Combinatorial Auctions

David C. Parkes

parkes@eecs.harvard.edu

Spring, 2002

Basics

Bids (p, S) for set $S \subseteq \mathcal{G}$ of items, "I only want A if I also get B". Auction mechanism determines the *allocation*, and agent *payments.*

Variants:

- Bidding languages:
  – OR-of-XORS, XOR-of-ORs, restricted bundles, etc.
- Iterative vs. Sealed-bid vs. Multi-round
- Linear, Non-linear, Non-anonymous ask prices
- First-price vs. Second-price (GVA)
- Reserve price

Application Domains

- **Take-off/landing slot auction** [RSB82].

  – Sealed bid, one-round, single-item prices, *secondary market* to clean up the outcome.

- **Chicago GSB course registration** [GSS93].

  – Multiple-round, combinatorial auction (limited expressivity). Computes linear item prices in each round. Fixed number of rounds.

- **Collaborative planning application**. Agents submit bids to perform combinations of sub-tasks, that are composed into an overall plan [HG00].

- **San Francisco Housing Auction**. Sealed-bid, constrained bids [Wired 2000].

FCC Spectrum Auction

- 51 Major Trading Areas (MTAs), 30 MHz spectrum per MTA. 492 Basic Trading Areas (BTAs), each with one 30 Mhz and four 10 MHz blocks.
  – $51 \times 1 + 492 \times (1 + 4) = 2511$ items
- Clear efficiencies to aggregating licenses.
  – fixed cost of infrastructure, marketing, roaming synergies, etc.
- Simultaneous ascending-price auction.
  – prices on items
  – complex *activity*, and *stopping* rules, *click-box* bidding.
- 1994–2001, more than $40 billion.

Auction 31: Combinatorial

- 700 MHz auction, 12 licenses (6 regions, 10 MHz and 20 MHz in each)
- Diverse preferences (30 MHz for high-speed data service; “fill holes”; build a “national footprint”).
- Limited number of bundles, XOR-across rounds, OR within round, stopping rules, activity rules.
- Still debated, and still not happened.

Multicast Cost-Sharing

David C. Parkes

parkes@eecs.harvard.edu

Spring, 2002

Multicast Cost-Sharing

[FPS01,JV01]

Network (N, E), cost $c_e \geq 0$ per-edge, $e \in E$, value v_i per-node, $i \in N$. Source node, $\alpha_S \in N$.

Task: Select receiver-set $R^* \subseteq N$, and multi-cast tree $T^* \subseteq E$, to maximize social welfare:

$$W(v) = \max_{R \subseteq N} \left[\sum_{i \in R} v_i - \min_{T \in T(R)} \sum_{e \in T} c_e \right] \qquad \text{[EFF]}$$

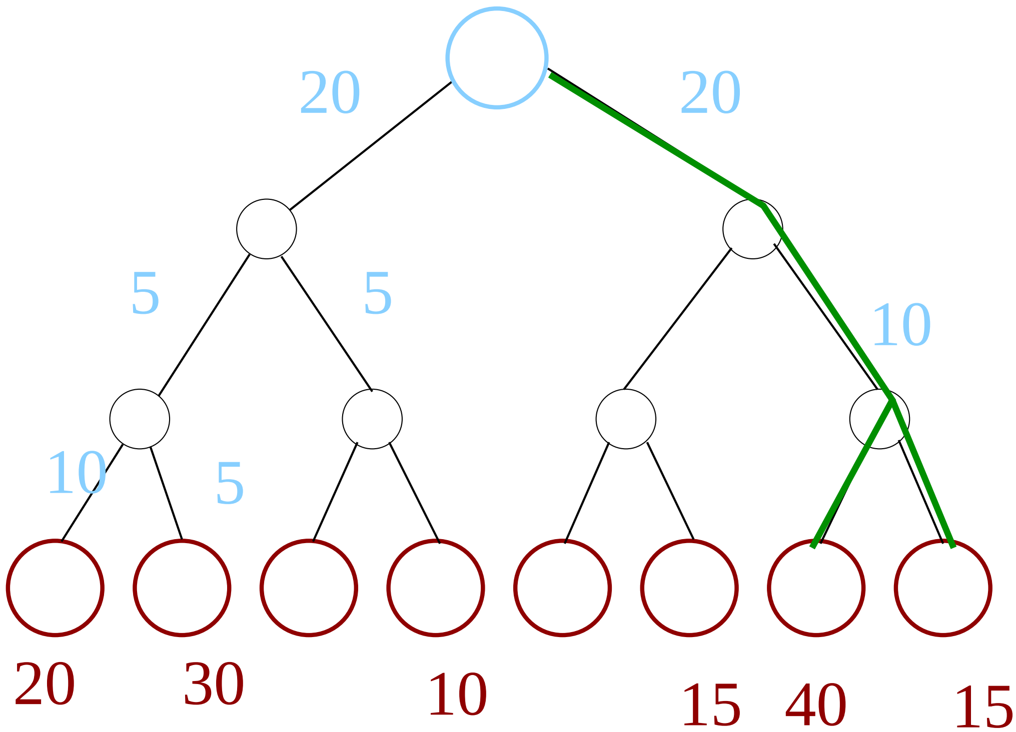
where $T(R)$ is the set of all trees that "touch" R.

Self-interested receivers, private information about values. Must collect payments to balance total cost to network.

Feigenbaum et al.: assume a Universal tree, propose *decentralized* algorithms to implement mechanisms.

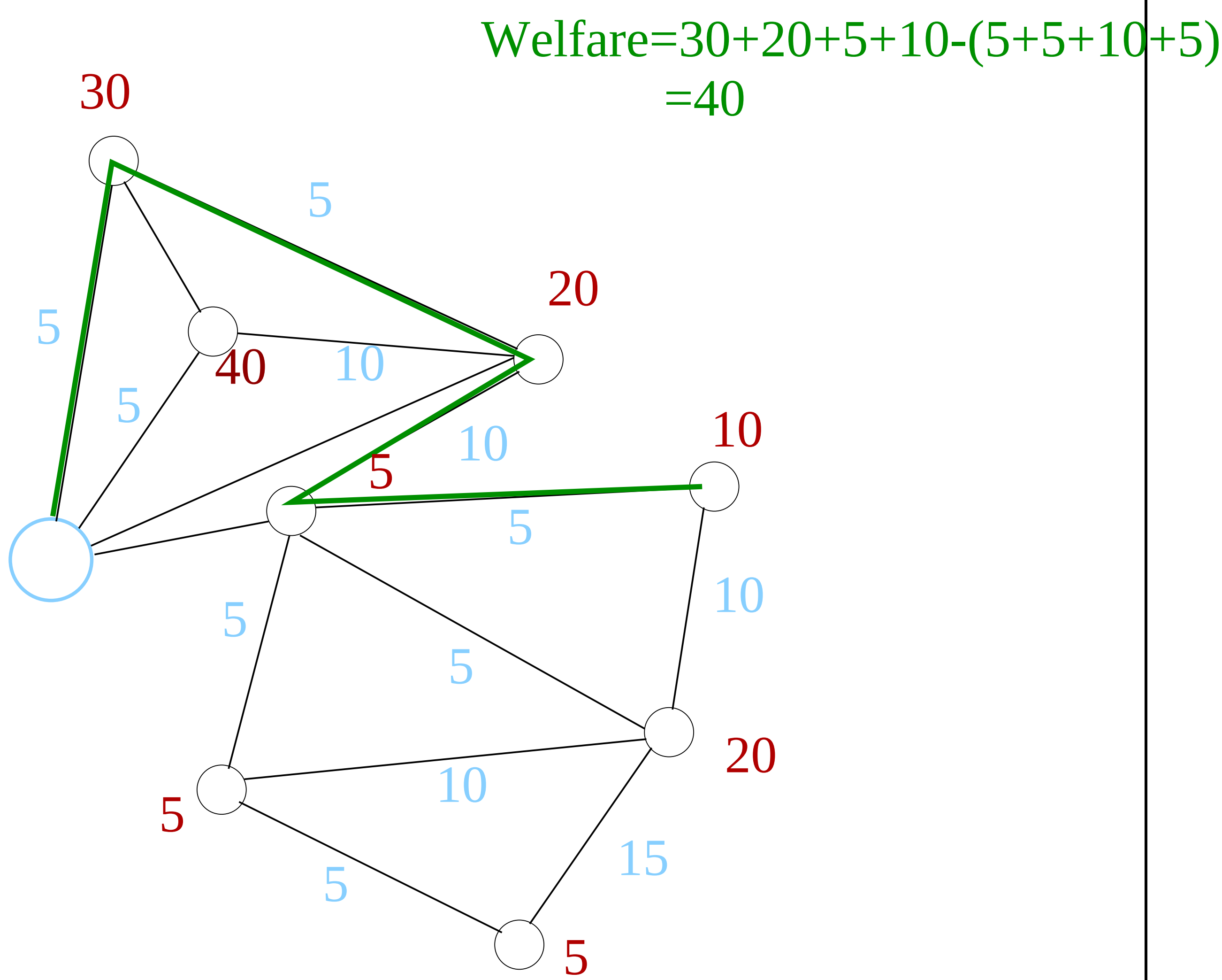
Jain & Vazirani: assume a general biconnected network, propose a centralized *approximation* mechanism.

Example: Universal Tree Multicast



Welfare=40+15-(20+10)=25

Example: General Multicast (Steiner)



Mechanism Design Problem

Values, v_i, of receivers are private information. Agents announce $\hat{\mathbf{v}} = (\hat{v}_1, \ldots, \hat{v}_{|N|})$. Propose a mechanism to compute receivers $r_i(\hat{\mathbf{v}}) \in \{0,1\}$, payments $x_i(\hat{\mathbf{v}}) \geq 0$, and tree $T(\hat{\mathbf{v}})$. Let $w_i(\hat{\mathbf{v}}) = q_i(\hat{\mathbf{v}})v_i - x_i(\hat{\mathbf{v}})$.

Desirable properties to achieve in equilibrium:

- [BB] $\sum_{i \in N} x_i(\hat{\mathbf{v}}) = \sum_{e \in T(\hat{\mathbf{v}})} c_e$
- [EFF] implement eff. outcome (R^*, T^*)
- [VP] $x_i(\hat{\mathbf{v}}) \leq q_i(\hat{\mathbf{v}})\hat{v}_i$
- [NPT] $x_i(\hat{\mathbf{v}}) \geq 0$
- [CS] $r_i(\hat{\mathbf{v}}) = 1$, if $\hat{v}_i$ large enough

What about the solution concepts?

Solution Concepts

- **Strategy-proof** [SP]

  $$w_i(v_i, \mathbf{v}_{-i}) \geq w_i(\hat{v}_i, \mathbf{v}_{-i}), \qquad \forall \mathbf{v}_{-i}, \forall \hat{v}_i$$

- **Group Strategy-proof** [GSP]

  for all coalitions $S \subseteq N$, we have:

  $\forall \mathbf{v}_{-S}, \forall \hat{\mathbf{v}}_S$

  $$w_i(\mathbf{v}_S, \mathbf{v}_{-S}) = w_i(\hat{\mathbf{v}}_S, \mathbf{v}_{-S}), \quad \forall i \in S$$

  $$\text{or} \quad \exists i \in S \quad \text{s.t.} \quad w_i(\mathbf{v}_S, \mathbf{v}_{-S}) > w_i(\hat{\mathbf{v}}_S, \mathbf{v}_{-S})$$

Impossibility Results

[BB] and [Eff] impossible to achieve (Moulin & Shenker, 01).

- Can achieve (VP), (EFF), (SP), with Groves mechanisms (with “truthful” network costs).
  – VCG unique amongst EFF, SP, *and* VP, NPT, & CS.

- Can achieve (VP), (BB), (GSP), with *cross-monotonic* sharing methods.
  – for *submodular* cost functions, $C(T)$, always exist, and Shapley minimizes worst-case eff. loss.

Def. Submodular:
$C(e \cup T_2) - C(T_2) \leq C(e \cup T_1) - C(T_1)$, for all $T_1 \subseteq T_2 \subseteq N$, and $e \notin T_1$.

Distributed Implementations

[Feigenbaum et al. 01]

- VCG, or “marginal cost”. (SP) and (EFF).
  – $p_{\mathrm{gva},i} = r_i(\hat{v})\hat{v}_i - (W(\hat{v}) - W(\hat{v}_{-i}))$
  – propose a mechanism to compute with 2 messages per-link (tree-traversal).

- Shapley. (GSP) and (BB).
  – cost-sharing function $\xi(R, i)$ to share cost of multicast to receivers R, with $\sum_i \xi(R, i) = C(R)$
  – select maximal number of receivers that can stand the cost-share
  – any implementation has worst-case *linear* number of messages per-link

Network complexity: total # msgs, max # msgs. on any link, max size msg, local comput. burden on agents.

Budget-Balanced Mechanisms for General Multicast Tree Problem

Moulin&Shenker, 01

Class of (GSP), (BB) mechanisms defined by *cross-monotone* cost-sharing methods, $\xi(Q,i)$, with properties:

$$\xi(Q,i) = 0, \forall i \neq Q$$

$$\sum_i \xi(Q,i) = C(Q)$$

$$\xi(Q,i) \geq \xi(R,i) \forall Q \subseteq R$$

$M(\xi)$ mechanism:

- Agents report values, $\hat{v}$; initialize $R \leftarrow N$.
- Select an agent $i \in R$ at random, if $\hat{v}_i < \xi(R,i)$ then drop i from R.
- Continue until $\hat{v}_i \geq \xi(R,i)$ for all $i \in R$.

Computational Considerations

Steiner tree problem is not feasible, unless P=NP. In addition, there are no *weak* cross-monotone cost-sharing methods, and so no GSP & BB mechanisms.

- Minimum Cost Spanning Tree (MCST) is a 2-approximation to optimal Steiner tree.
  – not submodular cost function, cannot use *Shapley*.
  – propose a linear-program formulation of MCST, and demonstrate that the *dual* solution provides a cross-monotone cost-sharing method.
- Leads to a (GSP), (BB), (VP) 2-approximation to the (EFF) multi-cast solution.

Open problems: decentralized implementations, better efficiency outcomes, etc.