A Bayesian Market Maker

Andrew Liu and Tianen Li

Discuss: Why do we love LMSR?

- What are its awesome properties?
  - Simple
  - Bounded worst-case loss
  - No arbitrage
    - Any trade can be a loser
  - Truthful information aggregation (separable securities)
    - Strict properness

Discuss: What are shortcomings of LMSR?

- MM frequently loses (why?)
- Price may not converge (when?)
- Loss-liquidity tradeoff (i.e. choosing correct *b*)

b and Loss-Liquidity Tradeoff

- Cost equation: $C(q) = b\ln(\sum_j e^{\frac{q_j}{b}})$
- Worst case loss: *b* ln *N*
- Higher *b* => higher worst-case loss, higher liquidity (why?)
- Why do we care about liquidity?
- How to address this tradeoff?

Solution 1: Liquidity-Sensitive LMSR

- Increase *b* in proportion to trade volume:

$$b(q) = \alpha \sum_j q_j$$

- Intuition: Start with low *b* to minimize loss; increase *b* later to raise liquidity.

Evaluating LS LMSR

- Solves loss-liquidity tradeoff
- Price still doesn't converge
- **New problem:** Higher *b* causes MM to be slow to adapt to true price shocks.
  - This sounds bad – why is it?
  - Is this a problem with classical LMSR?

Solution 2: Zero Profit MM

- Discuss: How have our previous MMs set prices?
  - MSRs – traders set price based on score
  - Cost function – traders set inventories
- Traders set prices. Is this bad?
- What if MM has own beliefs and Bayesian updates them based on trade information?

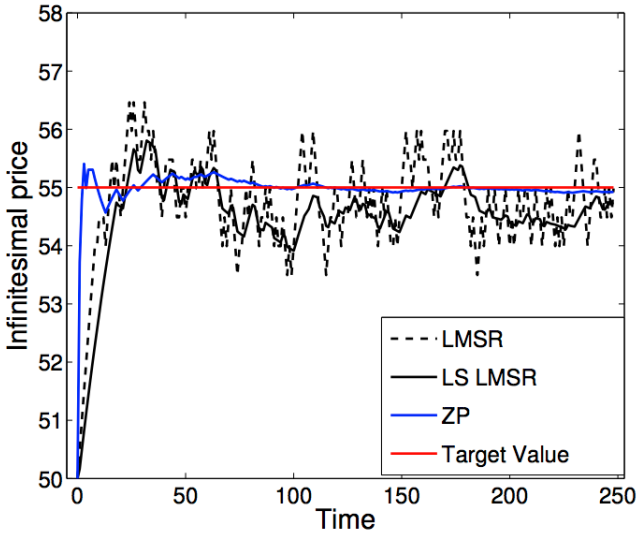
Solution 2: Zero Profit MM

- At each trade *t*,
  - MM has prior $P_t \sim N(\mu_t, \sigma_t^2)$ on true value V
  - Trader gets signal $s \sim N(V, \sigma_e^2)$
  - MM quotes spot price μ_t
  - Trader chooses buy/sell quantity
  - MM quotes bid/ask (center μ_t, spread $\propto \sigma_t/\sigma_e$)
  - Trader accepts/rejects
  - MM updates P_{t+1} based on accept/reject:

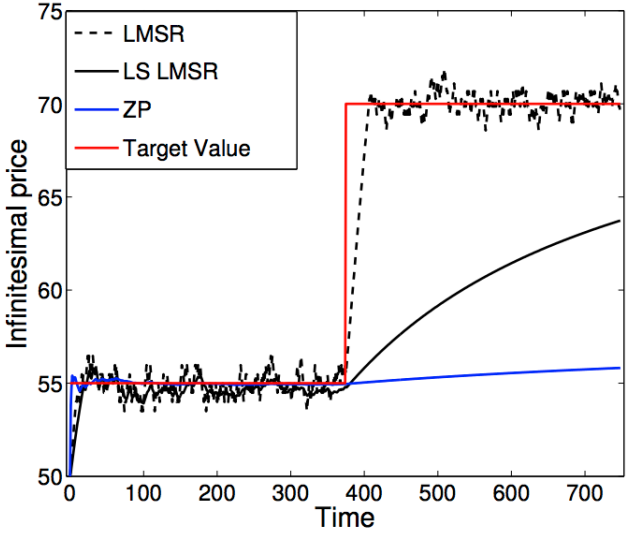
$$\mu_{t+1} = \mu_t + K(z_t)\sigma_t \qquad \sigma_{t+1}^2 = \sigma_t^2(1 - L(z_t))$$

Evaluating ZP

- Does price converge?
- How well does ZP adapt to shocks? Why?



(a) Behavior in a stable market.



(b) Adapting to a market shock.

A. Brahma, M. Chakraborty, S. Das, A. Lavoie, and M. Magdon-Ismail, ACM EC 2012.

Solution 3: Bayesian MM

- Modify ZP to increase variance when MM sees unlikely trades that imply change in true value.
- Likelihood function:

$$L(\mu,\sigma) = \int_{-\infty}^{\infty} N(v,\mu,\sigma) . \prod_{i=1}^{s} \left(\Phi(z_i^+, v, \sigma_\epsilon) - \Phi(z_i^-, v, \sigma_\epsilon) \right) dv$$

P(true value = v)

P(trade history | true value = v)

- Consistency index:

$$C(\textbf{history}) = L(\mu_t, 2\sigma_t) - L(\mu_t, \sigma_t)$$

- Double variance if it makes history more likely

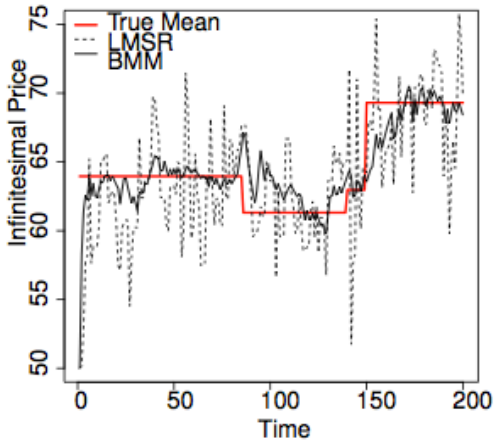
Evaluating BMM

- Does this solve?
  - Shocks
  - Price convergence
  - MM profitability

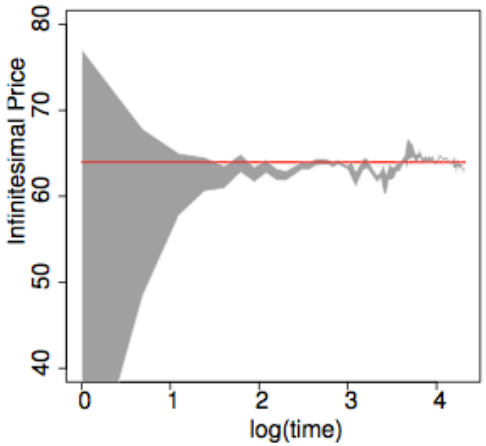
Test 1: Simulation

- Setup
  - Start with true value from initial distribution
  - At each trade, shock occurs with some probability
  - At each trade, traders arrive with beliefs of true value distributed normally around true value

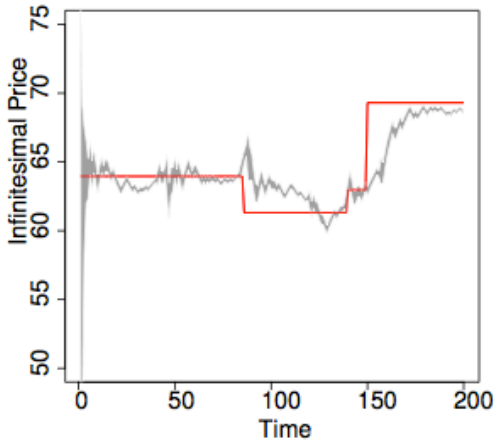
Test 1: Simulation Results



Spot price versus true market value.



Initial convergence of MM's beliefs (spread) shown by the width of the gray region (log scale).



Spread convergence for window size 10.

A. Brahma, M. Chakraborty, S. Das, A. Lavoie, and M. Magdon-Ismail, ACM EC 2012.

Test 1: Simulation Results

- Converges on price
- Adapts to shock
- What about profits?

Table I: Performance of BMM and LMSR in simulated trading

	Gaussian Shocks		Uniform Shocks	
	BMM	LMSR	BMM	LMSR
Profit	2081.35	−2457.30	603.40	−1897.98
Max Loss	9479.82	8662.32	50183.77	8384.42
Spread	1.42	1.35	1.79	1.40
RMSD	2.92	5.38	8.78	10.79

A. Brahma, M. Chakraborty, S. Das, A. Lavoie, and M. Magdon-Ismail, ACM EC 2012.

Test 2: Human Testing

- How might we test our MMs on live human trading?
  - Can we test MMs sequentially on same traders?
  - Can we test MMs on two different groups of traders?
  - How do we test two markets at once?

Solution: 2-d Random Walk

- Random walk on 2-d bounded grid
- 1 market predicts (# right hits / # LR wall hits); other predicts (# bottom hits / # TB wall hits)
- 1 market uses BMM; other uses LMSR
- Shocks on underlying walk probabilities occur

Test 2: Human Testing Results

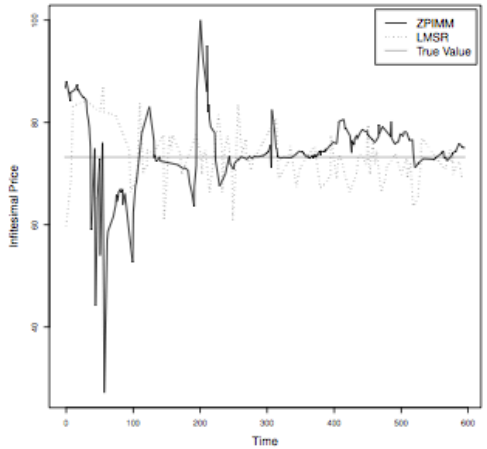
Table III: Summary Results of Human Subject Experiments

	Profit		Spread		RMSD		RMSDeq	
	LMSR	BMM	LMSR	BMM	LMSR	BMM	LMSR	BMM
Equilibrium(1)	-1350.12	47231.77	3.12	4.04	4.92	7.66	3.73	3.01
CommonInfoShock	-1510.89	8972.50	3.06	3.21	20.67	15.98	16.51	6.76
LimitedInformation	-1602.14	4083.95	1.61	0.49	14.43	2.15	14.56	0.93
Equilibrium(4)	-2619.07	-10588.86	1.81	0.95	21.67	23.13	14.82	17.05
Equilibrium(5)	-3168.55	9134.58	1.42	0.51	11.18	1.5	8.15	1
IndivInfoShock	-92.29	20226.44	1.89	0.89	8.87	6.47	6.88	6.6

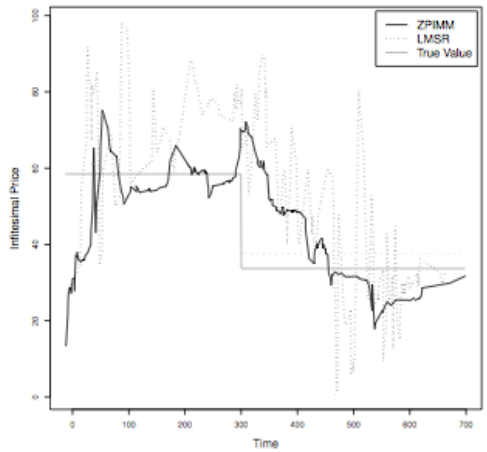
A. Brahma, M. Chakraborty, S. Das, A. Lavoie, and M. Magdon-Ismail, ACM EC 2012.

- Equilibrium 4???

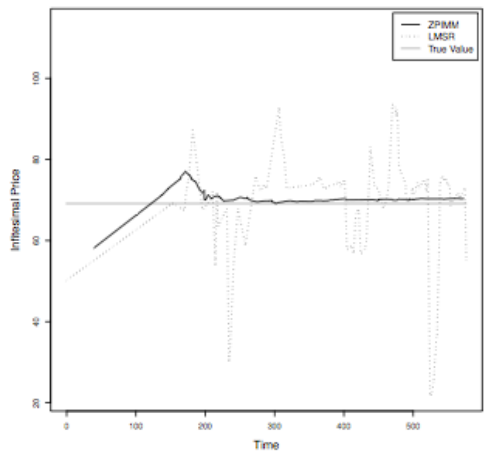
Test 2: Human Testing Results



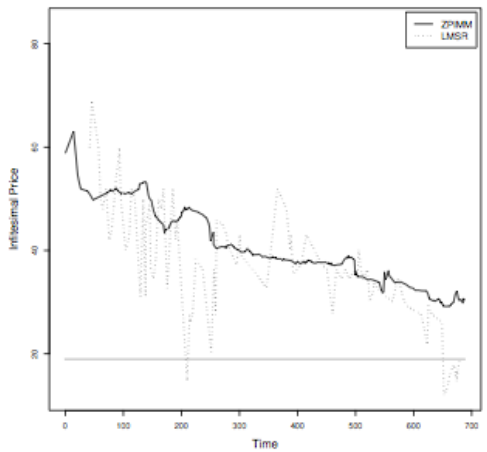
(a) Equilibrium(1)



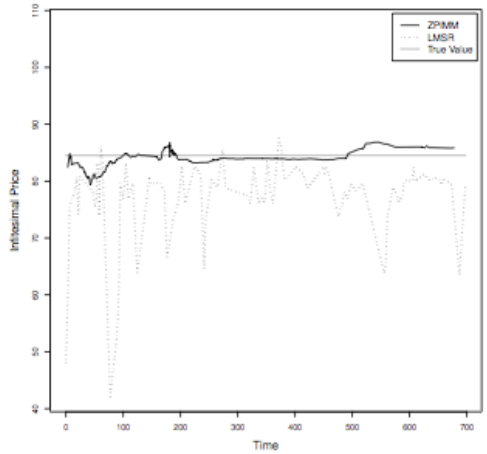
(b) CommonInfoShock



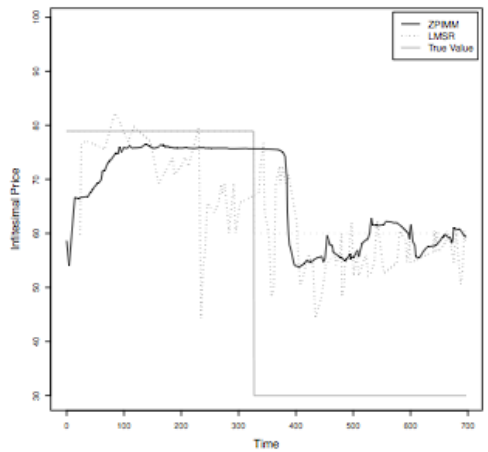
(c) LimitedInformation



(d) Equilibrium(4)



(e) Equilibrium(5)



(f) IndivInfoShock

What other problems does BMM face?

- Traders can mislead market (arbitrage?)
- Worst-case loss unbounded
- Assumptions – how to determine parameters?

Discuss: When should we use BMM?

- MM profits (generally)
- Price converges
- Adapts to shocks