CS286r: Assignment 2

Fall 2010

Due: 5pm, Friday, October 22 in Ann Marie King's office at MD 133

Points will be awarded for clarity, correctness and completeness of the answers. Submissions should be brought to class. You may work in a pair and submit only one solution, but you must understand your solution. Pairs are asked to work on your solution independently. Please show your work whenever mathematical derivation is required. *Good luck!*

Total points = 105 points + 10 bonus points.

Problem 1: Peer Prediction Method (60 points)

(1) (20 points) Consider the original peer prediction method proposed by Miller, Resnick and Zeckhauser (2005). Suppose that N agents participate sequentially, and agent $i+1$ is the reference rater for agent i, except for the last two agents who report simultaneously and be each other's reference rater.

In this setting, is truth-telling a unique equilibrium? If yes, explain why it's unique. If not, discuss how the agents might choose among the multiple equilibria.

Suppose that you can persuade a strict subset of the participants to collude with you (so your coalition has at least 2 agents and at most $N-1$ agents including you). Can you come up with a collusion strategy to benefit your coalition? Does your strategy depend on how many agents agree to collude with you?

If the mechanism asks for simultaneous reports and randomly assign a reference rater for each agent, what will be your answers to the above questions?

(2) (20 points) Consider the enhanced peer prediction method described in the paper "Mechanisms for Making Crowds Truthful" by Jurca and Falting (2009). Jurca and Falting enhanced the basic peer prediction method to use more than one reference rater for each agent, and therefore was able to derive results on collusion resistant peer prediction methods with truth-telling as the unique Nash equilibrium. For this question, assume that utilities are not transferable.

Describe a mechanism proposed in the paper for which there are n_{col} colluders who can only employ symmetric strategies and the rest of the agents are truthful. You don't need to write out the mathematical formulation. Instead, please explain what the formulation does in English. In this setting, is truth-telling a unique equilibrium? If yes, explain why it's unique. If not, discuss how the agents might choose among the multiple equilibria.

Suppose that you can persuade $n \neq n_{col}$ agents to collude with you, while the mechanism believes that there are n_{col} colluders. Can you come up with a collusion strategy to benefit your coalition?

(3) (10 points) Consider the basic survey mechanism described right before Theorem 2 in the paper "Truthful Surveys". Consider only the setting in which agents submit their reports simultaneously and do not assume that there are trusted individuals.

In this setting, is truth-telling a unique equilibrium? If yes, explain why it's unique. If not, discuss how the agents might choose among the multiple equilibria.

Suppose that you can persuade a strict subset of the participants to collude with you (so your coalition has at least 2 agents and at most $N-1$ agents including you). Can you come up with a collusion strategy to benefit your coalition? Does your strategy depend on how many agents agree to collude with you?

(4) (10 points) Consider the Bayesian Truth Serum mechanism. Assume that agents submit their reports to the center simultaneously.

In this setting, is truth-telling a unique equilibrium? If yes, explain why it's unique. If not, discuss how the agents might choose among the multiple equilibria.

Suppose that you can persuade a strict subset of the participants to collude with you (so your coalition has at least 2 agents and at most $N-1$ agents including you). Can you come up with a collusion strategy to benefit your coalition? Does your strategy depend on how many agents agree to collude with you?

Problem 2: Market Scoring Rules (45 points + 10 bonus points)

(1) (10 points) Robin Hanson invented the market scoring rule market makers, which he described as a sequential shared proper scoring rule. For a discrete random variable v that has n mutually exclusive and exhaustive outcomes, the market maker uses a proper scoring rule $s_i(\vec{r})$, where $\vec{r}$ is a reported probability distribution over the outcome space. The market maker starts the market with some initial probability distribution $\vec{p}^0$. Traders arrive interacting with a proper scoring rule in sequence. When a trader reports a probability distribution $\vec{p}$, he gets rewarded based on his own report and must pay the previous trader based on the previous report. Finally, the market maker pays the last trader the score of the last report based on the proper scoring rule.

Given this setting, suppose that a risk neutral trader has a subjective probability distribution $\vec{p}'$ and that he is allowed to participate in the market only once. What probability distribution will the trader report in order to maximize his expected profit?

If the market probability before the trader's report is $\vec{p}$, using this strategy, what is the profit of the trader when the i-th outcome is realized in the future?

(2) (10 points) Alternatively, a market scoring rule market maker can be run by letting traders trade contracts. For each outcome i ($1 \leq i \leq n$), a contract is defined which pays off \$1 per share if the i-th outcome is realized in the future. $\vec{q}$ is the quantity vector representing the total number of shares for each contract that has been purchased by all traders. The market maker uses a cost function $C(\vec{q})$ to compute transaction cost for trades. The market instantaneous price vector is $\vec{p} = \nabla C(\vec{q})$. The market maker starts the market with some initial quantity vector $\vec{q}^0$ corresponding to an instantaneous price vector $\vec{p}^0$. When a trader comes and wants to change the quantity vector from the current $\vec{q}^{t-1}$ to $\vec{q}^t$, he obtains $q_i^t - q_i^{t-1}$ shares of the contract for outcome i and must pay the market maker the amount $C(\vec{q}^t) - C(\vec{q}^{t-1})$.

Given this setting, suppose that a risk neutral trader has a subjective probability distribution $\vec{p}'$ and that he is allowed to participate in the market only once. What strategy will a risk neutral trader use in order maximize his expected profit? (Short sale is allowed.) If the market price before the trader's report is $\vec{p}$, using this strategy, what is the profit of the trader when the i-th outcome is realized in the future?

(3) (5 points) The two above interpretations of the market scoring rule market makers can be thought as equivalent from a risk neutral trader's perspective. A necessary condition for this equivalence is that a risk neutral trader with any subjective probability distribution receives the same profit in both settings not matter which outcome is realized in the future. Derive an equation to represent this necessary condition and explain your reasoning.

(4) (Optional – 10 points) Derive the cost and instantaneous price functions for the logarithmic market scoring rule. Show your work.

(5) (10 points) Consider the convex function $f(\vec{x}) = \sum_{i=1}^{n} x_i^2 - 1$. Derive the corresponding proper scoring rule, whose expected score function is f. Suppose that we run a market scoring rule market maker

mechanism using this scoring rule. If the market maker starts the market with the price vector $\vec{p}^0$ where p_i^0 is the initial market probability (instantaneous price) for the i-th outcome, then what is the worst case loss of the market maker? How can the market maker minimize this worst case loss by setting the initial price vector?

(6) (10 points) Consider the negative entropy function $f(\vec{x}) = \sum_{i=1}^{n} x_i \log(x_i)$. Derive the corresponding proper scoring rule whose expected score function is f. Suppose that we run a market scoring rule market maker mechanism using this scoring rule. If the market maker starts the market with the price vector $\vec{p}^0$ where p_i^0 is the initial market probability (instantaneous price) for the i-th outcome, then what is the worst case loss of the market maker? How can the market maker minimize this worst case loss by setting the initial price vector?