

**Harvard University**  
**Bits**  
**Professor Harry R. Lewis**  
**Midterm Examination, March 7, 2007**  
**SHOW YOUR WORK! PLEASE WRITE CLEARLY!**

PROBLEM 1 (15 points)

A source uses five different symbols.

- (a) How many bits would it take to represent the symbols in a simple, fixed-length block code?
- (b) If instead we encoded all possible **pairs** of source symbols using a fixed-length block code (25 codes for the 25 pairs of symbols), what would be the average number of bits **per source symbol**?
- (c) What is the smallest possible average number of bits per symbol for **any** code for this source, on the assumption that all five symbols are equally frequent?
- (d) Now suppose we know that three of the symbols account for 90% of the outputs from the source. Construct a variable-length Huffman code. (Assume the three more frequent symbols occur equally often, and the two less frequent symbols also occur equally often.)
- (e) What is the average number of bits per symbol using your Huffman code?

PROBLEM 2 (15 points)

- (a) What is 29 in binary? 319 in binary? Add the two binary numbers together in binary.
- (b) What is  $12^{23} \bmod 5$ ?

PROBLEM 3 (10 points)

Suppose all 6 million volumes in Widener library are on disk, and that it takes 5 milliseconds ( $5 \times 10^{-3}$  seconds) to read a book from disk.

- (a) In the worst case, how long would it take to search the entire digital library for a title?
- (b) Suppose you could build in primary memory an index of titles in alphabetical order. The index shows where on disk each book is located. If an access to primary memory takes 15 microseconds ( $15 \times 10^{-6}$  seconds), how long would it take to find and retrieve a book?

PROBLEM 4 (5 points)

Who in this course asked the question, “Where do bits go when they die?” and what is the answer?

PROBLEM 5 (5 points)

If it takes 3 minutes on your computer to break DES encryption with 40 bit keys, how long would it take to break the code if 80 bit keys were used?

**THE END**