Auction Theory

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Auctions: A Special Case of Mech. Design

- Allocation problems
  - finite set $\mathcal{G}$ of items to allocate
  - variations possible (e.g. information goods, configurable items)
  - 1:N settings typical, N:M possible.

- Agent models
  - private values vs. common values
  - "no externalities"
  - quasi-linear, i.e. $u_i(S, p) = v_i(S) - p$ for item(s)
    $S \subseteq \mathcal{G}$ at price $p$; i.e. risk-neutral

- Mechanism properties
  - Budget-balanced ("trading mechanisms")
  - efficiency (maximize total value), or revenue
    (maximize the utility of a single agent)
Private vs. Common Values

- Private values [e.g. antique collectors, contractors]
  - independently distributed, according to some prior, $F_i(\theta)$, for agent $i$; priors common knowledge [iid is special case]
  - models of information asymmetry also possible

- Common values [e.g. oil drilling rights]
  - common value, $V$, info. agent $i$, $v_i \sim H(V)$, independent draw from a common distribution.
  - learning about someone else’s value useful

- Interdependent values
  - e.g. inherent differences in production costs; but some shared “problem difficulty”

*Model of agent valuations changes auction prescriptions.*
Private Values

- Single-item variations
- Reverse auctions
- Iterative vs. sealed-bid
- Collusion, trust, privacy
- Variations: double auctions, multi-unit auctions, combinatorial auctions, multiattribute auctions, etc.
Single-item: Efficient

[Vickrey 61]

- English/Vickrey (second-price)
- Dutch/FPSB (first-price)

All efficient. (Why does Vickrey not break Green-Laffont imposs?)

Also, all revenue-equivalent (if IID, quasi-linear, symmetric).

Let $v_{(k)}$ denote the $k$-th order statistic.

- First-price Sealed-bid/Dutch
  - best-response, $B(v) = E[v_{(2)} | v_{(1)} = v]$; expected revenue, $E[B(v_{(1)})] = E[v_{(2)}]$

- Vickrey/English
  - revenue $E[V_{(2)}]$

**Thm.** [Rev. Equiv.] In any efficient auction, the expected payoff to every bidder, and the seller is the same.
Optimal Auction Design

[Myerson’81]

Consider a seller with value $v_0$, and suppose the seller can set a reservation price $r \geq 0$.

**Tradeoff:** between loss of revenue when $v_0 < v_1 < r$; and gain in revenue when $v_0 < r < v_1$.

**Thm.** The revenue-maximizing (optimal) single-item auction is a Vickrey auction with $r = B^{-1}(v_0)$.

i.e., the seller should set $r > v_0$, such that $B(r) = v_0$.

optimal auction $\neq$ efficient auction
eBay proxy agents

- Provide an “upper bid-limit” to the eBay agent, which competes in an English auction until price reached.

- Revelation principle!
  - English $\Rightarrow$ Vickrey

- Note: issue of trust.
Closing Rules

[Roth & Ockenfels 01]; eBay vs. Amazon (auctions now dead).

- **eBay** [hard closing rule]
  - industry in “sniping”, favors bidders with better technology
  - empirically, limits information revelation during the auction, many bidders do not use proxy agents [esp. experienced bidders]
  - bidders can implicitly collude and avoid price wars; uses the fact that there is a probability that bids will fail

- **Amazon** [soft closing rule]
  - removes this “arms race” for bidding technology
  - empirically, encourages bidding earlier in the auction
  - now it is hard to enforce implicit collusion
Multi-period Auctions

(e.g. Priceline, eBay, etc.)

You want a single item, and can participate in a sequence of Vickrey auctions. What should you do?

Notice, then that the strategyproofness of Vickrey auctions is quite brittle.

Some recent work considers the design of incentive-compatible *sequential auctions*, in which it is an equilibrium strategy for agents to bid truthfully in the first time-period in which they arrive. (Lavi & Nisan’00; Friedman & Parkes’02; Gallien’02)
Reverse Auctions/Private marketplaces

One buyer, multiple sellers. [e.g. *GM and its suppliers*]

- **Descending price** [second-price]
  - price starts high, continues to fall until only one supplier is left.

- **Ascending price** [first-price]
  - price starts low, continues to increase until one supplier accepts.
Iterative vs. Sealed-bid

- Cost of communication
- Cost of delay
- Cost of information revelation
- Common vs. Private values
- Cost of valuation
- Ability to manipulate
- Cost of participation
- Transparency
Collusion

E.g. **Bidder rings.** Group of bidders get together beforehand, and decide that only one will participate in the auction. Share gains afterwards. [Robinson 85]

- problems in reaching an agreement, sharing rewards
- first-price [Dutch, FPSB]
  - collusion is not self-enforcing because the selected bidder must submit a very small bid
- second-price [Vickrey, English]
  - collusion is self-enforcing, because deviators are punished.
- *shills*, “pulling bids off chandelier”
  - a tool for sellers to fight collusion
Trust

- **Vickrey auction.**
  - bidders must trust the auctioneer not to submit a false bid. [without risk]
  - *computational remedies?* [bid verif. mechanism, trusted 3rd party]

- **English auction.**
  - more transparent, although the auctioneer can still use a “shill” to increase the bid price [some risk]
  - how does this compare to setting a reservation price?
Information Revelation

[Rothkopf et al. 90]

• In a contracting example, the Vickrey auction awards a contract to the lowest bidder, but makes payment equal to the second-lowest bid.
  – Political problems? What if the FCC used this rule?

• Repeated auctions. In the context of repeated auctions, whenever I reveal my true value for an item, that can be used against me in the future.
  – Business implications, within a supply-chain context?

*perhaps English auctions have more desirable properties? computational remedies?*
Double Auctions

Multiple buyers, multiple sellers, each with private information. Suppose bids, \( b_1 \geq b_2 \geq \ldots \geq b_m \), and asks, \( s_1 \leq s_2 \leq \ldots s_n \). Compute \( l^* \), s.t. \( b_{l^*} \geq s_{l^*} \) and \( b_{l^*+1} < s_{l^*+1} \).

- strategyproof, efficient and budget-balanced impossible
  
  (Myserson & Satterthwaite 83)

- McAfee-Double auction
  
  – compute candidate trading price,
  \[
  p_0 = \frac{1}{2}(b_{l^*+1} + s_{l^*+1}),
  \]
  if \( s_{l^*} \leq p_0 \leq b_{l^*} \) clear first \( l^* \) bids and asks at this price, clear first \( l^* - 1 \) bids at price \( b_{l^*} \) and first \( l^* - 1 \) asks at price \( s_{l^*} \).
  
  – strategy-proof, BB, not EFF.

- \( k \)-DA
  
  – execute first \( l^* \) bids and asks; for a uniform price
  \[
  s_{l^*} + k(b_{l^*} - s_{l^*}),
  \]
  for some \( k \in [0, 1] \).
  
  – not strategyproof or EFF, but BB and good efficiency in practice, in particular for large markets.
Multi-unit Auctions: Single-item Bids

$N$ units of a homogeneous item. First, consider the special case in which each bidder demands a single unit. Let $v_i \geq 0$ denote the value of bidder $i$.

**Def.** The VCG auction for this special case sells the items to the $N$ highest bidders, each pays the $N + 1$st highest bid price.

$$p_{vick,i} = b_i - \left( \sum_{j \leq N} b_j - \sum_{j \leq N+1, j \neq i} b_j \right) = b_{N+1}$$
Multi-unit Auctions

Single bid, \((k_i, b_i)\), for \(k_i\) units, from each agent. Let \(x_i \in \{0, 1\}\) define whether bid \(i\) is accepted, and \(p_i\) denote payment by agent \(i\).

(1) compute \(x^*\) to solve (weighted knapsack) problem:

\[
V^* = \max_x \sum_i x_i p_i
\]

s.t. \(\sum_i x_i k_i \leq N\)

(2) compute payments, \(p_i = b_i - (V^* - V^{-i})\) if \(x_i = 1\), with \(p_i = 0\) otherwise; where \(V^{-i}\) is maximal value over subproblem induced by removing bid from agent \(i\).

**Note.** exclusive-or bid generalizations easy to define. See Suri et al. approximation to a multi-unit VCG mechanism later in course.
Iterative Multi-Unit Auctions

• Ausubel “clinching” auction, for decreasing marginal values
  – maintains a single ask price, but determines final payment of an agent along the path of the auction.
  – terminates with the efficient allocation, and the Vickrey payment; straightforward bidding is an equilibrium strategy.

• eBay “Yankee” auction.
  – maintains a per-unit price, agents submit bids for fixed quantities; auction terminates as soon as there is no overdemand.
  – terminates with Vickrey outcome in special cases; but in general not efficient.

Note: later in course, see a variation on Ausubel’s ascending-price auction that is robust to false-name bids [Iwaski et al.]
Multiple Heterogeneous Items

\( \mathcal{G} \) items, values \( v_i(S) \) for \( S \subseteq \mathcal{G} \). Outcome

\( S = (S_1, \ldots, S_N) \) is feasible if \( S_i \cap S_j = \emptyset \) for all \( i, j \).

efficient: maximize \( \sum_i v_i(S_i) \).

Examples: course registration; take-off/landing; logistics; bus routes, etc.

Computational challenges: winner-determination
(weighted set-packing), bidding languages, preference elicitation.
Possible Solutions

• Simultaneous ascending price auctions
  – work well if “gross-substitutes” property satisfied
  – in general, lead to exposure problem

• Combinatorial auctions [non-linear prices, contingent bids]
  – sealed-bid auctions, apply VCG mechanism.

• Ascending-price auctions
  – can be vulnerable to the threshold problem
    (coordination across small bidders)
  – revenue-maximizing designs (Milgrom&Ausubel’02)
  – efficient designs (Parkes&Ungar’02)
Fast & Strategyproof Comb. Auctions

Lehmann et al. 99

- single-minded bidders: there is a single set $S \subseteq \mathcal{G}$ demanded by each agent. (still NP-hard).

- greedy, monotonically allocation rule: sort bids by some criterion, then take bids in order if not in conflict.
  - e.g. scheme with norm $\alpha/|S|^{1/2}$ approximates within factor of $|\mathcal{G}|^{1/2}$.

- strategyproof auction: charge each winner the per-item price of the first unsuccessful bid.

Example: goods $A$, $B$. bidders Red (10, $A$); Green (19, $AB$); Blue (8, $B$).
Additional Auction Variations

- **Multiattribute auctions**
  - configure the attributes (e.g. quality, speed, color) of an item in addition to the price
  - optimal auction, two-attributes, continuous attribute levels (Che’93)
  - efficient price-based auction, multiple attributes, discrete attribute levels (Parkes&Kalagnamam’02)

- **Exchanges (combinatorial)**
  - multiple buyers & sellers, all with contingencies
  - important, for example, in the FCC wireless spectrum allocation setting
  - can consider a family of VCG-based payment rules (Parkes,Kalagnamam, Eso’01)
Common Value Settings

[Wilson 77; Kagel & Levin 86; Bazerman & Samuelson 83]

- $8 pennies in a jar; collect sealed bids
  - average bid $5.13, winning bid $10.01
  - winner’s curse, all get an unbiased estimate, $f(\cdot)$
  - bids increase in $f(\cdot)$ in equil.
  - winner is one with *most optimistic estimate*, “adverse selection bias”

- Simple model; signal $s_i \sim U(V - \epsilon, V + \epsilon)$
  - should bid $b_i \approx s_i - \epsilon$
Interdependent Values

[Milgrom & Weber 82]

- **Model:** if one agent has a high value, then other agents are more likely to have high values
  - ascending Vickrey; because the winning bidder’s surplus is due to private information
  - the more the price is related to the information of other agents, the lower the “information rent” of the winning bidder

- **Linkage principle**
  - if the seller has any private information, should precommit to releasing the information honestly
  - same argument; better to allow competition across bidders and drive price