CS286r Computational Mechanism Design
Homework 3: More Mechanism Design

Spring Term 2002
Prof. David Parkes
Division of Engineering and Applied Sciences
Harvard University

Feb 21, 2002

Due: Thursday 2/28/2002, in the beginning of class. You may use any sources that you want, but you must cite the sources that you use. Teamwork is not allowed.

1. Consider a generalized multi-cast routing problem, with a network $G = (N, E)$ (that may or may not have a tree structure). The server is located at node $\alpha_S \in N$, and cost $c_e \geq 0$ is incurred for sending data along edge $e \in E$. Users, $\mathcal{I}$, each with value $v_i \geq 0$, are located at nodes $\alpha_i \in N$. Define the *receiver set*, $R \subseteq \mathcal{I}$, as the set of users that will receive service. The *efficient* outcome, R^*, solves:

   $$\max_{R \subseteq \mathcal{I}} \sum_{i \in R} v_i - C(R)$$

   where $C(R) \geq 0$ is the cost of the minimal-cost tree connecting the server node, α_S, to all receiver nodes, $\{\alpha_i \ : \ i \in R\}$.

   (a) (15 pts) Consider that the users *and* the network are self-interested (i.e. rational) agents. Define the VCG mechanism for the problem. [**Hint:** the choice rule must define both the receiver set and the multi-cast tree, the transfer rule must define both the payment by each user and the payment to the network.]

   (b) (5 pts) We know that Groves mechanisms are strategyproof and efficient. Prove that the VCG mechanism is *ex post* individual-rational for both the users and the network.

   (c) (10 pts) Either prove that the VCG mechanism is *ex post* weak budget-balanced, or construct a simple counter-example for this multi-cast problem.

   Consider a modified mechanism, the *marginal cost* mechanism. The mechanism is unchanged from the VCG, except that the payment to the network by the mechanism is simply equal to its reported cost, $C(R^*)$, for providing service to receiver set R^*.

(d) (5 pts) Assume for the moment that the true costs of the network are already known to the mechanism (or equivalently, that the network is cooperative). Either prove that this marginal cost mechanism is *ex post* weak BB, or provide a simple counterexample.

(e) (5 pts) Is the marginal cost mechanism strategyproof for the network itself? Either prove, or provide a simple counterexample.

2. Consider a *double auction* (DA), with m buyers and n sellers, each trading a single item. Buyers and sellers submit bids and asks, and the DA determines the trade, and agents' payments. Let $b_1, \ldots, b_m$ denote the bid prices from buyers, and assume $b_1 \geq b_2 \geq \ldots \geq b_m \geq 0$. Let $s_1, \ldots, s_n$ denote the ask prices from sellers, and assume $0 \leq s_1 \leq s_2 \leq \ldots s_n$. In addition, define $b_{m+1} = 0$ and $s_{n+1} = \infty$. Later we refer to the following examples: (i) buyer values $9, 8, 7, 4$, seller values $2, 3, 4, 5$; (ii) buyer values $9, 8, 7, 4$, seller values $2, 3, 4, 12$.

   (a) (10 pts) Define the VCG mechanism to clear the auction, and show that the mechanism is not *ex post* weak BB for example (i). [**Hint:** it is useful to interpret a bid, or an ask, as an agent's claim about its value for the item. Define the trades implemented, payment by each buyer, payment to each seller.]

   Consider the following modified trading mechanism, the *McAfee-DA*:

   (1) select k, s.t. $b_k \geq s_k$ and $b_{k+1} < s_{k+1}$.

   (2) compute *candidate trading price*, $p_0 = 1/2(b_{k+1} + s_{k+1})$.

   (3) if $s_k \leq p_0 \leq b_k$, then the buyers/sellers from 1 to k trade at price p_0; otherwise, the buyers/sellers from 1 to $k-1$ trade, and each buyer pays b_k, each seller gets s_k.

   (b) (15 pts) Prove that the McAfee-DA is strategy-proof, and *ex post* weak budget-balanced.

   (c) (5 pts) Run the McAfee-DA on examples (i) and (ii). Is the DA efficient?

   (d) (10 pts) The McAfee-DA is vulnerable to *false-name bids*, where an agent submits an additional bid under another identity to influence the outcome. Provide an instance of successful manipulation with false-name bids in examples (i) and (ii).

3. Consider a special case of the multi-cast routing problem in Question 1, this time for a network with a *tree* structure, with the server at the root of the tree. We refer to the example in Figure 1.

   (a) (10 pts) Compute the outcome of the marginal-cost mechanism (def. in Q1) on this example (assuming that network costs are known), and demonstrate that the mechanism is not *group* strategyproof.

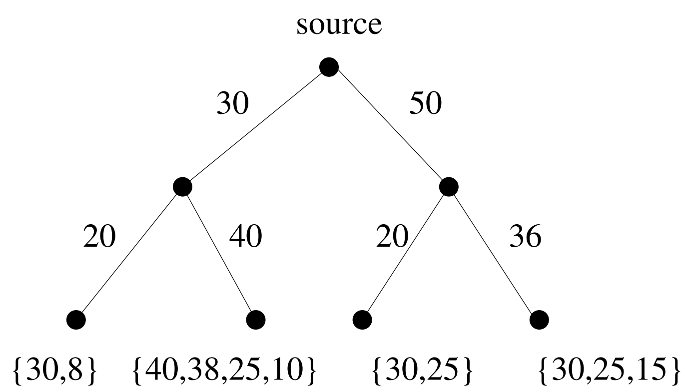

Figure 1: Multicast Network Problem

(b) (5 pts) Let $C(R)$ denote the total cost of the mimimal tree to provide service to receivers $R \subseteq \mathcal{I}$. Prove that $C(R)$ is *submodular*, s.t. $C(i \cup T) - C(T) \leq C(i \cup S) - C(S)$, for $S \subseteq T \subseteq \mathcal{I}$, and $i \notin S$.

The Shapley value for a submodular cost function, $C(R)$, defines a cross-monotonic cost-sharing method, $\xi_{\text{Shapley}}(R, i)$. In this multicast tree setting, the Shapley value has a simple interpretation: *the cost of an edge e in the multicast tree is shared equally by all receivers who are downstream of the edge.*

(c) (5 pts) Given this interpretation, prove that ξ_{Shapley} satisfies the cross-monotonicity property in the current setting; i.e. $\xi_{\text{Shapley}}(Q, i) \geq \xi_{\text{Shapley}}(R, i)$, for all $Q \subseteq R$.

Given a cross-monotonic cost-sharing method, ξ, we can define a mechanism, $\mathcal{M}(\xi)$, that is group strategyproof and *ex post* (strong) BB. Mechanism $\mathcal{M}(\xi)$ computes the receiver set, R^*, and then implements user payments, $\xi(R^*, i)$, and makes payment $C(R^*)$ to the network. $\mathcal{M}(\xi)$ is defined as:

(1) Agents report values, $\hat{v}$; initialize $R^* \leftarrow \mathcal{I}$.

(2) Select an agent $i \in R^*$ at random, if $\hat{v}_i < \xi(R^*, i)$ then drop i from R^*.

(3) Continue until $\hat{v}_i \geq \xi(R^*, i)$ for all $i \in R^*$.

(e) (10 pts) Use the cross-monotonicity of ξ to prove that the *order* with which agents are selected in $\mathcal{M}(\xi)$ does not change the outcome.

(f) (5 pts) Run the mechanism, $\mathcal{M}(\xi_{\text{Shapley}})$, on the example (assume that network costs are known), and compare the efficiency of the outcome with the VCG.

(g) (10 pts) Prove that $\mathcal{M}(\xi_{\text{Shapley}})$ is strategyproof.

(h) (15 pts) Provide a sketch of the proof that $\mathcal{M}(\xi)$ is *group* strategyproof. [**Hint:** First, consider an agent, x, that is not in the receiver set. Argue that such an agent can only improve the utility of other agents by reporting an inflated value, and then consider the effect on agent x itself. Second, consider an agent, y, that is currently in the receiver set. Argue that such an agent can only reduce the utility of other agents by misstating its value.]

4. **(Extra credit)** The Shapley value, $\xi_{\text{Shapley}}(R, i)$, is defined as:

$$\xi_{\text{Shapley}}(R, i) = \sum_{S \subseteq R_{-i}} \frac{|S|!(|R| - |S| - 1)!}{|R|!} [C(S \cup i) - C(S)]$$

where $R_{-i} = R \setminus i$ and $x!$ denotes the factorial. Note that, $\xi_{\text{Shapley}}(R, i) = 0$, for $i \neq R$. Derive the simple explanation of this quite forbidding formula in the multi-cast tree setting, i.e. *the cost of an edge e in the multicast tree is shared equally by all receivers who are downstream of the edge.*