Problem Set 2

CS 286r

1PM, Tuesday 10/23

Preamble

For Problem Set 2 you have two options and must complete one. You may do either the lit review described below or the problem set on the following page. If you do a lit review for this problem set you may not do the more in-depth lit review for your final, but must do a research project instead. You may work in pairs on the lit review or the problem set. If you are planning on doing a project it is recommended you do the lit review with your partner to start early, verify your interest, and craft a better project proposal.

Lit Review

Write a 4–5 page literature review on 3–6 related research papers. Ideally these papers will inform your project proposal and form the basis of your research paper’s related work. You should review the course website for recommendations on how to read a research paper before starting.

The literature review should begin by explaining the theme connecting the research papers, then summarize the salient points and differences of each paper, and conclude with the relevant open problems and challenges. You will be graded on the clarity of your exposition and how well you are able to craft a holistic perspective on the papers and their results. Think of this as an extended written presentation. Do not feel the need to spend equal time on each paper, or go into detail about each result, but spend your few pages smartly on the most relevant and important points. Minor details can be given passing reference.

Do not underestimate the space needed to introduce notation to state results formally. While every lit review is different, you will likely want to use clear English whenever possible and explain only the most relevant equations. If multiple papers all use slightly different notation you should think hard about how to unify, clarify, or reduce it.

Above all, do not leave things vague, imprecise or – worst of all – incorrect.
Problem Set Preamble

You may work in pairs and not discuss this problem set with anyone other than your (optional) partner. Turn in a single problem set for each pair.

All answers should be formal proofs unless an informal argument is specified. E.g., a question that asks for an informal answer may require a few lines of non-mathematical argument, but otherwise you should derive an answer formally and show your work.

Start early. Ask questions (email the course TF).

1 Cost Functions and Convex Optimization

This first set of questions refer to Abernethy et al.’s “Efficient Market Making via Convex Optimization, and a Connection to Online Learning.”

1. Prove that a continuous cost function satisfying Condition 3 must be convex.
2. Restate the “no arbitrage” condition as a constraint on the set of possible price vectors.
3. Prove the LMSR satisfies the “expressiveness” condition.

These questions refer to Dudik et al.’s “A tractable combinatorial market maker using constraint generation.” Consider a cost function using conjugate duality as described in the paper, but let $R(\mu) = 0$ everywhere, i.e. the cost function $C(\theta) = \sup_{\mu \in \mathbb{R}^d} \theta \cdot \mu$.

4. Is this cost function differentiable?
5. What is the worst-case loss of a market maker using this cost function?
6. Why are we interested in using non-trivial functions $R$? (Informally.)
7. Prove the growth of “market’s state” as described in Section 4 grows linearly with respect to the number of trades. (Hint: Be v. careful.)

2 Kelly Betting

Assume a trader is allowed to make even money bets with an unfair coin of known bias, but bets can only be made once a day and the trader has a daily discount factor $\delta \in (0, 1)$. Derive the “discounted Kelly” betting strategy. (Show your derivation.)
3 Separability

Following Ostrovsky’s assumptions (like the join consists of only singleton sets) in “Information Aggregation in Dynamic Markets with Strategic Traders,” prove that every distribution $P$ solving the equation in the definition of separability is uniform on all states it assigns positive probability to. (Hint: If there is no such distribution exists then the statement is vacuously true.)