CS 286r · Fall 2011
Assignment One

due Friday, October 7th

1 Preamble

You can submit this homework as a pdf (strongly recommended) by writing mruberry@seas.harvard.edu and yiling@seas.harvard.edu before midnight EST on Friday, October 7th and attaching your answers. Alternatively, you can submit handwritten answers to MD 242 (TF's office) by 5PM on Friday, October 7th. Yes, seven additional hours available for using tex.

You may work in pairs and submit a joint problem set. Justify your answers.

There are two problem sets in this course. This one covers game theory and voting theory, and will be 12.5% of your grade. We will agree to reasonable deadline extensions if requested sufficiently in advance of Friday, October 7th. It is strongly recommended you review these questions early since the last available teaching fellow's office hours are a few days before the deadline.

2 Folk theorems and a finitely repeated game

Consider a stage game with payoffs as in Figure 1.

	L	M	R
U	0, 0	5, 0	3, 3
C	0, 5	4,4	0, 5
D	1, 1	5, 0	0, 0

Figure 1: Stage game payoffs.

1. What are the pure strategy Nash equilibrium of the stage game?

Now let's talk about subgame perfection.

2. Prove that in every subgame perfect equilibrium of any finitely repeated stage game a Nash equilibrium is played in the final game.

3. Prove that playing a Nash equilibrium in every stage game is a subgame perfect equilibrium of any game.

Now assume the stage game above is played twice, with the actions from the first game being observed before the second is played.

4. Show that there is a subgame perfect equilibrium of this game where (C, M) is played in the first round. Hint! Consider how the players coordinate on a Nash equilibrium in the second round.

3 Single-peaked preferences

Consider n voters distributed on the interval $[0, 1]$ and a voting rule mapping from $[0, 1]^n \to [0, 1]$. Voters prefer outcomes closer to their position on the interval.[1] Let the "i-position voting rule" select the ith leftmost reported preference as the rules outcome. For example with reported preferences $.1, .3$ and $.5$ the 0-position voting rule selects .1 (zero indexed because this is a CS course) and the 2-position voting rule selects .5.

1. Prove that i-position voting rules, for all i, are strategyproof, onto and nondictatorial.

2. Prove the voting rule that selects the median voter's reported position is the unique positional voting rule that selects the Condorcet winner. Assume the median is always well-defined.

4 Voting with Partial Information

The paper "A Maximum Likelihood Approach Towards Aggregating Partial Orders" describes a weighted majority graph. Use this paper's model and notation when answering the questions below.

1. Describe how to compute the expected weighted majority graph given a particular outcome is correct.

2. Consider a voting rule that reviews the current weighted majority graph and selects an outcome by choosing the one most likely to have generated the current graph. Formally describe how to compute this voting rule's winning outcome.

3. Is this voting rule equivalent to the Kemeny rule? Why or why not?

5 Fighting Manipulation

The authors elide several proofs in "Approximately Strategy-Proof Voting."

1. Help them out by proving Lemma 2.4.

[1] L_1 norm if you want to precise.

6 Voting Games

The paper "Stackelberg Voting Games: Computational Aspects and Paradoxes" describes a sequential voting game.

1. Assume voters have uncertainty about the preferences of others. Model this uncertainty as a probability distribution over their preferences (utility functions). Revise the paper's Algorithm 1 to handle this uncertainty. Do not worry about the complexity. Note: as the paper mentions, you will have to assume utility functions.

2. In this game, is there a unique perfect Bayesian equilibrium?

7 Practical Voting Rules

The paper "Practical Voting Rules with Partial Information" describes a method of iteratively querying voters' preferences. Let the number of alternatives be n.

1. If there is a Condorcet winner, at most how many queries are needed to find it?