects (1-36) followed cut-off strategies. The predicted value was $\overline{m} = 4$. (Participant 23 was indifferent between 2 and 3, as maximum money holdings, and uniformly randomized between them.)

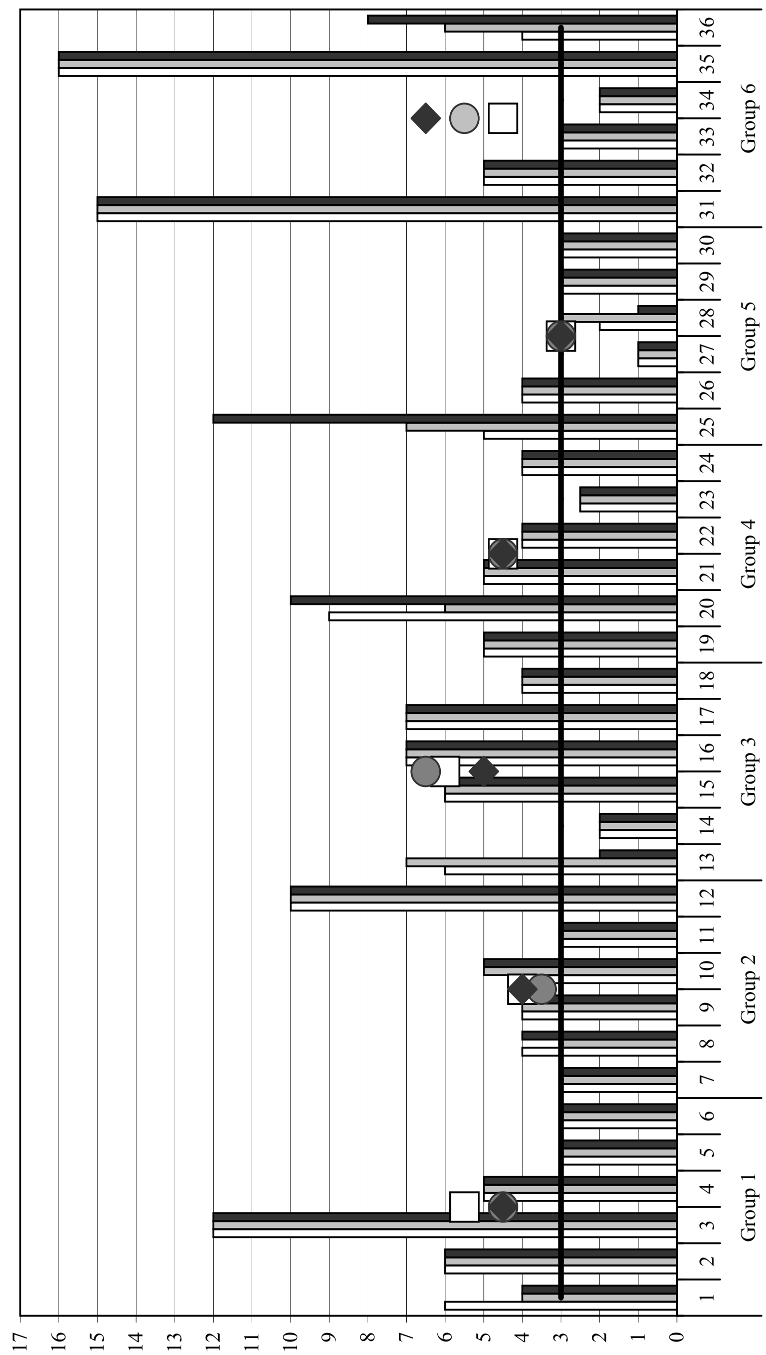

Figure 3: Game R (individual decision game). Results and legend are analogous to Figure 2. The predicted value was $\overline{m} = 3$.

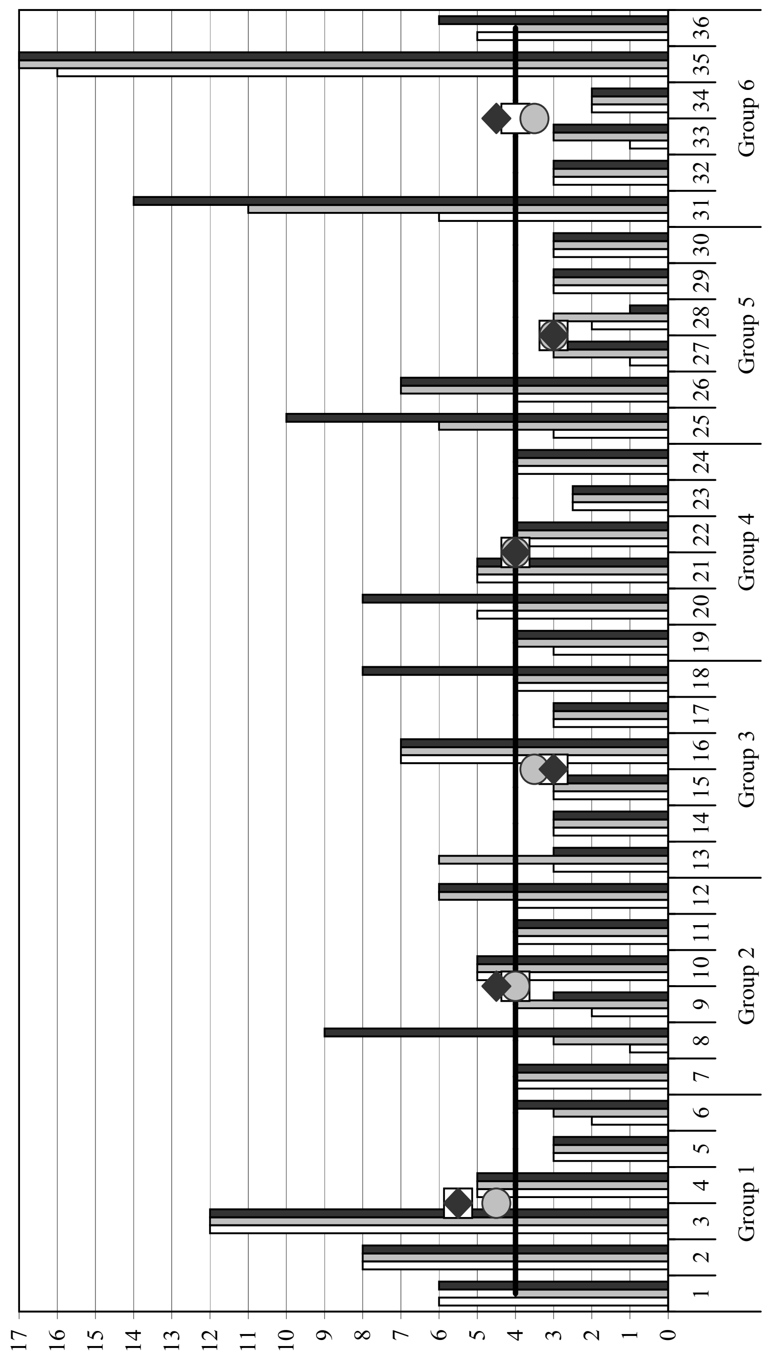

Figure 4: Game M.5 (market game). The predicted value for maximum money holdings was $\overline{m} = 4$. Legend as in Figure 2.

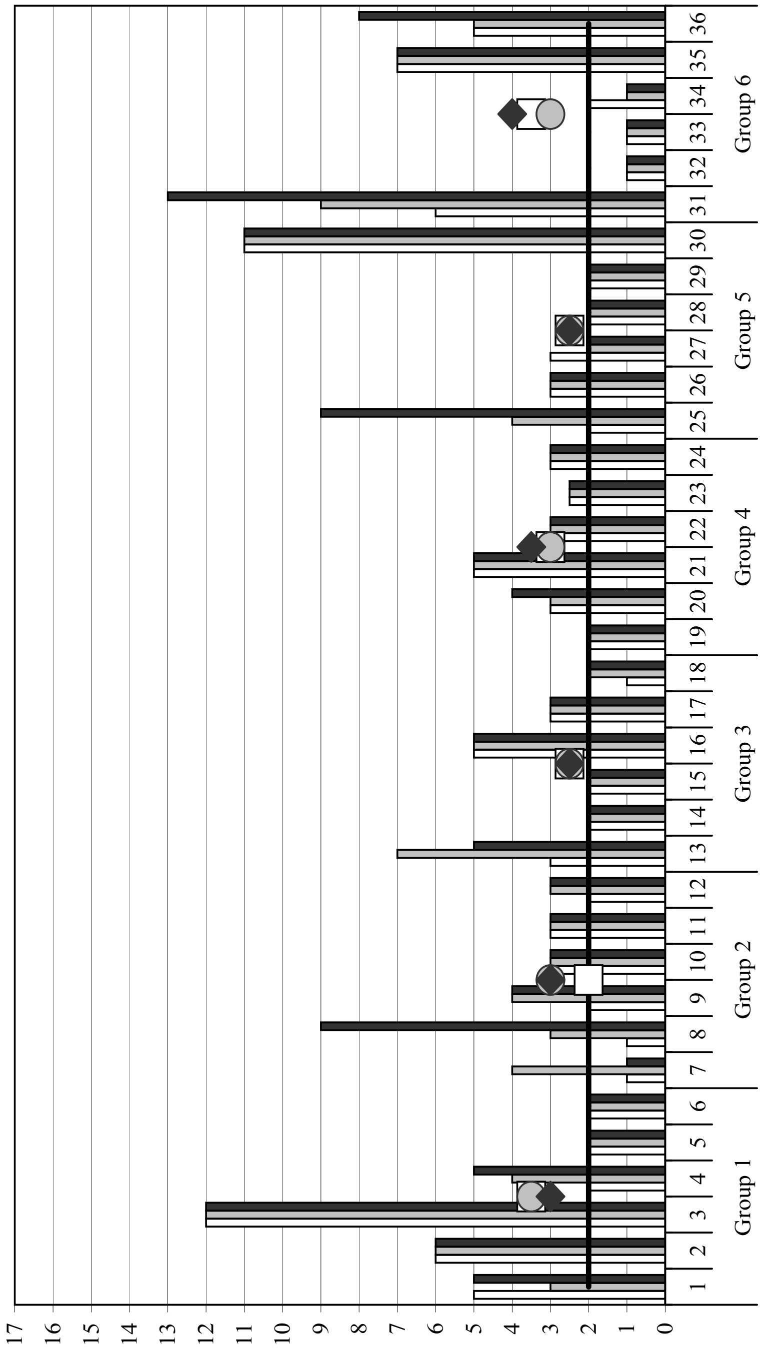

Figure 5: Game M.2 (market game). The predicted value for maximum money holdings was $\overline{m} = 2$. Legend as in Figure 2.

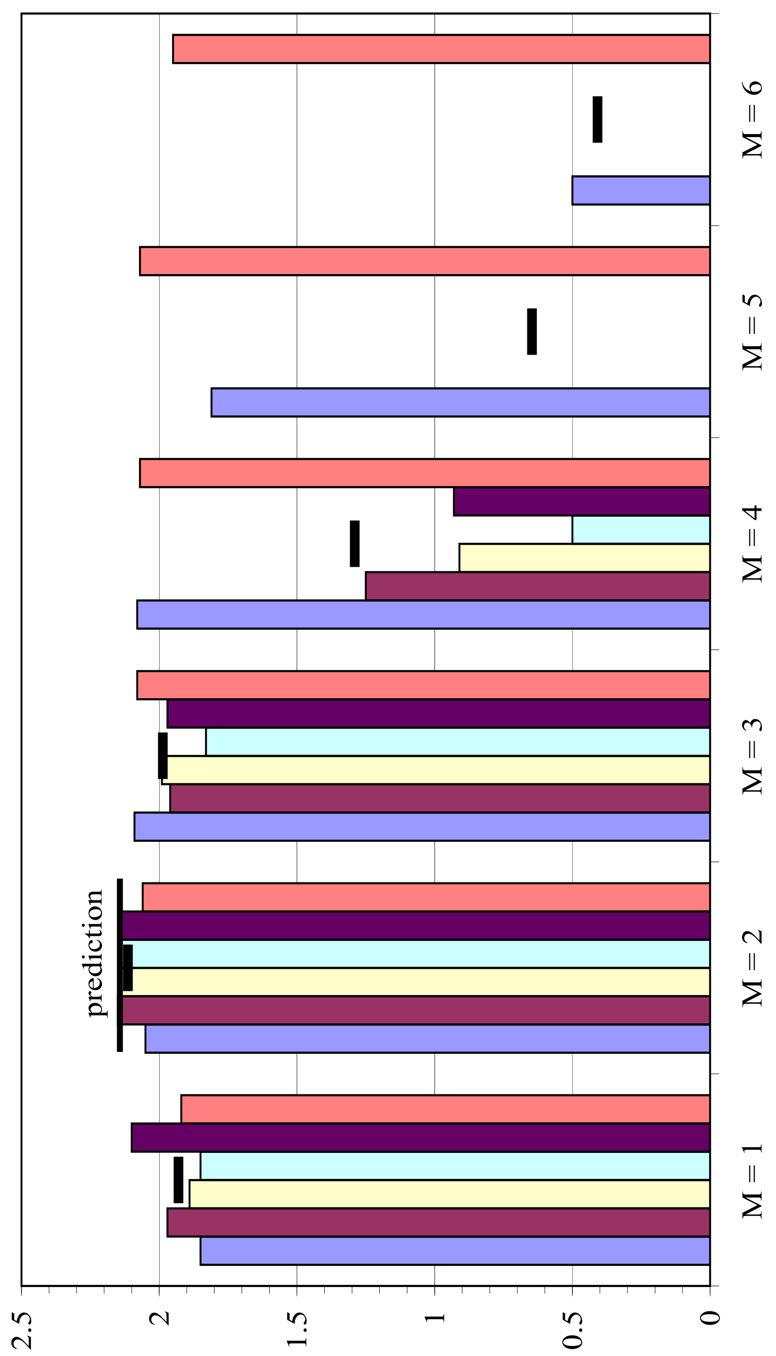

Figure 6: Optimum quantity of money, Game M.5. Average number of transactions for each subject and averages across groups for different quantities of money.

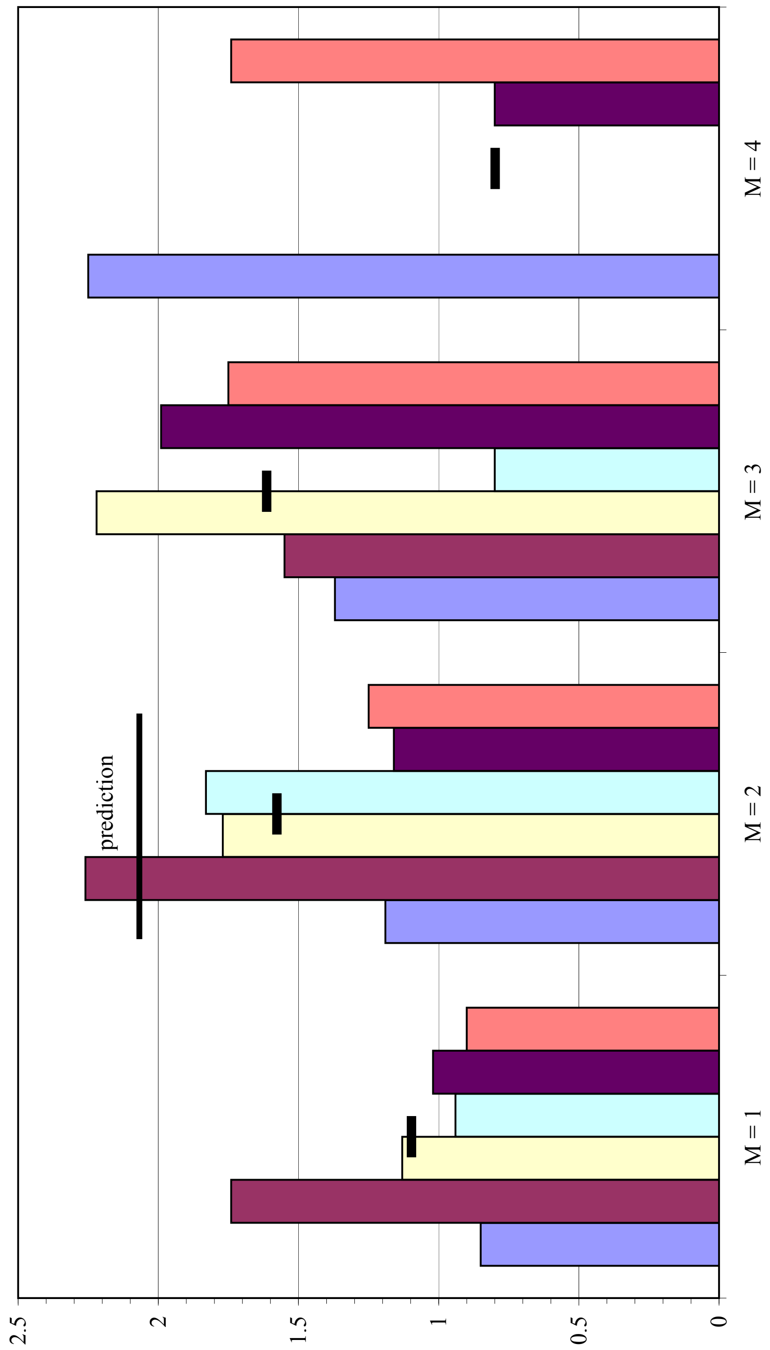

Figure 7: Optimum quantity of money, Game M.2. Average number of transactions for each subject and averages across groups for different quantities of money.