CS286r Computational Mechanism Design
Homework 1: Game Theory

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Due: Thursday 2/14/2002, in the beginning of class. You may use any sources that you want, but you must cite the sources that you use. Teamwork is not allowed.

1. (10 pts) In the following strategic-form game, what strategies survive iterated elimination of strictly-dominated strategies? What are the pure-strategy Nash eq.?

	L	C	R
T	2,0	1,1	4,2
M	3,4	1,2	2,3
B	1,3	0,2	3,0

2. (10 pts) Agents 1 and 2 are bargaining over how to split a dollar. Each agent simultaneously name shares they would like to have, s_1 and s_2, where $0 \leq s_1, s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the agents receive the shares they named; if $s_1 + s_2 > 1$, then both agents receive zero. What are the pure strategy equilibria of this game?

3. (5 pts) Show that there are no (non-trivial) mixed-strategy Nash eq. (i.e. with support greater than one) in the Prisoners' Dilemma game.

Prisoners' Dilemma

	C	D
C	1,1	-1,2
D	2,-1	0,0

4. (5 pts) Solve for the mixed-strategy Nash eq. in the game in Problem 1.

5. (15 pts) *Battle of the Sexes.* Pat and Chris must choose to go for dinner or go to the movies. Both players would rather spend the evening together than apart, but Pat would rather the go for dinner, and Chris would rather they go to the movies.

		Chris	
		Dinner	Movie
Pat	Dinner	2,1	0,0
	Movie	0,0	1,2

(a) What are the two pure-strategy Nash equilibria?
(b) Let $(q, 1-q)$ be the mixed strategy in which Pat plays Dinner with prob. q, and let $(r, 1-r)$ be the mixed strategy in which Chris plays Dinner with prob. r. Determine the best-response correspondences $q^*(r)$ and $r^*(q)$, and use a similar graphical method to that in class for the "matching pennies" game to determine the mixed-strategy Nash equilibrium.

6. (10 pts) Prove that if strategies, $s^* = (s_1^*, \ldots, s_I^*)$, are a Nash eq. in a strategic-form game $G = \{S_1, \ldots, S_I; u_1, \ldots, u_I\}$, then they survive iterated elimination of strictly dominated strategies. **(hint)** By contradiction, assume that one of the strategies in the Nash eq. is eliminated by iterated elimination of strictly dominated strategies.

7. (15 pts) Prove that if the process of iterated elimination of strictly dominated strategies in game $G = \{S_1, \ldots, S_I; u_1, \ldots, u_I\}$ results in a *unique* strategy profile, $s^* = (s_1^*, \ldots, s_I^*)$, that this is a Nash eq. of the game. **(hint)** By contradiction, suppose there exists some agent i for which $s_i \neq s_i^*$ is preferred over s_i^*, and show a contradiction with the fact that s_i was eliminated.