Assignment Two

CS 286r · Fall 2011

due Friday, Nov. 18

Prologue

You may work in pairs. Justify your answers and **keep them to one page per question (that means two pages (double-sided) or fewer for the entire problem set).** Extensions available. Submit as pdf to the course tf (mruberry@seas.harvard.edu).

1 Transitive (Un-?)Trust

Let n be a set of players who in successive rounds $t_0, t_1, \ldots, t_\infty$ are matched and play a Prisoner's Dilemma game (see Figure 1). Players are matched at random with each other (assume if n is a finite number it is always even so everyone always has a partner) and have some common and common knowledge discount factor $\delta \in (0, 1)$. Players observe (and remember) the actions of games they play, but no others.

Assume other players (not you!) play a grim-trigger strategy. That is, if you ever play "fink" against them, they will forever after play "fink" against you.

a What is your expected value for always playing "cooperate?"

b What is your expected value for always playing "fink?"

c Informally (argue, but you do not need to prove), would it ever be interesting to mix these strategies?

d For a given δ, which strategy is preferable as $n \to \infty$?

Now assume all players can observe if another player has ever played "fink", and play a generalized grim trigger. That is, if you have never played "fink"

	Cooperate	Fink
Cooperate	2, 2	-3, 4
Fink	4, -3	0, 0

Figure 1: The Prisoner's Dilemma game.

other players will cooperate with you, but if you play "fink" everyone will play "fink" against you.

e For a given δ, which strategy (always cooperating or always finking) is preferable as $n \to \infty$?

Finally, assume now players can observe if another player has played "fink" in a local neighborhood, so playing "fink" will cause cause some constant number of other players to play "fink" against you forever.

f For a given δ, which strategy (always cooperating or always finking) is preferable as $n \to \infty$?

g Informally, do any of these models seem similar to transitive trust? Why or why not? Which seems most similar? What might be a better way to model transitive trust (without trust)? (There is no expected or single correct answer to this question.)

2 Envy-Free Cake Cutting

Assume three players are enacting the envy-free protocol.

a Regardless of how other players choose to cut the cake, will a player following the protocol receive an envy-free allocation?

b Is the first player always better off when the cake is trimmed?

3 Online Cake Cutting

Assume a player knows the utility functions of players arriving later, and the cake cutter is using online cut-and-choose.

a Describe the player's optimal (expected value maximizing) strategy. You do not need to describe how to compute it.

Now assume players have a discount factor $\delta \in (0, 1)$.

b Assuming players are expected value maximizers, prove a player can get MORE utility under this mechanism than the mechanism without a discount factor.

Let an "immediate allocation" mechanism give a piece to a person in the round they arrive, and assume the cake cutter knows the number of future players but not their arbitrary utility functions. The cake cutter MUST give a piece (some portion of the cake with positive length) to each person.

c Prove there is an immediate allocation mechanism that has the online envy freeness and online order monotonicity properties.

d Prove there is a truthful immediate allocation mechanism.

f Is there a truthful immediate allocation mechanism satisfying online envy freeness and online order monotinicity? Prove or disprove.

4 Truthful and Fair Cake Cutting

Let agents express piecewise constant utility over portions of the cake and let the cake be considered in segments, where each segment is the largest contiguous piece of the cake with a constant number of agents expressing constant positive utility over that piece. For example if three agents have preferences over the cake and the agents one and two like [0, .5] and agent three likes (.5, 1], then there are at least two segments, and maybe more depending on the utility functions of the agents.

Now we consider two schemes to assign pieces of the cake. The first uses a fixed order of agents (like two, three, one) and the latter uses random orders. Both split each segment into n portions, where n is the number of agents with positive utility for that piece. For each segment, they then create an ordering of these n agents. The fixed portion assignment mechanism orders the n agents with respect to the fixed ordering, while the random portion assignment scheme creates a random order for each segment. The leftmost portion of the segment is then assigned to the first agent in the ordering, the second from left to the second agent in the ordering .... Whatever an agent is assigned becomes their piece of the cake.

a Formally describe the set of segments or how to compute it.

b Is the fixed portion assignment mechanism (weakly) truthful? Prove it or give a counterexample.

c Is the random portion assignment mechanism (weakly) truthful? Prove it or give a counterexample.