Uncertainty, Risk, & Information

- Uncertainty

 or 

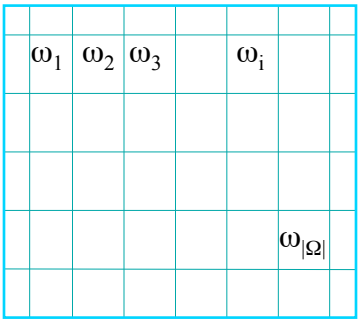
- Risk

Pr() Pr()

- Information

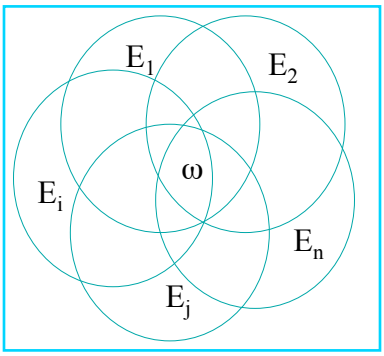
Pr( | info) Pr( | info)

Uncertainty & Risk, in General



- Ω: State Space
- ω are disjoint exhaustive *states of the world*
- ω_j: rain tomorrow & have umbrella & ...
- $\Pr(\omega)$

Uncertainty & Risk, in General



Alternatively,

- Overlapping events
  - E1: rain tomorrow
  - E2: have umbrella
- $|\Omega|=2^n$

Preference and Utility

- Preference

- Utility, $u(\omega)$

u()=10 >

u()=8 >

u()=-4 >

u()=-10

Decision Making Under Uncertainty

- Maximize expected utility
  - $E[u] = \Sigma_\omega \Pr(\omega) u(\omega)$
- Decisions (actions) can affect $\Pr(\omega)$ or $u(\omega)$

					E[u]
Don't Take umbrella	0.5	0	0	0.5	.5*10+.5*(-10) =0
Take umbrella (but I may leave it at the library)	0.25	0.25	0.25	0.25	.25*10+.25*8+.25*(-4)+.25*(-10) =1

Should take umbrella!

Utility of Money and Risk Attitude

- Outcomes are $
- Risk attitude:
  - risk neutral: $u(x) \sim x$
  - risk averse (typical):
    u concave ($u''(x) < 0$ for all x), e.g. $u(x)=\log(x)$
  - risk prone: u convex
- Absolute risk aversion:

$$r_u(x) = -u''(x) / u'(x)$$

Risk Attitude & Hedging

- I'm risk averse, u(x) = log (x), insurance company A is risk neutral, u(x)=x.
- I believe that my car might be stolen with prob. 0.01

ω_1: car stolen	ω_2: car not stolen
$u(\omega_1) = \log(10{,}000)$	$u(\omega_2) = \log(20{,}000)$

E[u]=.01 (4)+.99 (4.3) = 4.2980

- I buy $10,000 insurance for $125

$u(\omega_1) = \log(19{,}875)$	$u(\omega_2) = \log(19{,}875)$

E[u]=.01 (4.2983)+.99 (4.2983) = 4.2983

- Insurance company A also believes Pr(car stolen)=0.01

$u(\omega_1) = -9{,}875$	$u(\omega_2) = 125$

E[u]=.01 (-9875)+.99 (125) = 25 >0

I am happy to buy insurance. Insurance company A is happy to sell it. The transaction allocates risk.

Probability and Speculating

- Suppose that I'm also risk neutral, u(x)=x.
- But I think that the probability for my car being stolen is much higher than 0.01, say 0.1.
- A $10,000 car insurance is worth

  .1 (10,000)+.9 (0)=$1,000

  to me, but the insurance company only asks for $125. Too cheap!
- Buy the insurance, and I get $825 on expectation.

I am speculating the insurance company.

Risk-Neutral Probability

- Subjective probability: an agent's personal judgment
  - Always mixes with the agent's utility (risk attitude)
- Risk neutral probability: the probability that a risk neutral agent has to have the same expected utility

$$\Sigma_\omega \Pr^{RN}(\omega)\, u^{RN}(x_\omega) = \Sigma_\omega \Pr(\omega) u(x_\omega)$$

- Risk neutral probability is the normalized product of subjective probability and marginal utility

$$\Pr^{RN}(\omega) \sim \Pr(\omega) u'(x_\omega)$$