CS286r Multi-Agent Learning
Homework 1: Game Theory

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Due: Monday 2/13/2005, at the beginning of class. You may use any sources that you want, but you must cite the sources that you use. You can also work in a group, just list off the people you're working with. **If you took the class last year, or are an economics student please talk to me or the TFs about an alternative homework.** *Work hard on making the proofs clear, concise, and easy to read.* Total points: 130

1. (10 pts) (a) In the following strategic-form game, what strategies survive iterated elimination of strictly-dominated strategies?
   (b) What are the pure strategy Nash eq.?
   (c) Find a non-trivial (support > 1) mixed-strategy NE.

	L	C	R
T	2,0	1,1	4,2
M	3,4	1,2	2,3
B	1,3	0,2	3,0

2. (10 pts) Two agents are bargaining over how to split a dollar. Each simultaneously names the share it would like to have, s_1 and s_2, where $0 \leq s_1, s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the agents receive the shares they named; if $s_1 + s_2 > 1$, then the agents receive zero. What are the pure strategy Nash eq.?

3. (5 pts) Show that there are no (non-trivial) mixed-strategy Nash eq. (i.e. with support greater than one) in the Prisoners' Dilemma game.

	C	D
C	1,1	-1,2
D	2,-1	0,0

4. (15 pts) *Battle of the Sexes.* Pat and Chris must choose to go for dinner or go to the movies. Both players would rather spend the evening together than apart, but Pat would rather the go for dinner, and Chris would rather they go to the movies.

		Chris	
		Dinner	Movie
Pat	Dinner	2,1	0,0
	Movie	0,0	1,2

   (a) Find three Nash equilibria of this game.
   (b) Define a *correlated equilibrium* (CE)? What is the important difference between a CE and a mixed NE? Is there a CE in this game with higher payoffs to both agents than the (non-trivial) mixed NE?

5. (10 pts) Prove that if strategies, $s^* = (s_1^*, \ldots, s_n^*)$, are a Nash eq. in a strategic-form game $G =< N, (S_i), (u_i) >$, then they survive iterated elimination of strictly dominated strategies. **(hint)** *By*

contradiction, assume that one of the strategies in the Nash eq. is eliminated by iterated elimination of strictly dominated strategies.

6. (15 pts) (a) Define the *evolutionarily stable strategy* (ESS) of a game. Will an ESS always exist? Consider the Hawk-Dove game.

	Dove	Hawk
Dove	1/2,1/2	0,1
Hawk	1,0	1/2(1-c),1/2(1-c)

(b) Show that for ($c > 1$), that $\sigma_i = [1 - 1/c, 1/c]$ is a mixed NE and that this is the only ESS.
(c) Show that for ($c < 1$), the unique Nash eq. is (H, H), and that this is the only ESS.
(d) Provide a biologically plausable story to explain your result.

7. (15 pts) Suppose that three players share a pie as follows: First, player 1 proposes a division, then players 2 and 3 simultaneously respond either "yes" or "no." If both say "yes" then the division is implemented; otherwise no player receives anything. Each player prefers more of the pie to less. Formulate this as an *extensive game with simultaneous moves* and find its *subgame perfect equilibria*. Also demonstrate a Nash equilibrium that is **not** subgame perfect.

8. (15 pts) Construct a NE strategy (it need not be subgame perfect) for the infinitely-repeated Prisoner's Dilemma stage game of Q#3. Aim to implement the (C,C) strategy. Adopt the *common discounted* payoff model. Argue that your repeated-game strategy profile is an equilibrium with some discount, $\delta < 1$. *Why might it not be subgame perfect?*

9. (15 pts) Consider the following stage game:

	A	D
A	2,3	1,5
D	0,1	0,1

Show that $((A, A), (A, A), \ldots)$ is not a subgame perfect Nash equilibrium outcome path in an infinitely-repeated version of this game, and with common discount factor $\delta = 0.5$.

10. (15 pts) (a) Compute the set of feasible payoffs in the "battle of the sexes" stage game in Q# 4. Are these payoffs all individual-rational? What is the highest symmetric payoff?
(b) Consider discounting, $\delta = 9/10$, and find a deterministic strategy profile for the repeated game with payoffs $(3/2, 3/2)$. (**Careful:** it is not as obvious as you might initially imagine.)

11. (15 pts) Instantiate the construction of the perfect folk theorem with discounting (p.158-159) in Fudenberg and Tirole for the Prisoner's Dilemma. Provide some intuition for why it is a subgame perfect equilibrium.