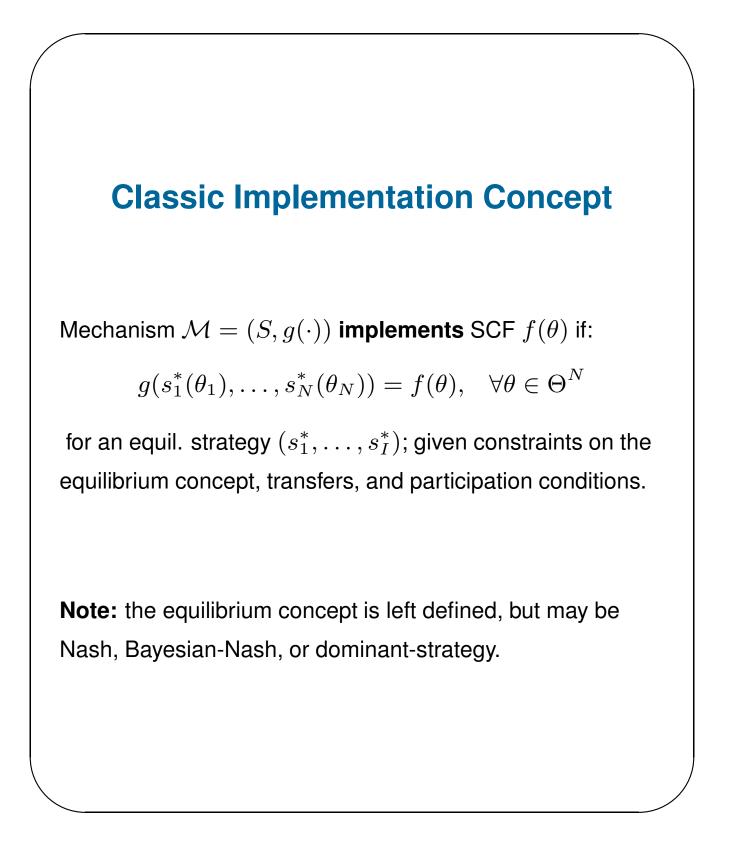


Basic Definitions

- Set of possible outcomes ${\cal O}$
- Agents $i \in \mathcal{I}$, with *preference types* $\theta_i \in \Theta_i$, and $|\mathcal{I}| = N$.
- Utility, $u_i(o, \theta_i)$, over outcome $o \in \mathcal{O}$

• Mechanism
$$\mathcal{M} = (S, g)$$
 defines:
- a strategy space $S = S_1 \times \ldots \times S_N$, s.t. agent i
chooses a strategy $s_i(\theta_i) \in S_i$, with $s_i : \Theta_i \to S_i$.
- an outcome function, $g : S^N \to \mathcal{O}$, s.t. outcome
 $g(s_1(\theta_1), \ldots, s_N(\theta_N))$ is implemented given strategy
profile $s = (s_1(), \ldots, s_N())$.

• Game: Utility to agent *i* from strategy profile *s*, is $u_i(g(s(\theta)), \theta_i)$, which we write as shorthand $u_i(s, \theta_i)$.



Mechanism Desiderata

Efficiency

- select the outcome that maximizes total utility

• Fairness

select the outcome that minimizes the variance in utility

Revenue maximization

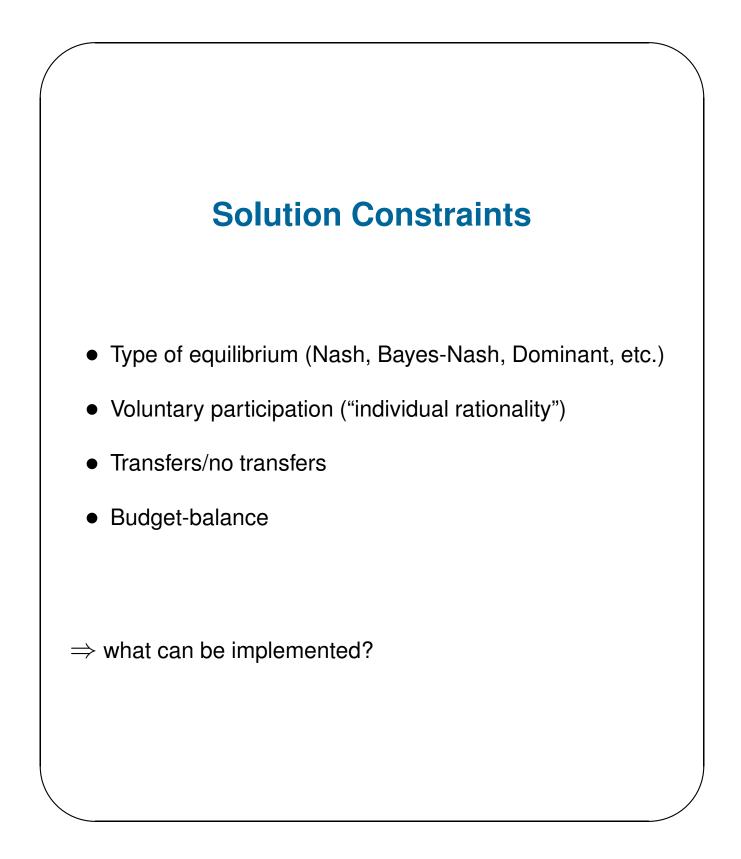
– select the outcome that maximizes revenue to a seller (or more generally, utility to one of the agents)

Budget-balanced

implement outcomes that have balanced transfers across agents

Pareto Optimal

- only implement outcomes o^* , for which for all $o' \neq o^*$, either $u_i(o', \theta_i) = u_i(o^*, \theta_i)$ for all i, or $\exists i \in \mathcal{I}$ with $u_i(o', \theta_i) < u_i(o^*, \theta_i)$.

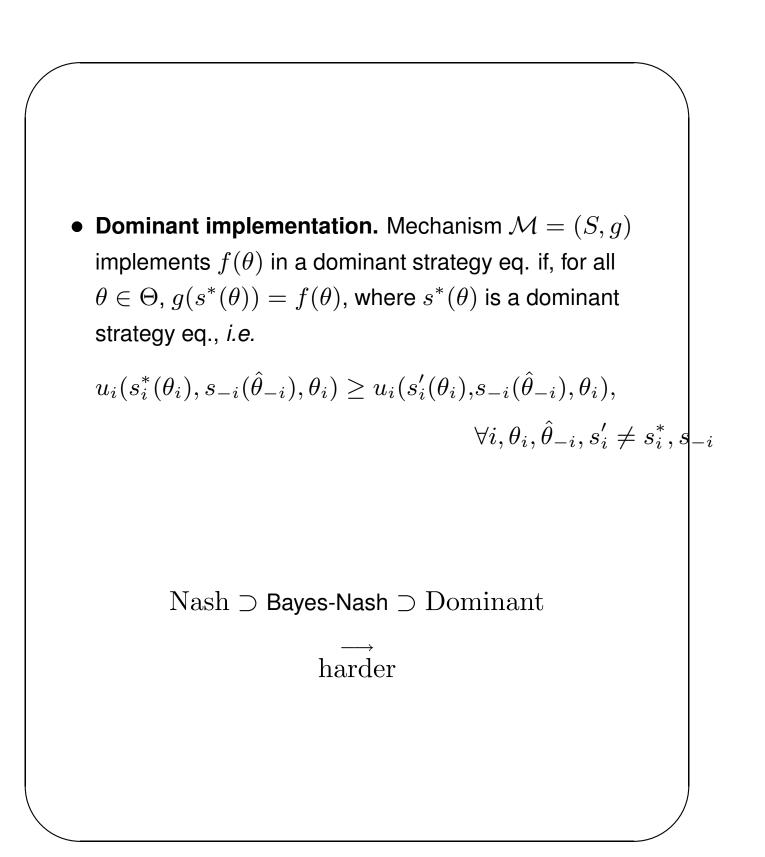


8

I: Equilibrium Concepts

- Nash implementation. Mechanism $\mathcal{M} = (S, g)$ implements $f(\theta)$ in Nash eq. if, for all $\theta \in \Theta$, $g(s^*(\theta)) = f(\theta)$, where $s^*(\theta)$ is a Nash eq., *i.e.* $u_i(s_i^*(\theta_i), s_{-i}^*(\theta_{-i}), \theta_i) \ge u_i(s_i'(\theta_i), s_{-i}^*(\theta_{-i}), \theta_i),$ $\forall i, \forall \theta, \forall s_i' \neq s_i^*$
- Bayes-Nash implementation. Common prior $F(\theta)$. Mechanism $\mathcal{M} = (S, g)$ implements $f(\theta)$ in Bayes-Nash eq. if, for all $\theta \in \Theta$, $g(s^*(\theta)) = f(\theta)$, where $s^*(\theta)$ is a Bayes-Nash eq., *i.e.*

$$\begin{split} \mathbf{E}_{\theta_{-i}} [u_i(s_i^*(\theta_i), s_{-i}^*(\theta_{-i}), \theta_i)] \geq \\ \mathbf{E}_{\theta_{-i}} [u_i(s_i'(\theta_i), s_{-i}^*(\theta_{-i}), \theta_i)], \\ \forall i, \forall \theta_i, \forall s_i' \neq s_i^* \end{split}$$



II: Participation

Let $\overline{u}_i(\theta_i)$ denote the (expected) utility to agent i with type θ_i of its outside option, and recall that $u_i(f(\theta), \theta_i)$ is the equilibrium utility of agent i from the mechanism.

• *ex ante* individual-rationality; agents choose to participate before they know their own types:

$$E_{\theta \in \Theta} \left[u_i(f(\theta), \theta_i) \right] \ge E_{\theta_i \in \Theta_i} \overline{u}_i(\theta_i)$$

- *interim* individual-rationality
 - agents can withdraw once they know their own type;

$$E_{\theta_{-i}\in\Theta_{-i}}\left[u_i(f(\theta_i,\theta_{-i}),\theta_i)\right] \ge \overline{u}_i(\theta_i)$$

• ex post individual-rationality

- agents can withdraw from the mechanism at the end; $u_i(f(\theta), \theta_i) \ge \overline{u}_i(\theta_i)$.

ex ante \supset interim \supset ex post

harder