Auction Theory

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Auctions: A Special Case of Mech. Design

- Allocation problems
  – finite set $\mathcal{G}$ of items to allocate
  – variations possible (e.g. information goods, configurable items)
  – 1:N settings typical, N:M possible.

- Agent models
  – private values vs. common values
  – "no externalities"
  – quasi-linear, i.e. $u_i(S,p) = v_i(S) - p$ for item(s) $S \subseteq \mathcal{G}$ at price p; i.e. risk-neutral

- Mechanism properties
  – Budget-balanced ("trading mechanisms")
  – efficiency (maximize total *value*), or revenue (maximize the *utility* of a single agent)

Private vs. Common Values

- Private values [e.g. antique collectors, contractors]
  – independently distributed, according to some prior, $F_i(\theta)$, for agent i; priors common knowledge [iid is special case]
  – models of information asymmetry also possible

- Common values [e.g. oil]
  – common value, V, info. agent i, $v_i \sim H(V)$, independent draw from a common distribution.
  – learning about someone else's value useful

- Correlated values
  – e.g. inherent differences in production costs; but some shared "problem difficulty"

Model of agent valuations changes auction prescriptions.

Private Values

- Single-item variations
- Reverse auctions
- Iterative vs. sealed-bid
- Collusion, trust, privacy
- Variations: double auctions, multi-unit auctions, combinatorial auctions, multiattribute auctions, etc.

Single-item: Efficient

[Vickrey 61]

- English/Vickrey (second-price)
- Dutch/FPSB (first-price)

All efficient. (Why does Vickrey not break Green-Laffont imposs?)

Also, all *revenue-equivalent* (if IID, quasi-linear, symmetric). Let $v_{(k)}$ denote the k-th order statistic.

- First-price Sealed-bid/Dutch
  – best-response, $B(v) = E[v_{(2)} | v_{(1)} = v]$; expected revenue, $E[B(v_{(1)})] = E[v_{(2)}]$
- Vickrey/English
  – revenue $E[V_{(2)}]$

Thm. [Rev. Equiv.] In any efficient auction, the expected payoff to every bidder, and the seller is the same.

Optimal Auction Design

[Myerson'81]

Consider a seller with value v_0, and suppose the seller can set a reservation price $r \geq 0$.

Tradeoff: between loss of revenue when $v_0 < v_{(1)} < r$; and gain in revenue when $v_0 < r < v_{(1)}$.

Thm. The revenue-maximizing (optimal) single-item auction is a Vickrey auction with $r = B^{-1}(v_0)$.

i.e., the seller should set $v_0 > r$, such that $B(r) = v_0$.

optimal auction $\neq$ efficient auction

As a Constrained Optimization Problem...

Consider a single-item allocation problem with N agents, let $p_i(v_1, \ldots, v_N)$ denote the expected payment from agent i to the mechanism and, $x_i(v_1, \ldots, v_N)$, denote the probability with which agent i is allocated the item. Let v_0 denote the value of the seller.

$$\max_{\{p_i, x_i\}} \sum_{i=1}^{N} E\left[p_i(v_1, \ldots, v_N)\right] - \sum_{i=1}^{N} E\left[x_i(v_1, \ldots, v_N)\right] v_0$$

$$\text{s.t.} \quad \sum_{i=1}^{N} x_i(v_1, \ldots, v_N) \leq 1, \quad \forall v \qquad \text{(feas)}$$

$$Eu_i(v_i) \geq Eu_i(\hat{v}_i), \quad \forall \hat{v}_i \neq v_i, \forall v_i, \forall i \qquad \text{(IC)}$$

$$Eu_i(v_i) \geq 0, \quad \forall v_i \qquad \text{(IR)}$$

where

$$Eu_i(\hat{v}_i) = E_{v_{-i}}\left[x_i(v_1, \ldots, \hat{v}_i, \ldots, v_N)v_i\right] - E\left[p_i(v_1, \ldots, \hat{v}_i, \ldots, v_N)\right]$$

Effect of an Aftermarket

[Ausubel & Cramton 99]

- Optimal auction designs makes two assumptions:
  – seller can prevent resale
  – seller can commit not to sell goods withheld after the auction
- Assume "perfect resale" (all gains from trade exhausted in resale)
  – seller's incentives to misassign goods now *destroyed*
  – *optimal to be efficient*

In addition: efficient marketplaces often will be the only markets to survive in long-term competition with other marketplaces ["larger pie to share"].

eBay proxy agents

- Provide an "upper bid-limit" to the eBay agent, which competes in an English auction until price reached.
- Revelation principle!
  – English $\Rightarrow$ Vickrey
- Note: issue of trust.

Closing Rules

[Roth & Ockenfels 01]; eBay vs. Amazon (now dead).

- eBay [hard closing rule]
  – industry in “sniping”, favors bidders with better technology
  – empirically, limits information revelation during the auction, many bidders do not use proxy agents [esp. experienced bidders]
  – bidders can implicitly collude; avoid price wars; “strategic demand reduction” in a long-term game
  – at the end there is a probability that bids will fail, helps commitment issue.
- Amazon [soft closing rule]
  – removes this “arms race” for bidding technology
  – empirically, encourages bidding earlier in the auction
  – now it is hard to enforce implicit collusion

Multi-period Auctions

(e.g. Priceline, eBay, etc.)

You want a single item, and can participate in a sequence of Vickrey auctions. What should you do?

- The strategyproofness of Vickrey is quite brittle.

[design of s'proof seq. auctions is an interesting open problem]

Reverse Auctions/Private marketplaces

One buyer, multiple sellers. [e.g. GM and its suppliers]

- Descending price [second-price]
  – price starts high, continues to fall until only one supplier is left.
- Ascending price [first-price]
  – price starts low, continues to increase until one supplier accepts.

Iterative vs. Sealed-bid

- Cost of communication
- Cost of delay
- Cost of information revelation
- Common vs. Private values
- Cost of valuation
- Ability to manipulate
- Cost of participation
- Transparency

Collusion

- **FCC auction.** Simultaneous ascending-price auction for multiple licenses. Collusion, “strategic-demand reduction” via trailing digits.
- **Bidder rings.** [Robinson 85] Group of bidders get together beforehand, and decide that only one will participate in the auction. Share gains afterwards.
  – problems in reaching an agreement, sharing rewards
  – first-price [Dutch, FPSB], this collusion is not self-enforcing because the selected bidder must submit a very small bid
  – second-price [Vickrey, English], this collusion is self-enforcing, because deviators are punished.
  – *shills*, “pulling bids off chandelier”, are a tool for sellers to fight collusion

Trust

- **Vickrey auction.**
  – bidders must trust the auctioneer not to submit a false bid. [without risk]
  – *computational remedies?* [bid verif. mechanism, trusted 3rd party]
- **English auction.**
  – more transparent, although the auctioneer can still use a “shill” to increase the bid price [some risk]
  – how does this compare to setting a reservation price?

Information Revelation

[Rothkopf et al. 90]

- In a contracting example, the Vickrey auction awards a contract to the lowest bidder, but makes payment equal to the second-lowest bid.
  – Political problems?
- **Repeated auctions.** In the context of repeated auctions, whenever I reveal my true value for an item, that can be used against me in the future.
  – Business implications, within a supply-chain context?

perhaps English auctions have more desirable properties?
computational remedies?

Double Auctions

Multiple buyers, multiple sellers, each with private information. Suppose bids, $b_1 \geq b_2 \geq \ldots \geq b_m$, and asks, $s_1 \leq s_2 \leq \ldots s_n$. Compute l^*, s.t. bids $i \leq l^*$ and asks $j \leq l^*$ trade; and determine payments.

- strategyproof, efficient and budget-balanced impossible
- McAfee-Double auction
  – compute a payment based on the bids not quite accepted, use this when IR; otherwise, implement one less trade.
  – strategy-proof, BB, not EFF.
- k-DA
  – clear double auction to maximize reported surplus
  – set a price equal to $s_{l^*} + k(b_{l^*} - s_{l^*})$, for some $k \in [0, 1]$.
  – not strategyproof or EFF, but BB and "good" EFF in practice, in particular for large markets.

Multi-unit Auctions: Sealed-bid

N units of a homogeneous item. First, consider the special case in which each bidder demands a *single unit*. Let $v_i \geq 0$ denote the value of bidder i.

Def. The VCG auction for this special case sells the items to the N highest bidders, each pays the $N+1$st highest bid price.

$$p_{\mathrm{vick},i} = b_i - (\sum_{j=1}^{N} b_j - \left[\sum_{j=1}^{N+1} b_j - b_i)\right] = b_{N+1}$$

Multi-unit Auctions

Single bid, (k_i, b_i), for k_i units, from each agent. Let $x_i \in \{0, 1\}$ define whether bid i is accepted, and p_i denote payment by agent i.

(1) compute x^* to solve (weighted knapsack) problem:

$$V^* = \max_x \sum_i x_i p_i$$

$$\text{s.t.} \sum_i x_i k_i \leq N$$

(2) compute payments, $p_i = b_i - (V^* - V^{-i})$ if $x_i = 1$, with $p_i = 0$ otherwise; where V^{-i} is maximal value over subproblem induced by removing bid from agent i.

Note. exclusive-or bid generalizations easy to define.

Multi-unit Auctions: Approx.

Use *greedy method* to select the winning bids:

(1) sort in decreasing per-unit bid price (2) greedily accept, with highest per-unit bid price first.

Then (a) compute price as per-unit price of first rejected bid; or (b) use VCG rule to compute price.

Prop. Payment rule (a) is strategy-proof; but VCG is no longer strategy-proof.

...good example of problems with introducing approximations into the VCG mechanism.

Iterative Multi-Unit Auctions

- Ausubel 97. “clinching mechanism”, for *decreasing marginal values*

  – maintains a single ask price, but determines final payment of an agent along the path of the auction.

  – terminates with the efficient allocation, and the Vickrey payment, if agents follow straightforward bidding strategies, *when bidders have decreasing marginal values* for items.

- eBay “Yankee” auction.

  – maintains a per-unit price, agents submit bids for fixed quantities; auction terminates as soon as there is no overdemand.

  – terminates with Vickrey outcome in special cases; but in general not efficient.

Multiple Heterogeneous Items

- Simultaneous ascending price auctions
  – work well if “gross-substitutes” property satisfied
  – in general, lead to exposure problem
- Combinatorial auctions [non-linear prices, contingent bids]
  – sealed-bid auctions, apply VCG mechanism.
- Ascending-price auctions
  – threshold problem (coordination across small bidders)
  – Vickrey payment might not be supported
  – revenue-maximizing designs [Milgrom&Ausubel]
  – efficient designs [Parkes&Ungar]

Additional Auction Variations

- **Multiattribute auctions**

  – configure the attributes (e.g. quality, speed, color) of an item in addition to the price

  – Che (93), Parkes&Kalagnanam (02)

- **Exchanges (combinatorial)**

  – multiple buyers & sellers, all with contingencies

  – important, for example, in the FCC wireless spectrum allocation setting

  – Parkes,Kalagnamam, Eso (01)

Common Value Settings

[Wilson 77; Kagel & Levin 86; Bazerman & Samuelson 83]

- \$8 pennies in a jar; collect sealed bids
  – average bid \$5.13, winning bid \$10.01
  – winner's curse, all get an unbiased estimate, $f(\cdot)$
  – bids increase in $f(\cdot)$ in equil.
  – winner is one with *most optimistic estimate*, "adverse selection bias"
- Simple model; signal $s_i \sim U(V - \epsilon, V + \epsilon)$
  – should bid $b_i \approx s_i - \epsilon$

Affiliated Values

[Milgrom & Weber 82]

- *Model:* if one agent has a high value, then other agents are more likely to have high values
  – ascending $\succ$ Vickrey; because the winning bidder's surplus is due to private information
  – the more the price is related to the information of other agents, the lower the "information rent" of the winning bidder
- Linkage principle
  – if the seller has any private information, should precommit to releasing the information honestly
  – same argument; better to allow competition across bidders and drive price