Eliciting Informative Feedback: The Peer-Prediction Method

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The Setting

Camera Quality: dist $D$ over $\{L, H\}$

$S^1 \sim S$

$S^2 \sim S$

$S$: dist over $\{l, h\}$ jointly w/ $D$

Center:
- Ebay
- NetFlix
- Amazon

Rater 1

Report $r^1$

Rater 2

Report $r^2$

Example:

Pr[$S = h | D = H$] = 0.85
Pr[$S = l | D = H$] = 0.15
Pr[$S = h | D = L$] = 0.45
Pr[$S = l | D = L$] = 0.55
Pr[$D = H$] = 0.5
Pr[$D = L$] = 0.5
The Setting

**Camera Quality:**

\[ \text{dist } D \text{ over } \{L, H\} \]

**Task:**

How to make rational raters report honestly \((r^i = s^i)\)

Naive Attempt: reward \(\tau = \$1\) if \(r^1 = r^2\), \(\tau = \$0\) otherwise.

Problem: rater 1 will report “more likely signal” of rater 2.
The Setting

Camera Quality: dist $D$ over $\{L, H\}$

Task (formulation):
Reporting $r_i = s_i$ is a Nash Equilibrium

$\forall$ rater $i$, signal $m$, $x \neq m$

$\mathbb{E}_{s^2}[\tau_1(s^1, s^2)|s^1 = m] \geq \mathbb{E}_{s^2}[\tau_1(x, s^2)|s^1 = m]$

$\mathbb{E}_{s^1}[\tau_2(s^2, s^1)|s^2 = m] \geq \mathbb{E}_{s^1}[\tau_2(x, s^1)|s^2 = m]$
Solution using Proper Scoring Rule

Camera Quality:
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Task:
choose payoff $\tau_1$ such that
\[
\mathbb{E}_{s_2}[\tau_1(x, s^2)|s^1 = m]
\]
is maximized at $x = m$

Equivalently,
\[
\mathbb{E}_{z \sim Z}[\tau_1(x, z)]
\]
where $Z = s_2|s_1 = m$

Recall: $f$ is **Proper Scoring Rule** if
\[
\mathbb{E}_{z \sim Z}[f(P, z)]
\]
is maximized at $P = Z$

So let $\tau_1(x, z) = f(s_2|s_1=x, z)$
Issue: costs to the rater

In reality,

- Evaluating and reporting honestly incur a **relative cost** $c > 0$
  (e.g. testing is time consuming, opportunity cost, ...)

- Can be offset by **scaling up** the payoffs

- **Problems:**
  - Paying too much for truthful information?
  - What if $c$ is unknown? [PRGJ08]
Issue: costs to the center

To reduce center’s cost:

- **Budget balancing:**
  - pair up raters, make each pair a zero-sum game

- Use linear optimization to find cost-optimal payoff function [JF06]
Issue: risk aversion

- When the center knows the raters’ utility function(s)
- When the center does not know the utility function(s)
- Using multiple reference raters reduces risk
Issue: collusion

- Using an honest reference rater
- Randomized reference rater
- Outside experts...
Issue: Unknown/diff. priors

Camera Quality: dist $D$ over $\{L, H\}$

- In reality, raters have diff. beliefs about $(D, S)$, unknown to the center
- Addressed in [WP12] for binary signals $S$