The sensitivity of competitive auctions to prior information

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Motivation

- worst-case (CS) vs. average-case (Econ) analysis
- two sampling-based truthful, randomized competitive auctions
  - ◇ SCS – worst-case 4-competitive
  - ◇ DSOT – worst-case 380-competitive
- empirical evaluation of average revenue
  - ◇ typical distributions
  - ◇ compare to posted-price, omniscient mechanisms
- single-item optimal auction
  - ◇ robustness to noise in the prior distribution

Dual-Price Sampling Optimal Threshold Auction (DSOT)

- Partition bids $\mathbf{b}$ at random into two sets $\mathbf{b}'$ and $\mathbf{b}''$.

- Let $p' = \text{opt}(\mathbf{b}')$ and $p'' = \text{opt}(\mathbf{b}'')$ be prices that yield the most revenue in fixed price auctions on $\mathbf{b}'$ and $\mathbf{b}''$.

- Use p' as threshold for all bids in $\mathbf{b}''$:

  ◇ all bids in $\mathbf{b}''$ of value below p' are rejected

  ◇ all remaining bids win at price p'.

- Use p'' as threshold for all bids in $\mathbf{b}'$.

Sampling Cost-Sharing Auction (SCS)

— based on CostShare$_C$.

CostShare$_C$: Find the largest k such that the highest k bidders' values are at least C/k. Charge each C/k.

$\mathcal{F}(\mathbf{b})$ = optimal profit of fixed price auction on $\mathbf{b}$.

- Divide vector $\mathbf{b}$ randomly into $\mathbf{b}'$ and $\mathbf{b}''$.

  Let $\mathcal{F}' = \mathcal{F}(\mathbf{b}')$ and $\mathcal{F}'' = \mathcal{F}(\mathbf{b}'')$.

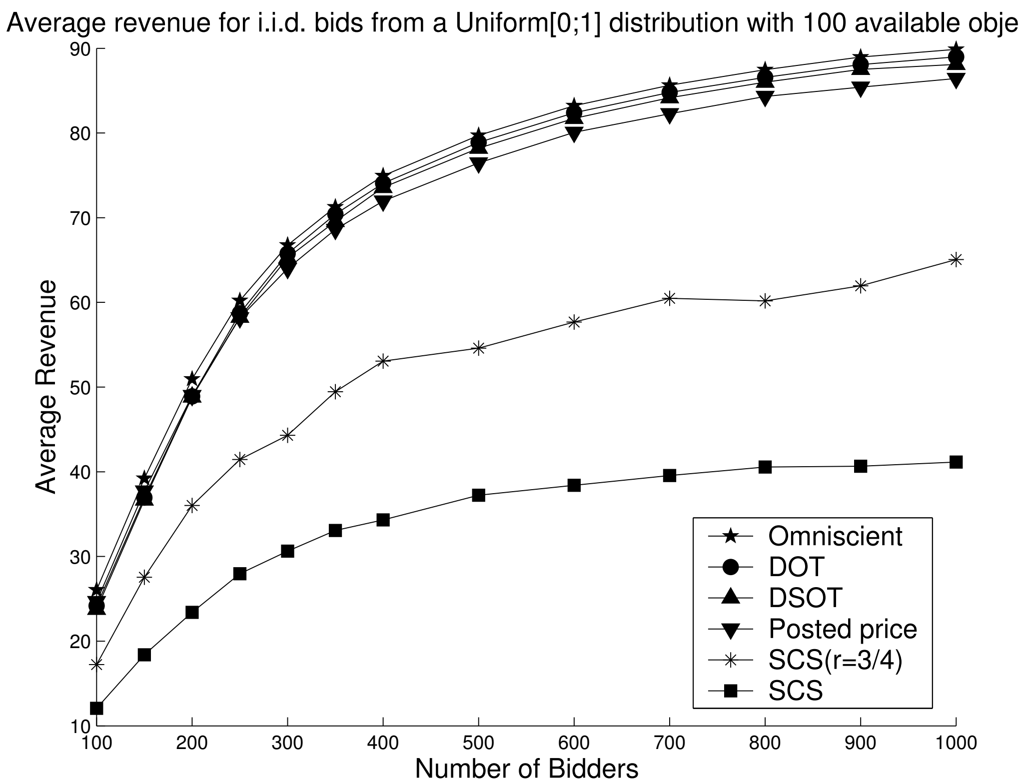
- Run CostShare$_{\mathcal{F}'}$ on $\mathbf{b}''$ and CostShare$_{\mathcal{F}''}$ on $\mathbf{b}'$.

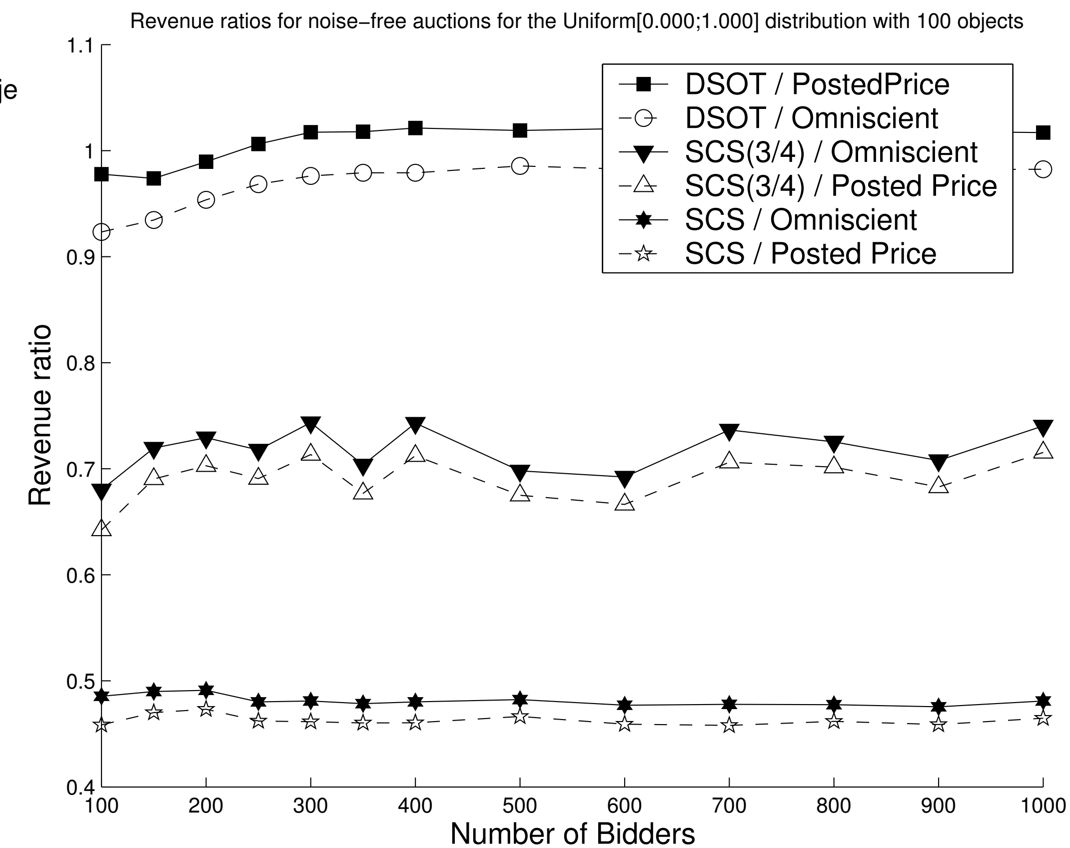
SCS has revenue $\min(\mathcal{F}', \mathcal{F}'')$.

Improving SCS

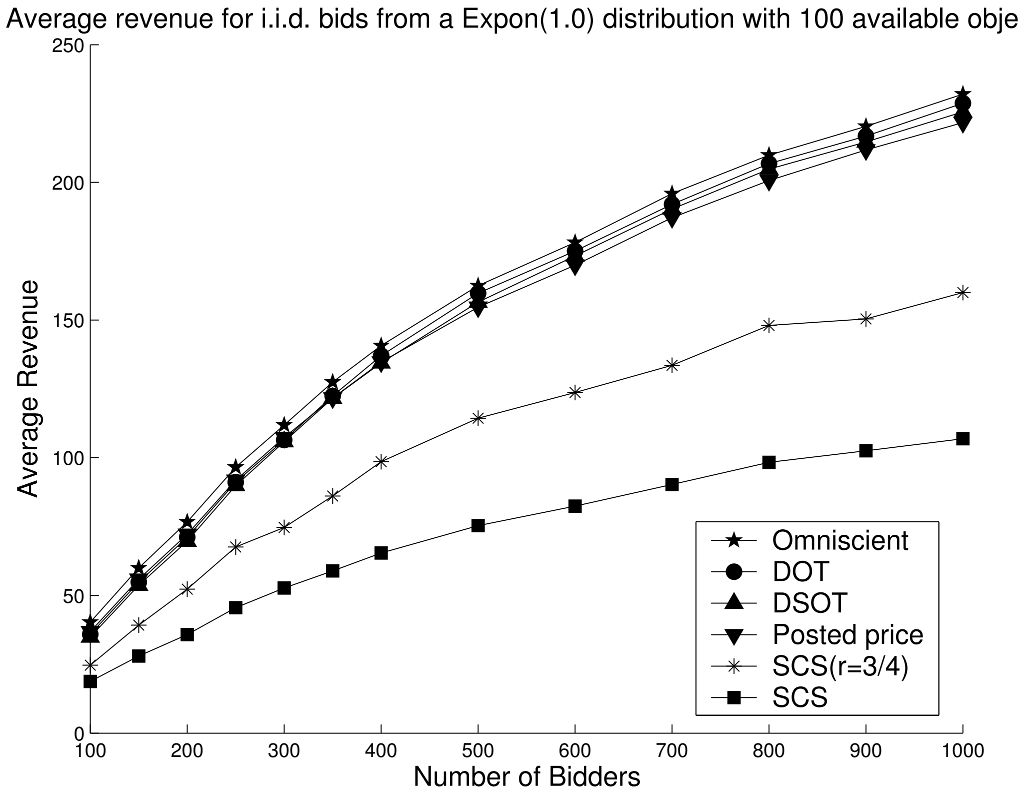
- $\mathbf{b}'$ can support cost ≤ 80; $\mathbf{b}''$ can support cost ≤ 100
- SCS asks $\mathbf{b}'$ to support 100 and $\mathbf{b}''$ to support 80 $\Rightarrow$ Profit **80**
- **Idea:** ask $\mathbf{b}'$ to support cost $\frac{3}{4} * 100 = 75$ and $\mathbf{b}''$ to support $\frac{3}{4} * 80 = 60$; $\Rightarrow$ Profit **135**
- **Intuition:** many iid bids $\Rightarrow$ $\mathbf{b}'$ and $\mathbf{b}''$ support close costs $\Rightarrow$ can use $\frac{3}{4}$
- We formalize intuition for U[0;1] distribution
  - $\diamond$ may work for continuous well-behaved distributions
  - $\otimes$ Don't try this at home!
- Empirically, improved SCS($\frac{3}{4}$) performs significantly better than standard SCS

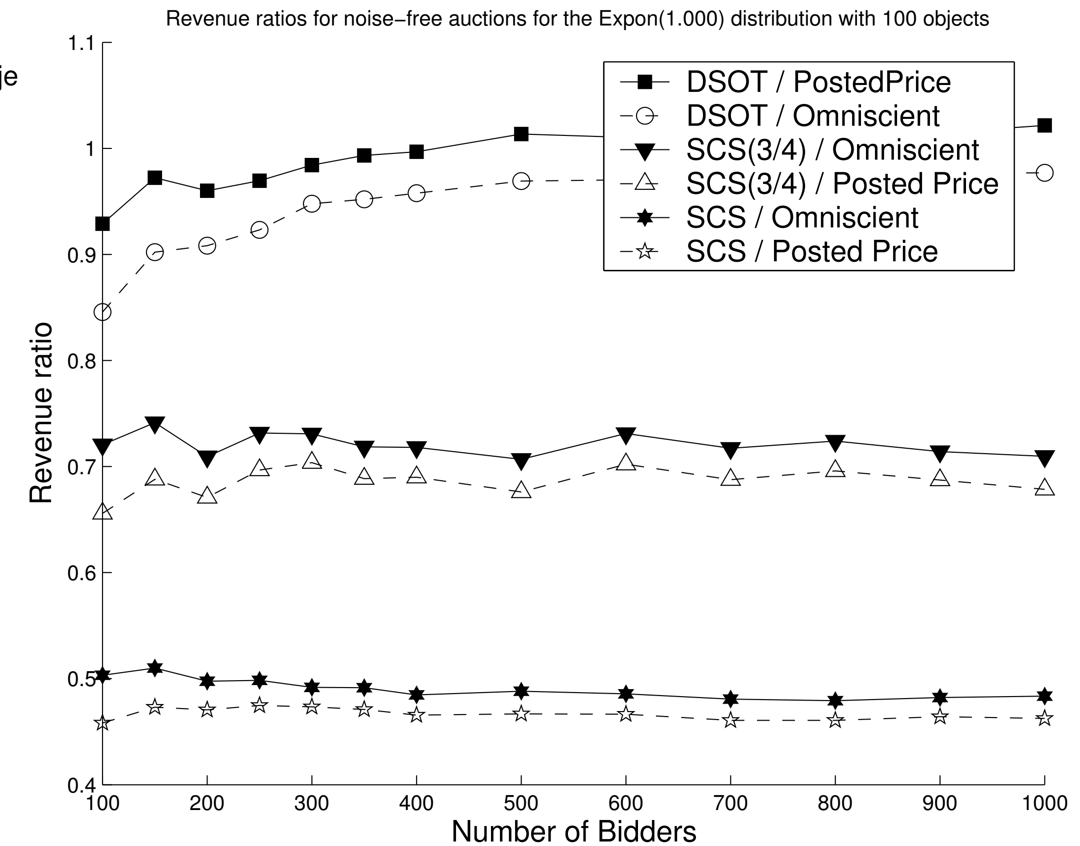
U[0;1] distribution, bounded supply



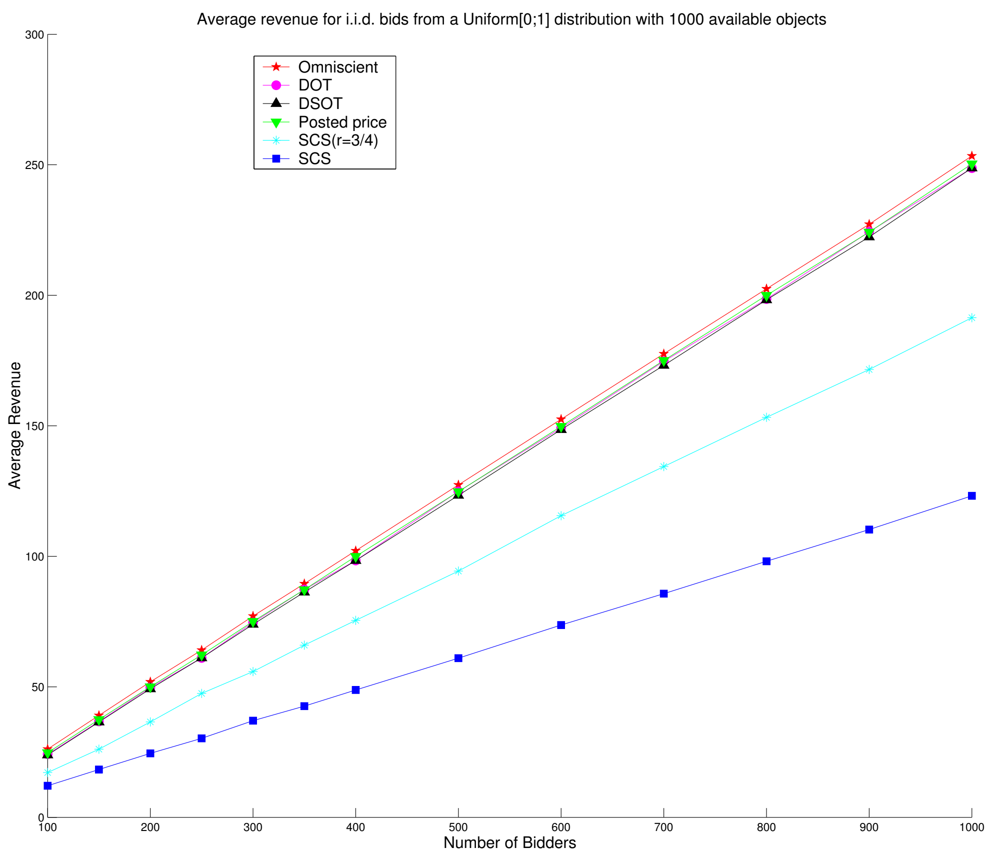


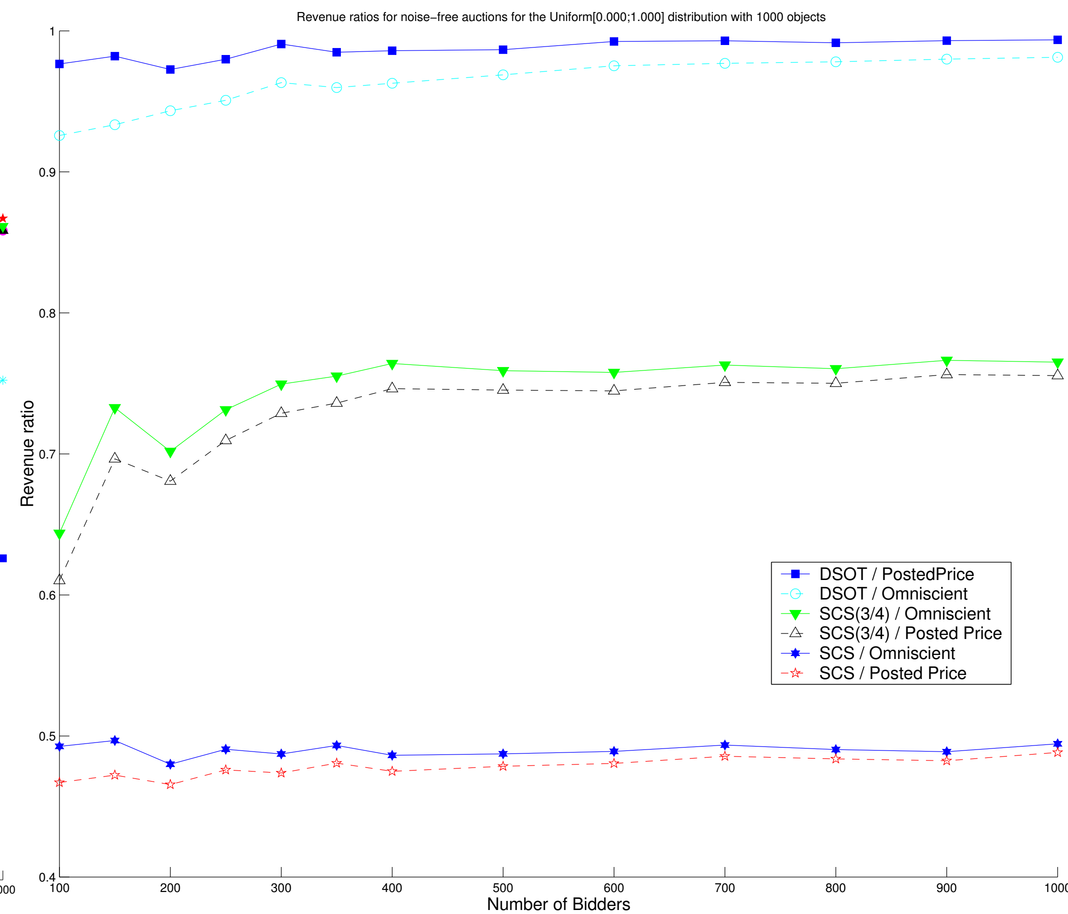
Exponential distribution, bounded supply



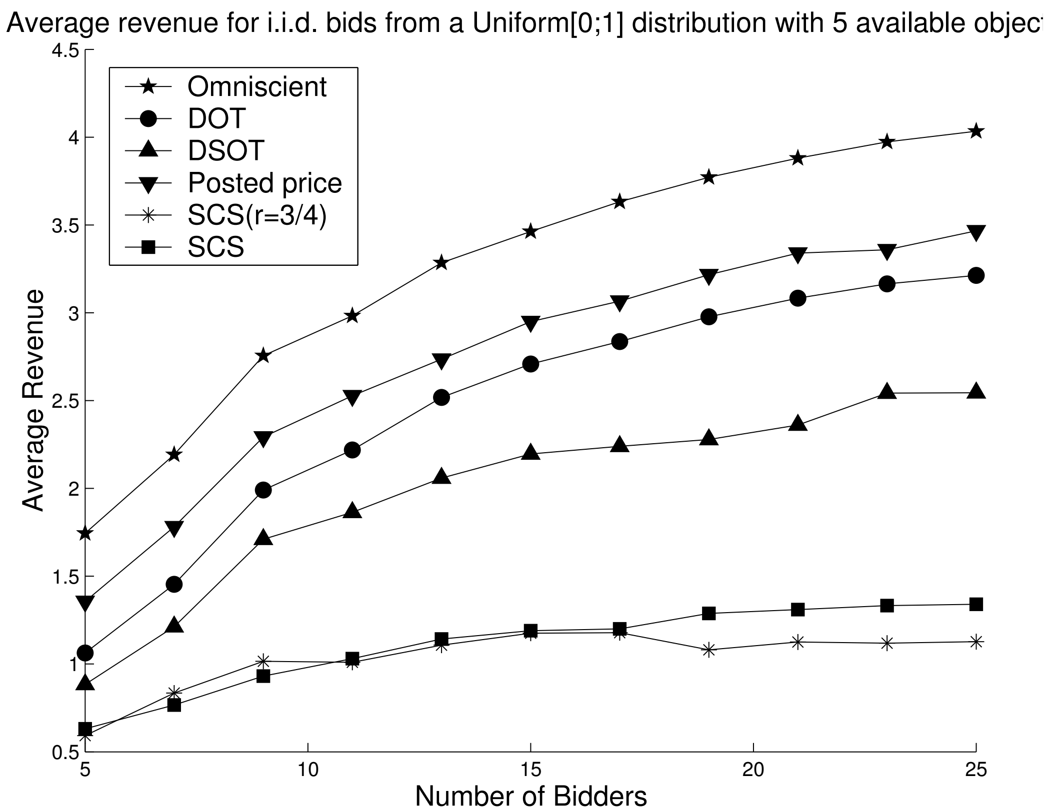


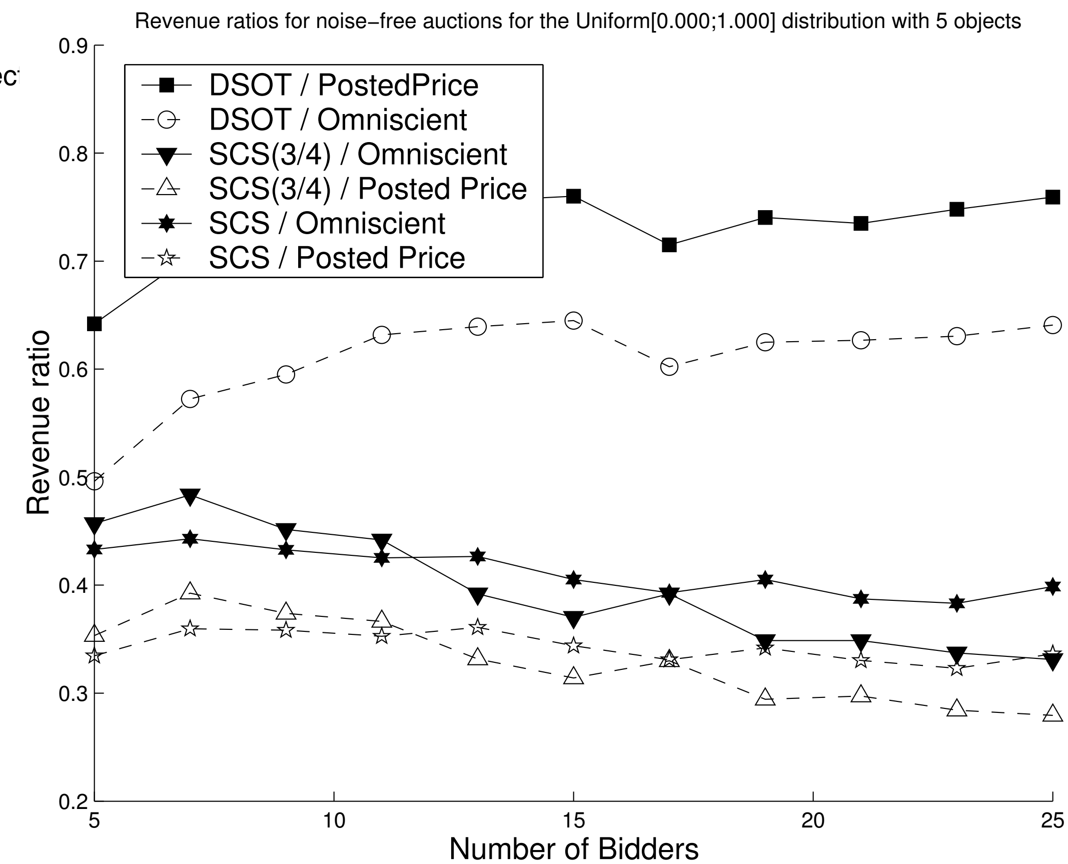
U[0;1] distribution, unbounded supply



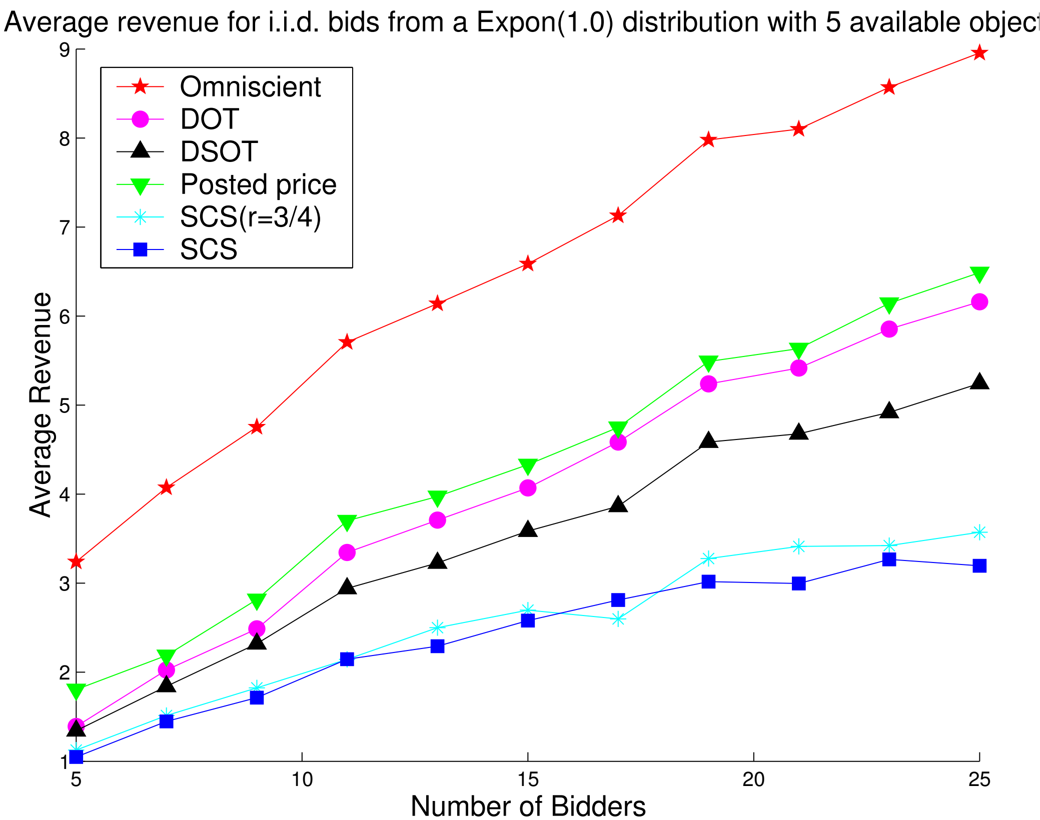


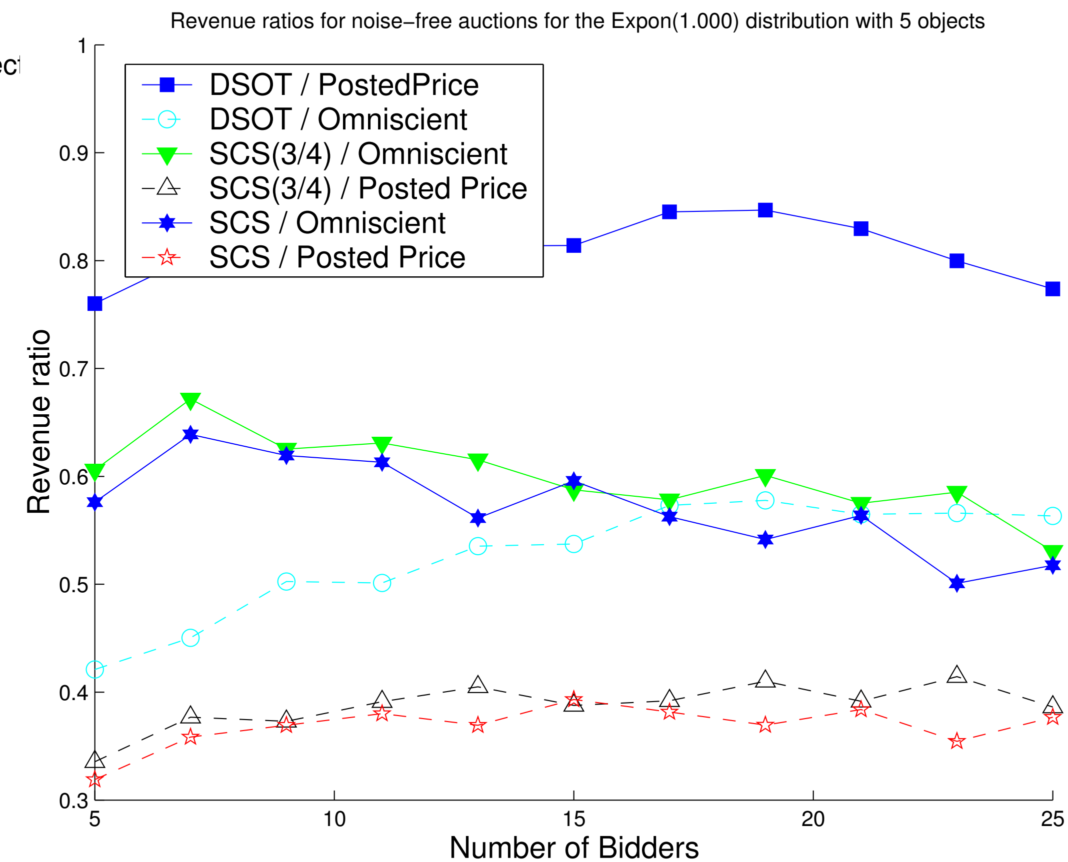
Few items, few bidders, U[0;1] distribution



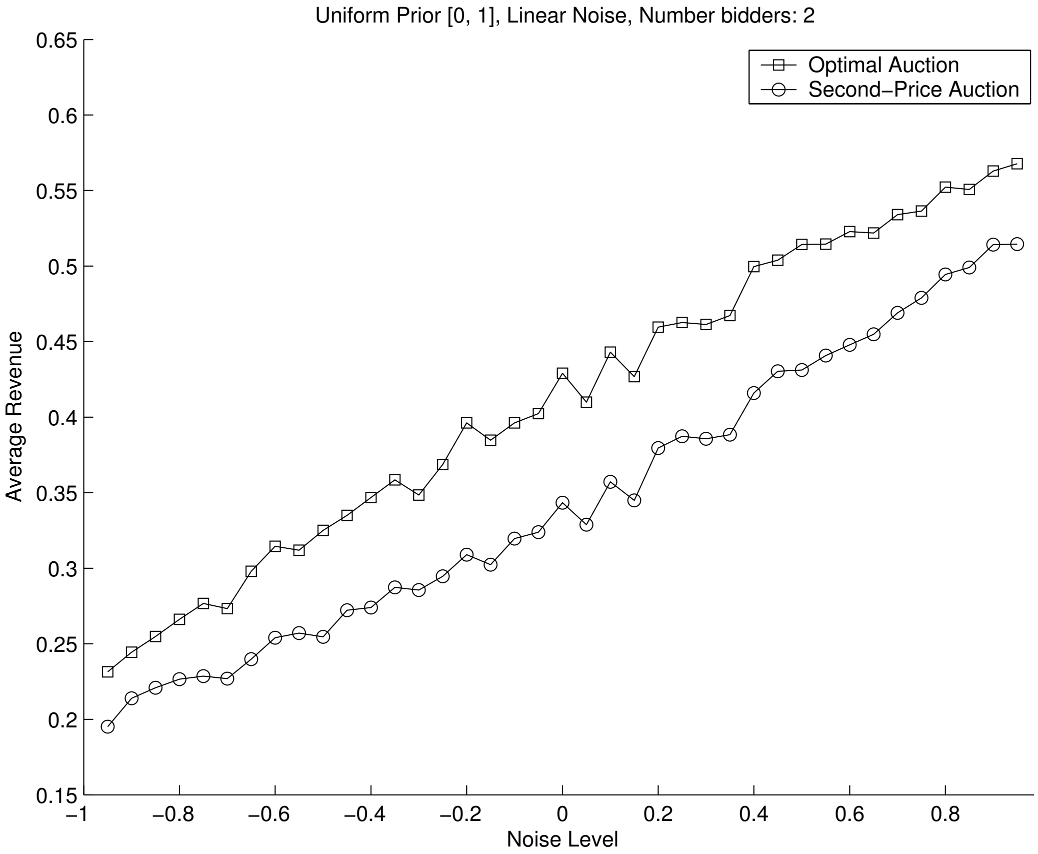


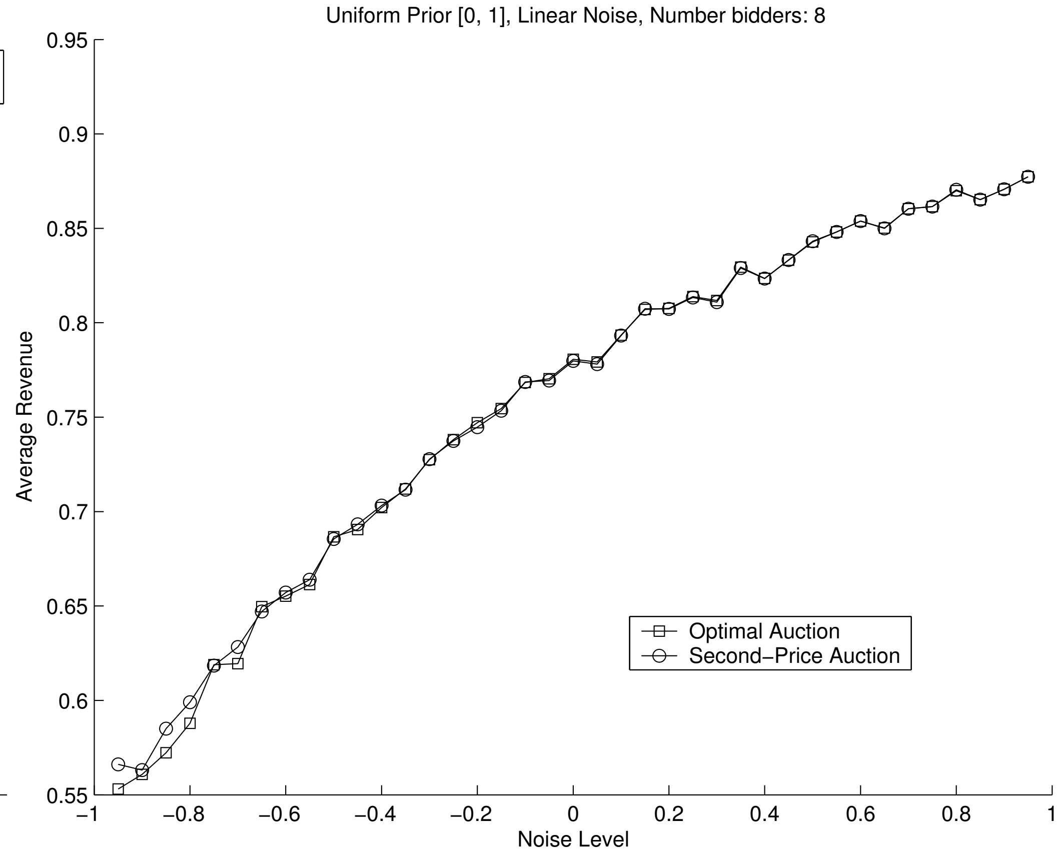
Few items, few bidders, Exponential distribution



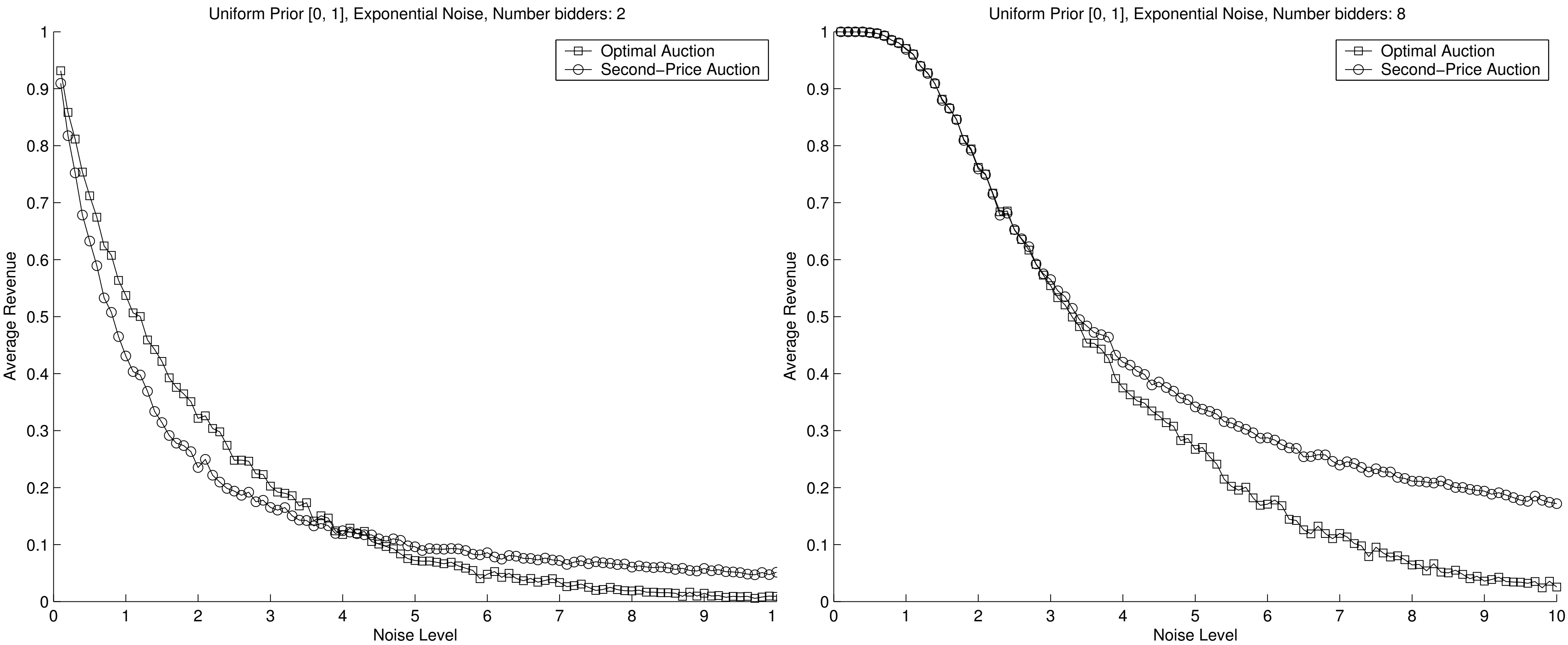


Linear Noise





Exponential Noise



Future Work

- mechanism with good worst-case performance and much better average-case performance

- worst-case optimal auction

- low revenue variance with high probability

- average-case theoretical analysis