CS286r Electronic Market Design
Homework 3: Auction Theory

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Due: Tuesday 3/4/2003, in the beginning of class. You may use any sources that you want, but you must cite the sources that you use. **You are encouraged to discuss this work with your peers, but your write-up must be your own and you must understand whatever you turn in!**. If you took the class last year you must choose your own topic to work on for this initial part of the course.[1] Please work hard on making the proofs clear, consise, and easy to read.

1. Consider a second-price sealed-bid (Vickrey) auction of one item, with bidders, i, with values, $v_i \in [0, \overline{v}]$, and quasilinear preferences, i.e. with $u_i(v_i, p) = v_i - p$, given price p.

   (a) (10 pts) Show that bid $b_i(\theta_i) = v_i$ for all values, $v_i \in [0, \overline{v}]$, is a weakly dominant strategy for each bidder i. [Prove this from first principles, do not use the fact that the Vickrey auction is a special case of the Groves mechanism].

   (b) (5 pts) Let $b_{(k)}$ denote the kth highest bid. Suppose that the seller introduces a reservation price, $r \in [0, 1]$, such that the item is only sold if $b_{(1)} \geq r$, for price $p = \max[r, b_{(2)}]$. Show that truthful bidding remains a weakly dominant strategy for bidders.

   (c) (5 pts) Consider the special case of an auction with a single bidder, with a Uniformly distributed value $v_1 \sim U(0, \overline{v})$. In addition, suppose that the seller has value, v_0, for the item. Verify that strategies, $r^*(v_0) = (v_0 + \overline{v})/2$, $b_1^*(v_1) = v_1$ form a Bayesian-Nash eq. of this reserve-price Vickrey auction.

   (d) (5 pts) In fact, $((v_0 + \overline{v})/2, v_1, \ldots, v_N)$, is also the Bayes-Nash eq. of the auction with N bidders, each with value v_i. Assuming, $\overline{v} = 1$ and $v_0 = 0$, determine the seller's expected revenue for the special case of **two** bidders. [**Hint:** construct an expression, by case analysis of the bids received, for the expected revenue to the seller. The fact, $E[v_{(2)} | v_{(2)} \geq 1/2] = 2/3$, where $v_{(2)}$ is the second-highest value across two bidders, will be useful.]

[1] This can involve choosing and answering questions from a GT text, or writing a review paper of some area of GT/MD about which you would like to learn more. Come talk to me!

(e) (5 pts) For this two-bidder case, compare the expected revenue in the reserve price Vickrey auction to that in the Vickrey auction with no reserve price, and provide an intuitive argument about the effect on allocative-efficiency. [**Hint:** The following fact is very helpful: the expected k^{th} highest value among n values independently drawn from the uniform distribution on $[\underline{v}, \overline{v}]$ is $\underline{v} + \left(\frac{n+1-k}{n+1}\right)(\overline{v} - \underline{v})$.]

2. Consider a *double auction* (DA), with m buyers and n sellers, each trading a single item. Buyers and sellers submit bids and asks, and the DA determines the trade, and agents' payments. Let $b_1, \ldots, b_m$ denote the bid prices from buyers, and assume $b_1 \geq b_2 \geq \ldots \geq b_m \geq 0$. Let $s_1, \ldots, s_n$ denote the ask prices from sellers, and assume $0 \leq s_1 \leq s_2 \leq \ldots s_n$. In addition, define $b_{m+1} = 0$ and $s_{n+1} = \infty$. Later we refer to the following examples: (i) buyer values $9, 8, 7, 4$, seller values $2, 3, 4, 5$; (ii) buyer values $9, 8, 7, 4$, seller values $2, 3, 4, 12$.

(a) (10 pts) Define the VCG mechanism for this problem, and show that the mechanism is not *ex post* weak BB for example (i). [**Hint:** it is useful to interpret a bid, or an ask, as an agent's claim about its value for the item. Define the trades implemented, payment by each buyer, payment to each seller.]

Consider the following modified trading mechanism, the *McAfee-DA*:

(1) select k, s.t. $b_k \geq s_k$ and $b_{k+1} < s_{k+1}$.

(2) compute *candidate trading price*, $p_0 = 1/2(b_{k+1} + s_{k+1})$.

(3) if $s_k \leq p_0 \leq b_k$, then the buyers/sellers from 1 to k trade at price p_0; otherwise, the buyers/sellers from 1 to $k-1$ trade, and each buyer pays b_k, each seller gets s_k.

(b) (15 pts) Prove that the McAfee-DA is strategy-proof, and *ex post* weak budget-balanced.

(c) (5 pts) Run the McAfee-DA on examples (i) and (ii). Is the DA efficient?

(d) (10 pts) The McAfee-DA is vulnerable to *false-name bids*, where an agent submits an additional bid under another identity to influence the outcome. Provide an instance of successful manipulation with false-name bids in examples (i) and (ii). [**Hint:** Consider that a seller can submit a false-name bid either as a buyer or a seller. The examples that you construct to demonstrate vulnerability to false-name manipulation for both problem i) and problem ii) will involve the addition of a single "false-name" bid.]