The Mandatory Disclosure of Trades and Market Liquidity

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Financial market regulations require various “insiders” to disclose their trades after the trades are made. We show that such mandatory disclosure rules can increase insiders’ expected trading profits. This is because disclosure leads to profitable trading opportunities for insiders even if they possess no private information on the asset’s value. We also show that insiders will generally not voluntarily disclose their trades, so for disclosure to be forthcoming, it must be mandatory. Key to the analysis is that the market cannot observe whether an insider is trading on private information regarding asset value or is trading for personal portfolio reasons.

Financial market regulations require certain individuals to disclose their trades. These disclosures are made after the trades have been completed. This paper analyzes the effects of such mandatory disclosure rules on the operation of a market. It is generally believed that more public information concerning the trades of a potentially informed trader limits the ability of the trader to profit on the basis of superior information and provides a more “level playing field” among market participants (for brevity, we refer to a potentially informed trader as an “insider”). Contrary to this view, we show that mandatory disclosure of an in-
sider's trades can increase expected trading profits for the insider relative to a market without disclosure. This increase comes at the expense of noninsiders who trade at a wider bid-ask spread. In this sense, the market is less liquid. Thus, mandatory disclosure can make insiders better off and noninsiders worse off. This results from the market's inability to observe an insider's motive for trading, which combined with mandatory disclosure, leads to profitable trading opportunities for insiders even if they possess no fundamental information. In addition, we show that for disclosure to be forthcoming, it must, in general, be mandatory. Even if ex ante, an insider is better off with disclosure, when the time comes to disclose, an insider will generally not voluntarily disclose his trades.

When the market observes a trade, the motive behind the trade cannot be observed. Traders who are subject to disclosure rules may sometimes trade on the basis of fundamental information about the firm's prospects or they may possess no fundamental information and trade for portfolio or liquidity reasons.\(^1\) Since a trader may possess fundamental information, a trader's disclosure is informative. In particular, when a trader discloses that he (sold) bought stock, the market infers that if the trader is informed, his information is probably (un)favorable and we would expect the stock price to (fall) rise.

The change in the stock price due to disclosure reduces the subsequent expected trading profit of a trader who actually is informed, but increases the subsequent expected trading profit of an uninformed trader. Whether the trader's ex ante (not conditional on whether he is informed) expected trading profit is higher or lower with disclosure depends on the likelihood that he becomes informed. If the likelihood is low (high), then his ex ante expected trading profit is higher (lower) with disclosure. To see the intuition, consider the following example which is a limiting case of the model presented below.\(^2\) Suppose that, with equal probability, a stock is worth 110 or 90 per share, and the initial posted price is 100. With probability \(q\), an insider knows the true stock value and makes an informed trade. With probability \(1 - q\), he is uninformed and makes a portfolio trade (suppose a buy or a sell is equally likely). Now suppose the insider sells stock and his trade is small enough so that it is not detected in the order flow. Hence, with no disclosure, the stock price remains at 100. Given the likelihood that the insider makes an informed trade, if the insider discloses his sell,

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\(^1\) Evidence that insiders sometimes do trade on the basis of fundamental information is presented by Jaffe (1974) and Seyhun (1986), among others. These studies find that insiders' trading returns exceed risk-adjusted benchmarks.

\(^2\) The example is the limit when the number of noninsiders becomes large. In this case, the order flow conveys no information on the insider's trade and the bid-ask spread is zero.
then the stock price drops to $\nabla_1 q + 100(1 - q)$. Now compare the insider's subsequent expected trading profit with and without disclosure of his initial sell.

First, consider the effect of disclosure on an informed insider's expected profit. If there is no disclosure and he sells one more share, based on the information that the stock is worth 90, the expected profit on his second trade is $100 - 90 = 10$. If there is disclosure and he sells one more share, the expected profit on his second trade is $\nabla_1 q + 100(1 - q) - 90 = 10(1 - q)$. Thus, the change in an informed insider's expected profit due to disclosure is $\Delta_I = -10q$. Now consider the effect of disclosure on an uninformed insider's expected profit. If there is no disclosure, his expected profit from trading is 0 since the stock is not mispriced relative to his information. If there is disclosure, an uninformed insider knows that buying stock is profitable. This is because he knows that his initial sell reflects no fundamental information, and thus $\nabla_1 q + 100(1 - q)$ is too low a price for the stock. If he buys one share his expected profit is $100 - 90q - 100(1 - q) = 10q$. Thus the change in an uninformed insider’s expected profit due to disclosure is $\Delta_U = 10q$. Since an informed insider’s loss from disclosure equals an uninformed insider’s gain from disclosure, the insider’s ex ante expected profit is higher with disclosure if the likelihood that he is informed, $q$, is less than 0.5. That is, $q \Delta_I + (1 - q) \Delta_U$ is positive if $q < 0.5$.

In effect, with disclosure, an uninformed insider can manipulate the market. The disclosure of one trade moves the price and creates a profitable subsequent trade. This is why an insider’s expected trading profit can be higher with disclosure.\textsuperscript{5}

A full assessment of the impact of mandatory disclosure regulations also requires an examination of the possibility of voluntary disclosure. This is because if an Insider would always voluntarily disclose his trades, then the regulations are irrelevant since the same outcome prevails in their absence. To consider this possibility, we study a market in which disclosure is voluntary. We find that an equilibrium in which the insider always voluntarily discloses his trades only exists if the probability that the insider possesses fundamental information is very low. Though an insider with no fundamental information has

\textsuperscript{5} A common view of insider trading reporting requirements is reviewed by Brudney (1979), "the more information insiders must disclose about their dealings in the corporation's securities, the less will be the temptation to manipulate the affairs of the company and the release of its information in order to create an impression of value that will facilitate insiders' personal trading in securities." Brudney associates this view with Louis Brandeis' view of disclosure in general, "Sunlight is said to be the best of disinfectants; electric light the most efficient policeman." See Brandeis (1967). While disclosure may eliminate some types of manipulations of corporate affairs, we show that disclosure facilitates other types of manipulations.
the incentive to disclose his trade—he wants to move the price—an insider with fundamental information does not have the incentive to disclose his trade—he does not want to move the price. Thus, mandatory disclosure regulations can have an effect on the market because voluntary disclosure will generally not be forthcoming.

While the specific issues are different, Benabou and Laroque (1992) and Bagnoli and Lipman (1993) study models with similar economic incentives—models with market manipulations that rely on the direct disclosure of information. Benabou and Laroque study a model in which an informed agent publicly discloses a forecast prior to trading. This agent might manipulate the market by disclosing a favorable (unfavorable) forecast when in fact he has observed unfavorable (favorable) news. Bagnoli and Lipman study a model in which a bidder might manipulate the market by buying stock, announcing a takeover bid, selling the stock, and canceling the takeover bid. In our article and in these articles, the presence of a noisy signal leads to profits for some agent. Here, an insider's disclosure of a trade is a noisy signal regarding his information, and disclosing an uninformed trade leads to trading profits for an uninformed insider. In Benabou and Laroque (1992), the agent's forecast is a noisy signal regarding his information, and disclosing a false forecast leads to trading profits for a dishonest agent. In Bagnoli and Lipman (1993), a bidder's announcement of a takeover bid is a noisy signal regarding his intention of whether to go through with the bid, and a false takeover announcement leads to trading profits for a bidder. Unlike these two articles, the manipulation of this article is not based on a fraudulent disclosure (like a false earnings forecast or a false takeover bid announcement). Here the manipulation is based on the truthful disclosure of the insider's actual trade. Further, in effect, these other two articles deal only with "mandatory disclosure." That is, in Benabou and Laroque, the agent must disclose a forecast and in Bagnoli and Lipman, the agent must disclose the takeover bid. Neither article analyzes the case of no disclosure or voluntary disclosure. We analyze all three cases: no disclosure, mandatory disclosure, and voluntary disclosure. This enables us to address the following questions: (i) who gains and who loses from the disclosure of this sort of information; and (ii) would agents voluntarily disclose this sort of information.4

A number of articles study market manipulations that do not rely on the direct disclosure of information. For instance, Kumar and Seppi

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4 Black (1991) also discusses trading opportunities of the sort considered here. He discusses how uninformed traders may attempt to profit by imitating informed traders. If successful, they move the price and they can profit since they know that the price change does not reflect a change in the fundamentals.
(1992) study a manipulation in which an uninformed trader profits by taking a position in a cash-settled futures contract and then trading in the spot market to influence the settlement value of the futures contract. Allen and Gale (1992) also study the issue of manipulation. There, however, the manipulator is actually an informed trader in the usual sense. He has private information that "bad news" is not forthcoming and therefore he buys stock. He profits on the subsequent price run up that results from the absence of bad news. Kyle (1984) studies a manipulation that takes the form of a futures market squeeze. The squeezer takes a sufficiently large long position so that those with short positions are forced to deliver a commodity grade other than the cheapest to deliver. As long as the squeeze is not fully anticipated, the squeezer profits by taking delivery of a higher-quality grade than that which the futures contract was priced against. For a survey of the financial market manipulation literature, see Cherian and Jarrow (1995).

Finally, John and Narayanan (1993) reexamine the model of this article for the case in which the insider is risk neutral and is certainly informed. Like us, they assume that the value of the asset being traded has a two-state distribution. They show that if the probabilities of the two states are sufficiently different, then the informed insider might manipulate the market by initially buying (selling) if he observes bad (good) news and then reversing his trade. In our analysis the two states are assumed to be equally likely and an informed insider does not manipulate the market.

We now briefly describe the regulations that mandate disclosure of financial market trades. In securities markets, Section 16(a) of the Securities and Exchange Act of 1934 imposes a disclosure requirement on corporate insiders, defined as shareholders who own 10 percent or more of the firm's stock, and the firm's officers and directors. These insiders must disclose their trades in the firm's stock within the first 10 days of the month following the trade. In addition, Section 13(d), added to the 1934 act in 1968 by the Williams Act and amended in 1970, requires any individual who acquires 5 percent of a firm's stock to disclose this position within 10 days. Further, material changes in shareholdings must be disclosed within 10 days (as long as the shareholder has at least 5 percent of the stock).5 All of these disclosures are made to the Securities and Exchange Commission (SEC). The SEC

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5 An increase or decrease in shareholdings amounting to 1 percent of the firm's stock is considered material. Smaller changes in shareholdings may be material, depending on the circumstances. Herzel and Shepro (1990) provide a good description and discussion of these disclosure regulations.
makes this information publicly available. Antitrust law also mandates the disclosure of stock trades—specifically, Section 7 of the Clayton Act, as amended in 1976 by the Hart–Scott–Rodino Antitrust Improvements Act. For acquisitions of stock with the intent to acquire control, the disclosure provision is triggered by the acquisition of either 15 percent of a firm's stock or $15 million of a firm's stock. The disclosure is made to the Department of Justice, the Federal Trade Commission, and to the target firm. Though this disclosure is not directly available to the public, the information is typically publicly disclosed by the target (since this is a material event for the target). Acquisitions of less than 10 percent of a firm's stock without the intent to acquire control are exempt from this disclosure requirement. See Herzel and Shepro (1990) for a discussion.

In our model, profitable trading opportunities for insiders who have no fundamental information involve buying and then selling, or the reverse. Individuals who are subject to disclosure under Section 13(d) of the 1934 act only (those who hold between 5 percent and 10 percent of a firm's stock and are not officers or directors of the firm) may buy and sell as they choose. In addition, individuals who are subject to disclosure under Section 7 of the Clayton Act only may buy and sell as they choose. Individuals who are subject to disclosure under Section 16(a) of the 1934 act, however, are also subject to Section 16(b) of the 1934 act, the short-swing profit rule. This rule constrains buy-sell strategies by requiring the affected individuals who buy and sell within a 6-month period to give up any resulting trading profit. While this rule may deter individuals who are subject to Section 16(a) from undertaking the manipulation strategy studied here, our results on voluntary disclosure suggest that the manipulation strategy might not even be feasible if there were no mandatory public disclosure of insiders' trades. Note also that short-term buy-sell strategies can be undertaken by two individuals. For instance, suppose the disclosure of a sale of stock by insider A leads to a drop in the stock price. If insider B knows that the sale was for portfolio/liquidity reasons, then insider B knows that buying stock is a profitable opportunity. As was the case with one insider, the disclosure of one trade creates a profitable subsequent trade.

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6 The disclosure documents are publicly available immediately upon filing. In addition, the SEC News Digest is published daily and lists the date, size, and price for all of the disclosed trades as well as traders' company name, title, and aggregate stockholdings. A casual perusal suggests that a typical lag from the date of the trade until publication is between 1 and 2 weeks. This publication is readily available to the public. In addition, there are commercial publications that publish this information. For instance, CDA Investment Technologies Inc. publishes the CDA/InvestNet Insider's Chronicle and the Quarterly Summary of Insider Trading which summarize the disclosed trades on a weekly and quarterly basis respectively.
Evidence that the disclosure of trade information does move prices has been found by Mikkelsen and Ruback (1985). For a sample of 230 Schedule 13D filings (as mandated by Section 13(d)) between 1978 and 1980, they find an average abnormal 2-day return at the time of the filing of 2.88 percent. Of the 230 filings, 132 indicated the motive for the transaction: 26 indicated that the acquirer was considering an acquisition and 106 indicated that the transaction was made for investment purposes. The average abnormal 2-day return at the time of the filing for these two subsamples is 7.74 percent and 3.24 percent respectively. Bagnoli and Lipman (1993) cite an example where an individual has indeed attempted to profit from the price response that accompanies announcements of share acquisitions and takeovers. In 1988, T. Boone Pickens announced that Mesa Petroleum had acquired 3.5 million shares (3.8 percent of the total) of Homestake Mining. The stock price of Homestake Mining increased from $14 to $18. Over the following week, Mesa sold about 3.3 million of Homestake’s shares and 3200 call options on Homestake’s shares. The SEC objected to this transaction and Mesa settled by giving up $2.3 million in profit (see Wall Street Journal, September 28, 1990).

This article is organized as follows. Section 1 presents the model. Section 2 derives an equilibrium for the case with no disclosure of an insider’s trades. Section 3 derives an equilibrium for the case with mandatory disclosure of an insider’s trades. Section 4 compares the no-disclosure equilibrium to the mandatory-disclosure equilibrium. Section 5 considers the possibility of voluntary disclosure. Section 6 concludes the article.

1. The Model

Consider a market in which shares of a risky asset are traded for cash. The per share value of the asset is \( \theta \), which with equal probability equals \( \theta_H \) or \( \theta_L \), where \( \theta_H > \theta_L > 0 \). Let \( \bar{\theta} = (\theta_H + \theta_L)/2 \). There are two trading dates, 1 and 2, and the realization of \( \theta \) is publicly observed after date 2.

There are \( 2N + 1 \) traders. Traders have a concave utility function over final wealth given by \( U(\cdot) \). Trader \( j \) is endowed with cash, \( w_j \), and shares of the risky asset, \( z_j \). Specifically, for \( j = 0, 1, \ldots, 2N \), with probability 1/2, \( w_j = w - \bar{\theta} \) and \( z_j = 1 \), and with probability 1/2, \( w_j = w + \bar{\theta} \) and \( z_j = -1 \); traders are either long or short one share of the risky asset. This parameterization ensures that buyers and sellers are symmetric; in particular, there are no differential wealth effects induced by an endowment that is either long or short shares. With no trading, trader \( j \)'s expected final wealth equals \( w \). Trader \( j \)'s
endowment is privately observed by trader \( j \), and endowments are independent of one another and independent of \( \theta \).

Trader 0 is an "insider." With probability \( q \), the insider is informed—he privately observes the realization of \( \theta \) before date 1. With probability \( 1 - q \), the insider is uninformed regarding \( \theta \). The parameter \( q \) is known by all; everyone has the same beliefs regarding the likelihood that the insider will observe \( \theta \). Whether the insider actually is informed is observed only by the insider and is independent of \( \Theta \) and independent of traders' endowments. Let \( I \in \{ \theta_L, \theta_H, \emptyset \} \) denote the insider’s information; he either observes \( \theta = \theta_L \), \( \theta = \theta_H \), or nothing at all. The insider can trade at both dates 1 and 2. At each date the insider can either buy one share, sell one share, or not trade. For \( t = 1, 2 \), let \( x_t \in \{-1, 0, 1\} \) denote the insider's date-\( t \) trade: a sell, no trade, or a buy.

The other \( 2N \) traders are uninformed regarding \( \theta \). We refer to these traders as "outsiders." Assume that outsiders \( j = 1, \ldots, N \) trade at date 1 and outsiders \( j = N + 1, \ldots, 2N \) trade at date 2. Also assume that outsiders hedge their positions; outsider \( j \) buys one share if \( z_j = -1 \) and sells one share if \( z_j = 1 \).

We want to consider a case in which the insider may trade for informational reasons or may trade for portfolio reasons. To ensure the latter, assume that

\[
U \left( w - \frac{q \theta_H - \theta_L}{N + 1} \right) > U \left( w + \frac{\theta_L - \theta_H}{2} \right) + U \left( w - \frac{\theta_L - \theta_H}{2} \right). 
\]  

(1)

The right side of Inequality (1) is an uninformed insider's expected utility if he does not trade. It will be shown that the left side of Inequality (1) is the uninformed insider’s expected utility if he trades to hedge his endowment risk. The left side is increasing in \( N \) and since \( U(\cdot) \) is concave, there is some sufficiently large \( N \) for which Inequality (1) holds. Thus Inequality (1) is equivalent to an assumption that there are sufficiently many outsiders. This is because, as will be seen, a larger \( N \) results in a narrower bid-ask spread.

The modeling of the trading process follows Admati and Pfleiderer (1989) and Easley and O’Hara (1990). A competitive risk-neutral market maker posts a bid and an ask price at each trading date. The market maker is prepared to satisfy all date-\( t \) buys (sells) at the date-\( t \) ask (bid).

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7 This assumption simplifies the analysis. If some outsiders can choose when to trade, patterns may arise in which outsiders' buys and sells are concentrated at particular trading dates. See Admati and Pfleiderer (1988, 1989) and Foster and Viswanathan (1990).
The sequence of events is as follows:

Date 1:

(i) The insider, if informed, observes $\theta = \theta_H$ or $\theta = \theta_L$.

(ii) The market maker posts date-1 bid and ask prices.

(iii) The $N$ outsiders who trade at date 1 and the insider submit their orders to the market maker.

(iv) Traders and the market maker observe the number of date-1 buys and sells.

(v) The insider either (truthfully) discloses his date-1 trade or discloses nothing.

Date 2:

(i) The market maker posts date-2 bid and ask prices.

(ii) The $N$ outsiders who trade at date 2 and the insider submit their orders to the market maker.

(iii) The realization of $\theta$ is observed.

Let $x_{b1}$ and $x_{a1}$ denote the total number of date-1 buys and sells. Let $d$ denote the insider’s disclosure, where $d \in \{x_1, \theta\}$; the insider either discloses his date-1 trade or discloses nothing. Based on the order flow and the insider’s disclosure (if any), the market maker updates his beliefs regarding the realization of $\theta$ and $I$. Let the joint probability density function $g(\theta, I | x_{b1}, x_{a1}, d)$ denote these updated beliefs conditional on $x_{b1}$, $x_{a1}$, and $d$.

Let $A_1$ and $B_1$ denote the market maker’s date-1 ask and bid pricing rules. Let $A_2(x_{b1}, x_{a1}, d)$ and $B_2(x_{b1}, x_{a1}, d)$ denote the market maker’s date-2 pricing rules as a function of the date-1 order flow and the insider’s disclosure. The actual date-1 prices are denoted $a_i$ and $b_i$.

The insider’s date-1 trading strategy is denoted $X_1(u_0, z_0, a_1, b_1, I)$ ($x_1$ is his actual date-1 trade). His date-1 trade is a function of his endowment, date-1 ask and bid prices, and his information. Let $u'_0$ and $z'_0$ denote the insider’s cash and stock position after his date-1 trade; $u'_0 = u_0 + b_1$ if he sells at date 1, $u'_0 = u_0 - a_1$ if he buys at date 1, $u'_0 = u_0$ if he does not trade at date 1, and $z'_0 = z_0 + x_1$.

The insider’s date-2 trading strategy is denoted $X_2(u'_0, z'_0, a_2, b_2, I)$ ($x_2$ is his actual date-2 trade). His date-2 trade is a function of his date-2 cash and stock position, date-2 ask and bid prices, and his information. Note that the insider’s date-2 trading strategy is not a function of the date-1 order flow or his own disclosure. Conditional on $I$, this data is not informative.

We now define an equilibrium with no disclosure and an equilibrium with mandatory disclosure. With no disclosure (mandatory disclosure) of the insider’s date-1 trade, an equilibrium consists of a trading strategy $X_1$, $X_2$, pricing rules $A_1$, $B_1$, $A_2$, $B_2$, and beliefs $g$ such
that (i) $X_1$ and $X_2$ maximize the insider's expected utility taking $A_1$, $B_1$, $A_2$, $B_2$, $g$, and $d = \emptyset$ ($d = x_1$) as given; (ii) $A_t$ and $B_t$ generate the minimum nonnegative expected profit on the ask and bid sides of the market respectively, for the market maker at date $t$ taking $X_1$, $X_2$, $g$, and $d = \emptyset$ ($d = x_1$) as given; and (iii) $g$ is consistent with Bayes' Rule in equilibrium.

2. Equilibrium with No Disclosure of the Insider's Trades

To construct an equilibrium with no disclosure, we hypothesize a date-1 and date-2 trading strategy for the insider and then solve for the market maker's pricing rules. Then we show that the hypothesized trading strategy is optimal given the derived pricing rules.

First, consider date 1. Suppose an informed insider buys if $\theta = \theta_H$ and sells if $\theta = \theta_L$. Suppose an uninformed insider hedges; he sells if $z_0 = 1$ and he buys if $z_0 = -1$. Formally,

$$ X_1(u_b, z_0, a_1, b_1, I) = \begin{cases} 
1 & \text{if } I = \theta_H \\
-1 & \text{if } I = \theta_L \\
-z_0 & \text{if } I = \emptyset.
\end{cases} \quad (2) $$

Taking Equation (2) as given, the equilibrium date-1 bid price is computed as follows. For a bid of $b_1$, the market maker's expected profit on the bid side of the market is

$$ \pi_{B1}(b_1) = \frac{1}{2}(\theta_H - b_1)E[\bar{x}_{s1} \mid b_1, \theta = \theta_H] \\
+ \frac{1}{2}(\theta_L - b_1)E[\bar{x}_{s1} \mid b_1, \theta = \theta_L] \\
= \frac{1}{2}(\theta_H - b_1) \left( q \frac{N}{2} + (1 - q) \frac{N + 1}{2} \right) \\
+ \frac{1}{2}(\theta_L - b_1) \left( q \left( \frac{N}{2} + 1 \right) + (1 - q) \frac{N + 1}{2} \right), $$

where the calculation of the expected number of sells conditional on $b_1$ and $\theta$ accounts for the fact that with probability $q$, the insider buys (sells) if $\theta = \theta_H$ ($\theta = \theta_L$), with probability $1 - q$, the insider buys or sells with equal probability and outsiders each buy or sell with equal probability. Solving $\pi_{B1}(b_1) = 0$ yields the date-1 zero expected profit bid of

$$ B_1 = \bar{\theta} - \frac{q}{N + 1} \frac{\theta_H - \theta_L}{2}. \quad (3) $$
An analogous derivation of the date-1 zero expected profit ask yields

$$A_1 = \tilde{\theta} + \frac{q}{N+1} \frac{\theta_H - \theta_L}{2}. \quad (4)$$

The bid and ask satisfy $B_1 < \tilde{\theta} < A_1$, and the market maker's expected profit from outsiders offsets the market maker's expected loss to the insider. The bid-ask spread, $q(\theta_H - \theta_L)/(N+1)$, is increasing in $q$, the likelihood that the insider is informed, is increasing in $\theta_H - \theta_L$, the degree of uncertainty regarding $\theta$, and is decreasing in $N$, the number of outsiders.

With no disclosure, the market maker updates his beliefs on $\theta$ and $I$ based on the date-1 order flow, $x_{b1}$ and $x_{s1}$. Using Bayes' Rule, for $x_{b1} + x_{s1} = N + 1$, $g(\theta_H, \theta_L | x_{b1}, x_{s1}, \emptyset) = q x_{b1} / (N+1)$, and $g(\theta_H, \emptyset | x_{b1}, x_{s1}, \emptyset) = (1-q)/2$. Note that the order flow conveys no information on the probability that the insider is informed. This is because the insider is equally likely to buy or sell, whether informed or not, and thus the likelihood of observing any particular order flow is independent of whether he is informed.

Now consider date 2. Suppose an informed insider buys if $\theta = \theta_H$ and sells if $\theta = \theta_L$. If an uninformed insider follows the hypothesized date-1 strategy given by Equation (2), then his position at the beginning of date 2 consists of $u_0' = w - \tilde{\theta} + B_1 = w + \tilde{\theta} - A_1 = w - q(\theta_H - \theta_L)/2(N+1)$ in cash and $x_0' = 0$ shares (he hedged at date 1). Suppose, at date 2, an uninformed insider sells if $b_2 > B_R$ and buys if $a_2 < A_R$, where $A_R(B_R)$ denotes the reservation price at which an uninformed insider is willing to reestablish a long (short) position after hedging at date 1. That is, $A_R$ and $B_R$ satisfy

$$U \left( w - \frac{q}{N+1} \frac{\theta_H - \theta_L}{2} \right)$$

$$= U \left( w - \frac{q}{N+1} \frac{\theta_H - \theta_L}{2} + \theta_H - A_R \right) + U \left( w - \frac{q}{N+1} \frac{\theta_H - \theta_L}{2} + \theta_L - A_R \right)$$

$$= \frac{U \left( w - \frac{q}{N+1} \frac{\theta_H - \theta_L}{2} + \theta_H - A_R \right) + U \left( w - \frac{q}{N+1} \frac{\theta_H - \theta_L}{2} + B_R - \theta_H \right)}{2}.$$

Since the insider is risk averse, $A_R < \tilde{\theta} < B_R$. Formally, this date-2
trading strategy for the insider satisfies

\[ X_2(w'_0, z'_0, a_2, b_2, I) = \begin{cases} 
1 & \text{if } I = \theta_H \\
1 & \text{if } I = \emptyset, z'_0 = 0, \text{ and } a_2 < A_R \\
-1 & \text{if } I = \theta_L \\
-1 & \text{if } I = \emptyset, z'_0 = 0, \text{ and } b_2 > B_R \\
0 & \text{if } I = \emptyset, z'_0 = 0, a_2 \geq A_R, \text{ and } b_2 \leq B_R.
\end{cases} \tag{5} \]

Taking Equations (2) and (5) as given, the equilibrium date-2 bid price is computed as follows. Let \(\pi_{B2}(b_2, x_{b1}, x_{s1}, d)\) denote the market maker’s date-2 expected profit on the bid side of the market as a function of the date-2 bid, the date-1 order flow and the insider’s disclosure. With no insider disclosure,

\[
\begin{align*}
\pi_{B2}(b_2, x_{b1}, x_{s1}, \emptyset) & = (\theta_H - b_2)E[\tilde{x}_{s2} \mid b_2, \theta = \theta_H, I = \emptyset]g(\theta_H, \theta_H \mid x_{b1}, x_{s1}, \emptyset) \\
& \quad + E[\tilde{x}_{s2} \mid b_2, \theta = \theta_H, I = \emptyset]g(\theta_H, \emptyset \mid x_{b1}, x_{s1}, \emptyset) \\
& \quad + (\theta_L - b_2)E[\tilde{x}_{s2} \mid b_2, \theta = \theta_L, I = \emptyset]g(\theta_L, \theta_L \mid x_{b1}, x_{s1}, \emptyset) \\
& \quad + E[\tilde{x}_{s2} \mid b_2, \theta = \theta_L, I = \emptyset]g(\theta_L, \emptyset \mid x_{b1}, x_{s1}, \emptyset) \\
& = (\theta_H - b_2)\left[\left(\frac{N}{2} + 1\right)q \frac{N}{N+1} + \frac{N - 1}{2}\right] \text{ if } b_2 \leq B_R \\
& \quad + (\theta_L - b_2)\left[\left(\frac{N}{2} + 1\right)\frac{1 - q}{2}\right] \text{ if } b_2 \leq B_R \\
& \quad + (\theta_H - b_2)\left[\frac{N}{2} + 1\right]q \frac{N}{N+1} + \frac{N - 1}{2} \text{ if } b_2 > B_R \\
& \quad + (\theta_L - b_2)\left(\frac{N}{2} + 1\right)\frac{1 - q}{2} \text{ if } b_2 > B_R.
\end{align*}
\]

The calculation of the expected number of sells conditional on \(b_2, \theta\), and \(I\) accounts for the fact that if the insider is informed, he sells only if \(\theta = \theta_H\), if the insider is uninformed, he sells only if \(b_2 > B_R\), and each outsider sells with probability 1/2.

Now consider the date-2 bid that yields the market maker the minimum nonnegative expected profit. If \(\pi_{B2}(B_R, x_{b1}, x_{s1}, \emptyset) \leq 0\), then there is a zero expected profit bid which is less than or equal to \(B_R\). This bid is given by

\[
B_2^{NT}(x_{b1}) = \frac{\theta_H N}{2} \left(\frac{\frac{N}{N+1}}{\frac{N}{N+1}} + \frac{1 - q}{2}\right) + \theta_L \left[\left(\frac{N}{2} + 1\right)q \frac{N}{N+1} + \frac{N - 1}{2}\right] \tag{6}
\]

This is found by solving \(\pi_{B2}(b_2, x_{b1}, x_{s1}, \emptyset) = 0\) for \(b_2\) when \(b_2 \leq B_R\).
The superscript $NT$ denotes that an uninformed insider would not trade at this bid. If $\pi_{B2}(B_R + \varepsilon, x_b1, x_s1, \emptyset) \geq 0$ for some $\varepsilon > 0$, then there is a zero expected profit bid which is greater than $B_R$. This bid is given by

$$B_2^T(x_b1) = \frac{\theta_H \left[ \frac{N}{2} \frac{q_x a}{N+1} + \left( \frac{N}{2} + 1 \right) \frac{1-q}{2} \right] + \theta_L \left( \frac{N}{2} + 1 \right) \left[ q \frac{N+1-x_s a}{N+1} + \frac{1-q}{2} \right]}{\frac{N}{2} + 1 - \frac{q_x a}{N+1}}$$

(7)

This is found by solving $\pi_{B2}(b_2, x_b1, x_s1, \emptyset) = 0$ for $b_2$ when $b_2 > B_R$. The superscript $T$ denotes that an uninformed insider would trade at this bid. Finally, if $\pi_{B2}(B_R, x_b1, x_s1, \emptyset) > 0 > \pi_{B2}(B_R + \varepsilon, x_b1, x_s1, \emptyset)$ for all $\varepsilon > 0$, then the minimum nonnegative expected profit for the market maker is positive and is attained with a bid of $B_R$. In summary, for $x_b1 + x_s1 = N+1$, the date-2 bid that yields the market maker the minimum nonnegative expected profit is given by

$$B_2^{NT}(x_b1) = \begin{cases} B_2^T(x_b1) & \text{if } B_2^{NT}(x_b1) \leq B_R \\ B_R & \text{if } B_2^T(x_b1) > B_R \end{cases}$$

(8)

An analogous derivation applies to the date-2 ask side of the market. For $x_b1 + x_s1 = N+1$, the date-2 ask that yields the market maker the minimum nonnegative expected profit is given by

$$A_2^{NT}(x_b1) = \begin{cases} A_2^T(x_b1) & \text{if } A_2^{NT}(x_b1) \geq A_R \\ A_R & \text{if } A_2^T(x_b1) < A_R \end{cases}$$

(9)

where

$$A_2^{NT}(x_b1) = \frac{\theta_H \left[ \left( \frac{N}{2} + 1 \right) \frac{q_x a}{N+1} + \frac{N}{2} \frac{1-q}{2} \right] + \theta_L \frac{N}{2} \left[ q \frac{N+1-x_s a}{N+1} + \frac{1-q}{2} \right]}{\frac{N}{2} + \frac{q_x a}{N+1}}$$

(10)

and

$$A_2^T(x_b1) = \frac{\theta_H \left( \frac{N}{2} + 1 \right) \left[ \frac{q_x a}{N+1} + \frac{1-q}{2} \right] + \theta_L \left[ \frac{N}{2} q \frac{N+1-x_s a}{N+1} + \left( \frac{N}{2} + 1 \right) \frac{1-q}{2} \right]}{\frac{N}{2} + 1 - q + \frac{q_x a}{N+1}}$$

(11)

Summarizing the results thus far, if the insider’s trading strategy is given by Equations (2) and (5), then date-1 bid and ask prices are given by Equations (3) and (4), and date-2 bid and ask prices are given by Equations (8) and (9). Proposition 1 verifies that there is an
equilibrium with no disclosure in which the insider's strategy satisfies Equations (2) and (5).

**Proposition 1.** There is an equilibrium with no disclosure in which (i) the insider's strategy satisfies Equations (2) and (5); (ii) date-1 bid and ask prices are given by Equations (3) and (4); and (iii) for equilibrium date-1 order flows, date-2 bid and ask pricing rules are given by Equations (8) and (9).

**Proof of Proposition 1.** See the Appendix.

In the equilibrium of Proposition 1, if the insider is informed he trades with the information at both dates. If he is uninformed, he hedges at date 1. At date 2, he buys if the ask is sufficiently low, sells if the bid is sufficiently high, and does not trade otherwise.

It is assumed that outsider $j$ hedges his position. Given the choice to buy one share, sell one share, or not trade, under what conditions would outsiders endogenously choose to hedge? Assumption (1) implies that date-1 outsiders would choose to hedge. Date-2 outsiders would choose to hedge if

$$U(w - \tilde{\theta} + B_2(x_{b1}, x_{s1}, 0)) > E[U(w - \tilde{\theta} + \tilde{\theta})]_{x_{b1}, x_{s1}, d = 0}$$

and

$$U(w + \tilde{\theta} - A_2(x_{b1}, x_{s1}, 0)) > E[U(w + \tilde{\theta} - \tilde{\theta})]_{x_{b1}, x_{s1}, d = 0},$$

(12)

for all $x_{b1}, x_{s1}$.

Both Inequalities (1) and (12) are equivalent to conditions that the number of outsiders at each date, $N$, is sufficiently large, and thus bid and ask prices are sufficiently close to the expected share value.\(^8\)

In equilibrium, the date-2 bid and ask are high if there are many date-1 buys and low if there are many date-1 sells. An insider who did not observe $\theta$ knows that $\tilde{\theta}$ is a fair price for the shares, and this is not known by the market maker. So if the date-1 order flow leads the market maker to set too high or too low a price, an “uninformed” insider may make an “informed” trade. The highest date-2 bid follows a date-1 order flow of $N + 1$ buys and zero sells, and if

$$B_2(N + 1, 0, 0) > B_R,$$

(13)

then at this bid, an uninformed insider sells at date 2. Since $B_R$ and $A_R$ are symmetric about $\tilde{\theta}$, and $B_2(N + 1, 0, 0)$ and $A_2(0, N + 1, 0)$ are

---

\(^8\) Inequalities (1) and (12) imply that outsiders would prefer to buy or sell one share rather than not trade. Of course, since outsiders are purchasing unfairly priced insurance, they would prefer to underinsure—that is, buy or sell less than one share.
symmetric about $\bar{\theta}$, Inequality (13) implies

$$A_2(0, N + 1, \emptyset) < A_R.$$  \hspace{1cm} (14)

Inequality (14) implies that an uninformed insider buys at date 2 following a date-1 order flow of zero buys and $N + 1$ sells. If inequalities (13) and (14) are not satisfied, an uninformed insider never trades at date 2.  

3. Equilibrium with Mandatory Disclosure of the Insider's Trades

We now construct an equilibrium with mandatory disclosure of the insider's date-1 trade. This disclosure is made prior to date-2 trading. As before, we hypothesize a date-1 and date-2 trading strategy for the insider and solve for the market maker's pricing rules. Then we show that the hypothesized trading strategy is optimal given the derived pricing rules.

Suppose the insider follows the same date-1 and date-2 trading strategy as with no disclosure as given by Equations (2) and (5). With the same date-1 trading strategy, the market maker's date-1 bid and ask prices are given by Equations (3) and (4), the same as with no disclosure.

The market maker updates his beliefs on $\theta$ and $I$ based on the insider's disclosure—the order flow is no longer informative. Using Bayes' Rule, $g(\theta_H, \theta_H \mid x_{b1}, x_{s1}, 1) = g(\theta_L, \theta_L \mid x_{b1}, x_{s1}, -1) = q$, $g(\theta_H, \theta_H \mid x_{b1}, x_{s1}, -1) = g(\theta_L, \theta_L \mid x_{b1}, x_{s1}, 1) = 0$, and $g(\theta_H, \emptyset \mid x_{b1}, x_{s1}, d) = g(\theta_L, \emptyset \mid x_{b1}, x_{s1}, d) = (1 - q)/2$. The information conveyed by the disclosure of an insider buy (sell) is equivalent to the information conveyed by $N + 1$ (0) total buys with no disclosure. Thus, if the insider follows the same date-2 strategy as with no disclosure, the market maker's date-2 prices following the disclosure of a buy (sell) are the same as the date-2 prices following $N + 1$ (0) buys with
no disclosure. Specifically, using Equations (8) and (9), if the insider discloses a date-1 buy, the market maker's date-2 bid and ask prices are given by

$$
B_2(x_{b1}, x_{s1}, 1) = \begin{cases} 
B_2^{NT}(N + 1) & \text{if } B_2^{NT}(N + 1) \leq B_R \\
B_2^T(N + 1) & \text{if } B_2^T(N + 1) > B_R \\
B_R & \text{otherwise}
\end{cases} 
$$

$$
A_2(x_{b1}, x_{s1}, 1) = A_2^{NT}(N + 1). 
$$

(15)

If the insider discloses a date-1 sell, the market maker's date-2 bid and ask prices are given by

$$
B_2(x_{b1}, x_{s1}, -1) = B_2^{NT}(0) 
$$

$$
A_2^{NT}(0) & \text{if } A_2^{NT}(0) \geq A_R \\
A_2^T(0) & \text{if } A_2^T(0) < A_R \\
A_R & \text{otherwise.}
\end{cases} 
$$

(16)

Summarizing, if the insider's trading strategy satisfies Equations (2) and (5), then date-1 bid and ask prices are given by Equations (3) and (4), and date-2 bid and ask prices are given by Equations (15) and (16). Proposition 2 verifies that there is an equilibrium with mandatory disclosure in which the insider's strategy satisfies Equations (2) and (5).

**Proposition 2.** There is an equilibrium with mandatory disclosure in which (i) the insider's strategy satisfies Equations (2) and (5); (ii) date-1 bid and ask prices are given by Equations (3) and (4); and (iii) for equilibrium date-1 insider trades, date-2 bid and ask pricing rules are given by Equations (15) and (16).

**Proof of Proposition 2.** See the Appendix. ■

With mandatory disclosure, the date-2 bid and ask are above $\tilde{\theta}$ if the insider discloses a date-1 buy and below $\tilde{\theta}$ if the insider discloses a date-1 sell. Since an uninformed insider knows that $\tilde{\theta}$ is a fair price for the shares, he knows that reversing his date-1 trade has a positive expected profit. If Inequalities (13) and (14) are satisfied, then the expected profit is sufficiently high so that he is willing to reestablish a risky position (he had hedged his position at date 1). If he bought (sold) at date 1, he sells (buys) at date 2.

If Inequalities (13) and (14) are satisfied, then an uninformed insider is, in effect, manipulating the market. The disclosure of his first trade moves the price enough so that he profits by subsequently trad-
ing in the opposite direction. Though an uninformed insider’s date-1 trade can be said to be a legitimate portfolio trade, he is willing to make the same trade even without such a motive. That is, suppose the insider’s initial endowment consists only of cash and thus the insider has no portfolio motive for trading. There is a mixed strategy equilibrium with mandatory disclosure in which at date 1 an uninformed insider buys with probability 1/2 and sells with probability 1/2, and at date 2, he makes the opposite trade. The date-1 and date-2 prices and the informed insider’s trading strategy are identical to those of Proposition 2. An uninformed insider has the incentive to make a date-1 trade, even though it is unprofitable, in order to make a profitable date-2 trade. The total profit is riskless and equals

\[ B_2(x_{b1}, x_{s1}, 1) - A_1 = B_1 - A_2(x_{b1}, x_{s1}, -1) > 0. \]

While disclosure improves the trading opportunities for an uninformed insider, it worsens the trading opportunities for an informed insider. For instance, an informed insider who observes \( \theta = \theta_H \) buys at both dates. Disclosing the date-1 buy leads to the insider paying a higher price, relative to no disclosure, when he buys at date 2, that is, \( A_2(x_{b1}, x_{s1}, 1) \geq A_2(x_{b1}, x_{s1}, \emptyset) \). The overall effect on an insider’s expected profit depends on the likelihood that he is informed.\(^{10}\)

The insider’s trading behavior and market prices were illustrated in the introduction for the limiting case in which the number of outsiders, \( N \), becomes large. Suppose that Inequalities (13) and (14) are satisfied for large \( N \). As \( N \to \infty \), the bid-ask spread at each date collapses to zero about the market maker’s expected value of a share. At date 1, \( A_1 = B_1 = \tilde{\theta} \). At date 2, with no disclosure \( A_2 = B_2 = \tilde{\theta} \), and with mandatory disclosure

\[
A_2(x_{b1}, x_{s1}, d) = B_2(x_{b1}, x_{s1}, d) = \begin{cases} 
(\theta_H(1+q) + \theta_L(1-q))/2 & \text{if } d = 1 \\
(\theta_H(1-q) + \theta_L(1+q))/2 & \text{if } d = -1 
\end{cases}
\]

An insider’s date-1 expected profit does not depend on whether there is disclosure. An informed insider’s date-2 expected profit equals \( \theta_H - \tilde{\theta} \) with no disclosure and \( \theta_H - (\theta_H(1+q) + \theta_L(1-q))/2 \) with mandatory disclosure (his expected profit is the same whether he observes \( \theta_H \) or \( \theta_L \)). The change in an informed insider’s expected profit due to mandatory disclosure equals \( \Delta I = -q(\theta_H - \theta_L)/2 \). An uninformed

\(^{10}\) An interesting possibility arises with more than one insider. Mandatory disclosure may lead to a situation in which the market learns nothing from insiders’ disclosures, yet insiders learn something. For instance, suppose there are two insiders and one discloses a buy while one discloses a sell (which implies that at least one of them is uninformed). The market learns nothing from the disclosures, but the uninformed insider updates on the basis of the other insider’s trade.
insider's date-2 expected profit equals 0 with no disclosure and \( \Delta_U = \tilde{\theta} - (\theta_H (1-q) + \theta_L (1+q))/2 = q(\theta_H - \theta_L)/2 \) with mandatory disclosure (his expected profit is the same whether he buys or sells at date 1). Since \( \Delta_U = -\Delta_L \), the change in the insider's \textit{ex ante} expected profit due to disclosure, \( q\Delta_U + (1-q)\Delta_L \), is positive (negative) if \( q < (>) 0.5 \). The next section compares mandatory disclosure to no disclosure for a finite \( N \).

4. A Comparison of No Disclosure to Mandatory Disclosure

We now examine the effect of mandatory disclosure on (i) the liquidity of the market, measured by the bid-ask spread; (ii) the expected utility of outsiders; and (iii) the expected utility of the insider. To facilitate the comparison, consider the following example. Share value is either \( \theta_H = 2 \) or \( \theta_L = 1 \). There are \( N = 40 \) outsiders at each date. Traders' utility functions are \( U(\cdot) = \ln(\cdot) \) and traders' cash endowments are either \( w + \tilde{\theta} = 11.5 \) or \( w - \tilde{\theta} = 8.5 \). For this example, if \( q \geq 0.0252 \) and there is mandatory disclosure, an uninformed insider buys (sells) at date 2 if he sold (bought) at date 1. That is, Inequalities (13) and (14) are satisfied if \( q \geq 0.0252 \). For \( q < 0.0252 \), the likelihood that the insider is informed is so low that the disclosure of the insider's trade moves prices too little to induce an uninformed insider to reverse his trade. The expected trading profit is too low to justify taking on the risk. So if \( q < 0.0252 \), an uninformed insider hedges his position at date 1 and does not trade at date 2. Also, for this example, Inequalities (1) and (12) are satisfied for all \( q \).

4.1 Bid-ask spreads

The date-1 bid-ask spread, \( A_1 - B_1 \), does not depend on whether there is disclosure. This is because the insider's equilibrium date-1 trading strategy is the same in either case.

With no disclosure, the equilibrium expected date-2 bid-ask spread equals

\[
E[A_2(\tilde{x}_{b1}, N + 1 - \tilde{x}_{b1}, 0) - B_2(\tilde{x}_{b1}, N + 1 - \tilde{x}_{b1}, 0)].
\]

With disclosure, the equilibrium date-2 bid-ask spread equals

\[
A_2(x_{b1}, x_{s1}, d) - B_2(x_{b1}, x_{s1}, d),
\]

which is the same whether \( d = 1 \) or \( d = -1 \). If Equations (13) and (14) are satisfied, then \( A_2(x_{b1}, x_{s1}, -1) = A_2^F(0) \). Using Inequalities (6),

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(11), and (16), we can express (18) as
\[
A_2^T(0) - B_2^{NT}(0) = (\theta_H - \theta_L) \left( \frac{N}{2} + 1 - q \right) \left( 1 - \frac{1}{2} \right) \left( \frac{N^2}{2} - q \right)
\]
This bid-ask spread is a concave function of \( q \), maximized at \( q = 0.5 \).
The intuition is as follows. The insider's disclosure of his date-1 trade is a signal that conveys information on what his date-2 trade will be. A more informative signal leads to a narrower bid-ask spread because the market maker's expected loss to the insider is lower. The insider's disclosure is most informative when \( q \) is close to zero or one. In the former case, it is likely that the insider is uninformed and will make the opposite trade as in date 1 (assuming Inequalities (13) and (14) are satisfied). In the latter case, it is likely that the insider is informed and will make the same trade as in date 1. The insider's disclosure is least informative when \( q = 0.5 \). In this case, the insider is as likely to buy or sell at date 2 and the bid-ask spread that compensates the market maker for his expected loss to the insider is at its highest.

Figure 1 plots (17) and (18) for the example as a function of \( q \). With no disclosure, the expected bid-ask spread is increasing in \( q \); the more likely that the insider is informed, the more the market maker expects to lose to him and thus the wider the bid-ask spread to compensate for the expected loss. For \( 0.0240 < q < 0.4758 \) the expected date-2 bid-ask spread is higher with disclosure—the market is less liquid.

The market maker's expected profit is approximately zero.\(^{11}\) Thus, the insider's expected profit approximately equals outsiders' expected losses which are proportional to the bid-ask spread. Thus, the effect of disclosure on traders' date-2 expected profits can be seen (approximately) from Figure 1. For the values of \( q \) for which disclosure increases the expected bid-ask spread, outsiders' expected trading profits are lower and the insider's expected trading profit is higher.\(^{12}\)

---

\(^{11}\) Recall that the minimum nonnegative expected profit for the market maker may be positive. This is the case for \( 0.0240 < q < 0.0252 \).

\(^{12}\) Mandatory disclosure affects date-2 liquidity but not date-1 liquidity. This is because of the assumption that the insider either buys one share, sells one share, or does not trade. Suppose, though, that the insider can buy or sell any quantity. Also suppose that the insider, who is risk averse, observes a noisy signal so that he trades a finite amount. How is date-1 liquidity likely to change with mandatory disclosure? Suppose mandatory disclosure induces a separating equilibrium at date 1 in which an informed insider trades a different quantity than an uninformed insider. Since the insider's date-1 trade fully reveals his information, an informed insider has the incentive to trade more at date-1 with mandatory disclosure. If so, mandatory disclosure leads to lower liquidity at date 1 and greater liquidity at date 2 as compared to the case with no disclosure. Now suppose mandatory disclosure induces a pooling equilibrium at date 1 in which an informed insider trades the same quantity as an uninformed insider. The net effect on date-1 liquidity is not clear. For example, to pool with the uninformed, an insider who is initially short one share and who observes good news may buy fewer shares relative to the no disclosure case (this increases liquidity), and an insider who is initially long one share and who observes good news may buy...
Table 1 illustrates the minimum $q$ for which an uninformed insider reverses his date-1 trade and illustrates the range of $q$ for which the expected bid-ask spread is wider with mandatory disclosure for the example with various parameter values. Asset values $(\theta_k, \theta_H) = (1, 2), (0.5, 2.5), \text{ and } (0, 3) \text{ and the number of outsiders } N = 40, 60, \text{ and } 80 \text{ are considered. An increase in the variability of the asset value leads to an increase in the minimum value of } q \text{ for which an uninformed insider reverses his date-1 trade at date-2 (i.e., the minimum } q \text{ for which Inequalities } (13) \text{ and } (14) \text{ are satisfied). This is because higher asset value variability means that reversing a date-1 trade is riskier. An increase in the number of outsiders leads to a decrease in the minimum value of } q \text{ for which an uninformed insider reverses his trade. This is because more outsiders implies a more liquid market, which implies a narrower bid-ask spread and thus greater expected profits for an}

more shares (this decreases liquidity). In addition, if the date-1 price were sensitive to the order flow, as in Kyle (1985), then the above logic suggests that the date-1 price could become more or less informationally efficient with mandatory disclosure relative to no disclosure.
### Table 1
Example from Section 4 for various parameters values

<table>
<thead>
<tr>
<th>$(\theta_U, \theta_H)$</th>
<th>$N$</th>
<th>Minimum $q$ for which an uninformed insider reverses his date-1 trade</th>
<th>Range of $q$ for which expected bid-ask spread is wider with mandatory disclosure</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 2)</td>
<td>40</td>
<td>0.0252</td>
<td>(0.0240, 0.4758)</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.0248</td>
<td>(0.0240, 0.4788)</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>0.0246</td>
<td>(0.0240, 0.4830)</td>
</tr>
<tr>
<td>(0.5, 2.5)</td>
<td>40</td>
<td>0.0524</td>
<td>(0.0501, 0.4808)</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.0516</td>
<td>(0.0500, 0.4855)</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>0.0512</td>
<td>(0.0500, 0.4885)</td>
</tr>
<tr>
<td>(0, 3)</td>
<td>40</td>
<td>0.0782</td>
<td>(0.0749, 0.4863)</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.0770</td>
<td>(0.0748, 0.4892)</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>0.0764</td>
<td>(0.0748, 0.4937)</td>
</tr>
</tbody>
</table>

In the example, it is assumed that $w = 10$ and $U(\cdot) = \ln(\cdot)$.

An uninformed insider who reverses his trade. Now consider the range of $q$ for which the bid-ask spread is higher with mandatory disclosure. The lower bound on the range is lower but tracks the minimum value of $q$ for which an uninformed insider would reverse his trade. The upper bound on the range approaches 0.5 as $N$ increases (Section 3 discusses the limiting case of $N \rightarrow \infty$).

### 4.2 Expected utility of outsiders

We compute the expected utility of outsiders conditional on information available prior to date-1 trading. Since date-1 prices are unaffected by disclosure, the expected utility of a date-1 outsider is unaffected by disclosure. The expected utility of a date-2 outsider with $w_j = w + \tilde{\theta}$ and $z_j = -1$ equals

$$E[U(w + \tilde{\theta} - A_2(\bar{x}_{b1}, \bar{x}_{z1}, \bar{\theta})]$$

if there is no disclosure, and

$$E[U(w + \tilde{\theta} - A_2(\bar{x}_{b1}, \bar{x}_{z1}, \tilde{d})]$$

if there is disclosure. Given the symmetry of the problem, the expected utilities of a date-2 outsider with $w_j = w - \tilde{\theta}$ and $z_j = 1$ are the same. Prior to date-2, the uncertainty facing date-2 outsiders concerns date-2 prices. As discussed above, the expected bid-ask spread may be higher or lower with disclosure. For the example with $\theta_H = 2, \theta_L = 1$, and $N = 40$, if $0.0240 < q < 0.4758$, outsiders expect to pay a higher ask or receive a lower bid with disclosure. There is another effect. Consider the ask side of the market. With no disclosure, date-2 prices are determined by the date-1 order flow and there are $N + 2$ possi-
ble ask prices—using Equation (9), $A_2(0, N + 1, \emptyset) \leq A_2(1, N, \emptyset) \leq \cdots \leq A_2(N + 1, 0, \emptyset)$. With disclosure, date-2 prices are determined by the insider’s disclosure and there are two possible ask prices—using Equations (15) and (16), $A_2(x_{B_1}, x_{S_1}, -1) < A_2(x_{B_1}, x_{S_1}, 1)$. Recall that $A_2(x_{B_1}, x_{S_1}, -1) = A_2(0, N + 1, \emptyset)$ and $A_2(x_{B_1}, x_{S_1}, 1) = A_2(N + 1, 0, \emptyset)$. Thus, with disclosure, the outsider faces one of the two extreme prices that are possible with no disclosure (the bid side of the market is analogous). In that sense, a market with disclosure is riskier for date-2 outsiders.\footnote{Hirshleifer (1971) provides a general discussion of how the revelation of information prior to trading is detrimental for risk sharing.}

Figure 2 plots Inequalities (19) and (20) for the example as a function of $q$. For this example the expected utility of date-2 outsiders is lower with disclosure for all $q$. If the example had used a utility function for outsiders that exhibited less risk aversion, then there might have been values of $q$ for which outsiders are better off with mandatory disclosure. If outsiders are less risk averse, comparing their expected utility for the two cases looks more like a comparison of the expected bid-ask spreads.

4.3 Expected utility of the insider

We compute the expected utility of the insider conditional on information available prior to date-1 trading. Given the symmetry of the problem, the insider's expected utility is the same regardless of his initial endowment. Suppose $u_0 = w + \tilde{\theta}$ and $z_0 = -1$, and let $y$ denote the number of date-1 outsider buys.

First, consider the case when there is no disclosure. If the insider is informed, then his expected utility equals

$$u_{\text{informed}} = \frac{E[U(u_0 + \theta_H - A_1 - A_2(\tilde{y} + 1, N - \tilde{y}, \emptyset))]}{2} + \frac{E[U(u_0 - 3\theta_L + B_1 + B_2(\tilde{y}, N + 1 - \tilde{y}, \emptyset))]}{2}.$$

This reflects the fact that an informed insider is equally likely to observe $\theta = \theta_H$ or $\theta = \theta_L$. If the insider is uninformed and buys at date 1 (since $z_0 = -1$), then his expected utility equals

$$u_{\text{uninformed}} = E \max\{ U(u_0 - A_1), E[U(u_0 + \tilde{\theta} - A_1 - A_2(\tilde{y} + 1, N - \tilde{y}, \emptyset)) | \tilde{y}] , E[U(u_0 - \tilde{\theta} - A_1 + B_2(\tilde{y} + 1, N - \tilde{y}, \emptyset)) | \tilde{y}] \}.$$

The three arguments of the $\max\{ \cdot, \cdot, \cdot \}$ function are the insider's ex-
The expected utility of date-2 outsiders
In the example, from Section 4, it is assumed that $\theta_H = 2$, $\theta_L = 1$, $N = 40$, $w = 10$, and $U(\cdot) = \ln(\cdot)$.

Expected utility (as of date 2) if he does not trade at date 2, if he buys at date 2, or if he sells at date 2, respectively. Note that as of date-2, his expected utility is computed conditional on the date-1 order flow and the expectation is taken over share value. The expectation of the maximum of the three choices is taken over the date-1 order flow.

Now consider the case when there is mandatory disclosure. If the insider is informed, then his expected utility equals

$$u_{\text{informed}} = \frac{U(u_0 + \theta_H - A_1 - A_2(x_{b1}, x_{s1}, 1))}{2} + \frac{U(u_0 - 3\theta_L + B_1 + B_2(x_{b1}, x_{s1}, -1))}{2}.$$

If the insider is uninformed and buys at date 1, then his expected utility equals

$$u_{\text{uninformed}} = \max\{U(u_0 - A_1), E[U(u_0 - \tilde{\theta} - A_1 + B_2(x_{b1}, x_{s1}, 1))]\}.$$

The two arguments of the $\max\{\cdot, \cdot\}$ function are the insider's expected
utility if he does not trade at date 2 or if he sells at date 2 (with disclosure, an uninformed insider would not buy at both dates).

If the insider is informed, then his expected utility is lower with disclosure, $u_{informed} > u_{informed}$. This is because disclosure makes his date-2 trade less profitable. In effect, if the insider is informed, disclosure leads to date-2 prices that are a better reflection of share value. By contrast, if the insider is uninformed, then disclosure leads to date-2 prices that are a poorer reflection of share value. For if the insider is uninformed, the fair share price is $\tilde{\theta}$ and disclosure leads to a greater difference between prices and $\tilde{\theta}$ than is the case with no disclosure. Thus, an uninformed insider has a more profitable date-2 trading opportunity and higher expected utility with disclosure, $u_{uninformed} < u_{uninformed}$. The insider's *ex ante* expected utility equals

$$q u_{informed} + (1 - q) u_{uninformed} \quad (21)$$

with no disclosure, and

$$q u_{informed} + (1 - q) u_{uninformed} \quad (22)$$

with disclosure.

Figure 3 plots (21) and (22) for the example with $\theta_H = 2$, $\theta_L = 1$, and $N = 40$ as a function of $q$. The insider's expected utility is higher with disclosure for $0.0270 \leq q \leq 0.4868$. For $0.4758 \leq q \leq 0.4868$, the insider's expected utility is higher with disclosure even though his expected trading profit is lower. For $0.0252 \leq q < 0.0270$, the insider's expected utility is lower with disclosure even though his expected trading profit is higher. The difference reflects the effects of disclosure on the risk facing the insider. Disclosure decreases the insider's expected income when he is informed and increases his expected income when he is uninformed. Since the insider is risk averse and since his income is higher when he is informed, this is good for the insider. The increase in his expected income when he is uninformed comes by way of a risky gamble—the uninformed insider must reestablish a risky position. This is bad for the insider. When $q$ is high (low), an insider's disclosure moves the price a lot (little) and thus the risky gamble is more (less) favorable. This implies that when $q$ is high, the beneficial effect of disclosure for the insider dominates the harmful effect, and vice-versa when $q$ is low.

Note that with disclosure the insider's expected utility is decreasing in $q$ when $q$ is high. Differentiating (22) with respect to $q$ yields

$$\left(u_{informed} - u_{uninformed}\right) + q \frac{\partial u_{informed}}{\partial q} + (1 - q) \frac{\partial u_{uninformed}}{\partial q}.$$
The expected utility of the insider

In the example, from Section 4, it is assumed that $\theta_H = 2$, $\theta_L = 1$, $N = 40$, $w = 10$, and $U(\cdot) = \ln(\cdot)$.

The first term is positive; the insider is better off if informed. The second term is negative; an informed insider is better off when the market believes that it is likely that he is uninforme. The third term is positive; an uninformed insider is better off when the market believes that it is likely that he is informed. For high $q$ the negative effect dominates, and thus the insider prefers that the likelihood of being informed not be too high. 14

14 For the example developed throughout this section, the equilibrium mentioned in footnote 9, in which, with no disclosure, an uninformed insider trades only at date 2, does not exist. Given the prospective equilibrium prices, it would be optimal for an informed insider to follow the proposed trading strategy, but it would not be optimal for an uninformed insider to follow the proposed strategy. The uninformed insider would prefer to trade at date 1 and then, based on the date-2 prices, decide whether to trade again. For cases when this equilibrium exists, an uninformed insider's expected trading profit is zero and an informed insider's expected trading profit is positive and decreasing in $q$. Further, the insider's ex ante expected trading profit is increasing in $q$.  

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5. Voluntary Disclosure

The previous analysis characterized the impact of disclosure on the market. We now ask whether in the absence of mandatory disclosure regulations insiders would voluntarily disclose their trades. For if one is to argue that disclosure regulations of any sort have an impact, it must be established that full disclosure is not forthcoming without such regulations. We find that equilibria in which the insider voluntarily discloses his date-1 trade exist only if the probability that the insider is informed is very small.

To accommodate voluntary disclosure, we expand the strategy of the insider to include the decision of whether to disclose his date-1 trade. Let $D(w'_0, z'_0, x_1, x_{b1}, x_{s1}, I)$ denote the insider's disclosure strategy, where the function takes the value $x_1$ (disclosure of his date-1 trade) or $\emptyset$ (no disclosure). An equilibrium with voluntary disclosure consists of a trading strategy $X_1, X_2$, a disclosure strategy $D$, pricing rules $A_1, B_1, A_2, B_2$, and beliefs $g$ such that (i) $X_1, X_2,$ and $D$ maximize the insider's expected utility taking $A_1, B_1, A_2, B_2$, and $g$ as given; (ii) $A_t$ and $B_t$ generate the minimum nonnegative expected profit on the ask and bid sides of the market respectively, for the market maker at date $t$ taking $X_1, X_2, D,$ and $g$ as given; and (iii) $g$ is consistent with Bayes' Rule in equilibrium.

We state the following proposition without proof, which is straightforward.

**Proposition 3.** There is an equilibrium in which (i) the insider never discloses his date-1 trade, i.e., $D(w'_0, z'_0, x_1, x_{b1}, x_{s1}, I) = \emptyset$, and (ii) the insider's trading strategy and the bid and ask pricing rules are identical to those of the equilibrium described in Proposition 1.

This result is not surprising. Out-of-equilibrium beliefs for the market maker that support this equilibrium are those that ignore any disclosure, $g(\theta, I \mid x_{b1}, x_{s1}, 1) = g(\theta, I \mid x_{b1}, x_{s1}, -1) = g(\theta, I \mid x_{b1}, x_{s1}, \emptyset)$, and thus the insider is indifferent between disclosing and not disclosing his trade. We now consider the possibility of an equilibrium in which the insider always voluntarily discloses his date-1 trade.

**Proposition 4.** There is an equilibrium in which (i) the insider always discloses his date-1 trade, i.e., $D(w'_0, z'_0, x_1, x_{b1}, x_{s1}, I) = x_1$, and (ii) the insider's trading strategy and the bid and ask pricing rules are identical to those of the equilibrium described in Proposition 2, if and only if $q \leq 1/(N + 3)$.

**Proof.** See the Appendix.
In such an equilibrium, disclosing must result in more favorable date-2 bid and ask prices than not disclosing. That is,

\[ A_2(x_{b1}, x_{s1}, d) \leq A_2(x_{b1}, x_{s1}, \emptyset) \]  \hspace{1cm} (23)

and

\[ B_2(x_{b1}, x_{s1}, d) \geq B_2(x_{b1}, x_{s1}, \emptyset) \]  \hspace{1cm} (24)

for all \( x_{b1} \) and \( x_{s1} \), and \( d \). Out-of-equilibrium beliefs in response to the out-of-equilibrium move of not disclosing must be such that the insider is sufficiently “punished” with a wider bid-ask spread. The necessary and sufficient condition for such out-of-equilibrium beliefs to exist is \( q \leq 1/(N + 3) \). This is because, for a low \( q \), the market maker believes it very likely that the insider is uninformed. Thus, if the insider discloses his date-1 trade, this disclosure will not move prices much. The date-2 bid-ask spread will be narrow with prices close to \( \theta \). If the insider does not disclose his date-1 trade and the beliefs in response to no disclosure are that the insider is certainly informed, then the date-2 bid-ask spread will be relatively wide and the prices less favorable. The insider’s punishment for not disclosing is that the market maker believes him to be informed. And the lower is \( q \), the greater is such punishment.

Reconsider the example from Section 4: \( \theta_L = 1, \theta_H = 2, N = 40, w = 10, U(\cdot) = \ln(\cdot) \), and let \( q = 0.02 \). For these parameter values \( q \leq 1/(N + 3) \). Hence, an equilibrium in which the insider voluntarily discloses his trade exists. The equilibrium date-2 bid and ask prices are computed using Equations (15) and (16), and equal

\[ B_2(x_{b1}, x_{s1}, 1) = 1.51 \quad A_2(x_{b1}, x_{s1}, 1) = 1.5105 \]
\[ B_2(x_{b1}, x_{s1}, -1) = 1.4895 \quad A_2(x_{b1}, x_{s1}, -1) = 1.49 \]

for the disclosure of a date-1 buy and sell, respectively. Suppose the beliefs in response to no disclosure are that the insider is certainly informed, having observed \( \theta = \theta_H \) or \( \theta = \theta_L \) with equal probability. That is, \( g(\theta_H, \theta_H \mid x_{b1}, x_{s1}, \emptyset) = g(\theta_L, \theta_L \mid x_{b1}, x_{s1}, \emptyset) = 1/2 \) and \( g(\theta_H, \emptyset \mid x_{b1}, x_{s1}, \emptyset) = g(\theta_L, \emptyset \mid x_{b1}, x_{s1}, \emptyset) = 0 \). Then, by Equation (46) (in the proof of Proposition 4), the out-of-equilibrium bid and ask prices equal

\[ B_2(x_{b1}, x_{s1}, \emptyset) = 1.4878; \quad A_2(x_{b1}, x_{s1}, \emptyset) = 1.5122 \]

Whether the insider buys or sells at date 1, he faces more favorable date-2 bid and ask prices if he discloses his date-1 trade.

Now consider the same example except with \( q > 1/(N + 3) \), say \( q = 0.5 \). Then, an equilibrium in which the insider discloses his date-1 trade would (if it existed) entail date-2 bid and ask prices of (computed
using Equations (15) and (16))

\[ B_2(x_{b1}, x_{s1}, 1) = 1.744 \quad A_2(x_{b1}, x_{s1}, 1) = 1.756 \]

\[ B_2(x_{b1}, x_{s1}, -1) = 1.244 \quad A_2(x_{b1}, x_{s1}, -1) = 1.256 \]

for the disclosure of a date-1 buy and sell, respectively. For this example, there are no out-of-equilibrium beliefs that lead to \( B_2(x_{b1}, x_{s1}, \emptyset) \leq B_2(x_{b1}, x_{s1}, -1) \) and \( A_2(x_{b1}, x_{s1}, \emptyset) \geq A_2(x_{b1}, x_{s1}, 1) \) for all \( x_{b1} \) and \( x_{s1} \). The most severe punishment, that the insider is believed to be certainly informed, is not sufficient.

The more outsiders, \( N \), there are, the lower \( q \) must be for an equilibrium with voluntary disclosure to exist. To see why, suppose the insider sells at date 1. Both the equilibrium bid, \( B_2(x_{b1}, x_{s1}, -1) \), and the out-of-equilibrium bid, \( B_2(x_{b1}, x_{s1}, \emptyset) \), are increasing in \( N \); the market maker offers more favorable prices because his losses to the insider are spread across more outsiders. For \( q > 0 \), \( B_2(x_{b1}, x_{s1}, \emptyset) \) is increasing in \( N \) faster than is \( B_2(x_{b1}, x_{s1}, -1) \). Thus, for a given \( q > 0 \), if \( N \) is "too large," then \( B_2(x_{b1}, x_{s1}, \emptyset) > B_2(x_{b1}, x_{s1}, -1) \) for any out-of-equilibrium beliefs. If so, then an informed insider does not have the incentive to disclose a date-1 sell (an analogous argument applies if the insider buys at date 1).

In other contexts it is argued that mandatory disclosure rules are irrelevant because parties always have the incentive to voluntarily disclose private information; see, for example, Grossman (1981) and Milgrom (1981). This is because withholding information induces uninformed parties to believe that the withheld information is "unfavorable." For example, if a firm that is selling stock to the public withholds information regarding past profitability, then potential buyers of the stock infer that profitability was low. This gives the firm the incentive to disclose the information and disclosure rules are not necessary. This type of argument fails here. In securities markets there is no unfavorable inference that induces insiders to always disclose their trades. Insiders may possess good or bad news about a firm. If market makers interpret an insider's withholding of information to mean that the insider has good (bad) news about the stock, then the insider with bad (good) news can withhold information and profitably sell (buy). For our model, this reasoning implies that an equilibrium in which the insider voluntarily discloses his trade exists if and only if \( q \) is very low. Thus, voluntary disclosure is not a substitute for mandatory disclosure.\(^{15}\)

\(^{15}\) For cases in which, \textit{ex ante}, the insider is better off with disclosure, but the equilibrium with voluntary disclosure does not exist, the insider would like to precommit (\textit{ex ante}) to disclose his date-1 trade (if there were no mandatory disclosure). It is not clear, however, how one could precommit. If, \textit{ex ante}, the insider simply announces that he will disclose his trade, but this is not
6. Concluding Remarks

The two major results of this paper are (i) the disclosure of insider trades can lead to a less liquid market in which the insider is better off and noninsiders are worse off; and (ii) such a consequence is generally only possible if disclosure is mandated—if disclosure is voluntary, then disclosure will not in general be forthcoming.

These results have implications for the debate on the short-swing profit rule. This rule, Section 16(b) of the Securities and Exchange Act of 1934, covers the insiders who are subject to the mandatory disclosure rules of Section 16(a) of the same act. Section 16(b) requires insiders who buy and sell within a 6-month period to give up any resulting trading profit. It has been argued that Section 16(b) has various adverse effects on matters of corporate ownership and governance. For example, Black (1990) argues that institutional investors could benefit by investing in large share positions in individual firms and then monitoring the management, but Section 16(b) deters such shareholder activism. This is because the liability for short-swing profits reduces the liquidity of such institutions' investments. Roe (1991) argues that Section 16(b) imposes similar costs on institutional investors who attempt to act as a group in influencing control over corporations. In spite of such adverse consequences, our results indicate that, consistent with the general view of Manne (1966), Section 16(b) may deter manipulation. It limits an insider's ability to profit by disclosing a trade that moves the stock price and then trading in the opposite direction.\(^{16}\)

Our results also indicate that such an antimanipulation device may only be necessary as a complement to Section 16(a), which mandates disclosure. Since voluntary disclosure is generally not forthcoming, such manipulations may not be possible without mandatory disclosure regulations. Therefore, if either insiders' disclosures were not mandated or were mandated but not made available to the public, then the short-swing rule may not be necessary. In futures markets, traders who establish large long or short futures positions must disclose these positions (the position size that triggers disclosure varies across con-

\(^{16}\) The legislative intent underlying the short-swing profit rule was to eliminate both typical insider trading and manipulations. In Senate Hearings on Stock Exchange Practices (before the Committee on Banking and Currency, U.S. Senate, 73rd Congress, 2nd Session, February 28, 1934), Mr. Thomas Corcoran, one of the drafters of the legislation, testified that the purpose of the law was “to prevent directors receiving benefits of short-term speculative swings on the securities of their own companies, because of inside information.” Senator Albert Gore, Sr. stated that the legislation “is aimed at the general evil of officers and directors rigging their stock up and down, and squeezing out their own stockholders,” to which Mr. Corcoran agreed.
tracts) to the Commodity Futures Trading Commission (CFTC). The CFTC, however, does not make the information on traders' positions publicly available. A similar rule might limit manipulations in securities markets.

For individuals who are subject to the mandatory disclosure rules under the Williams Act but not Section 16 (those holding between 5 percent and 10 percent of a firm's stock and who are not managers or directors of the firm), the short-swing profit rule does not apply. Therefore, it would be useful to examine these trader's trades to look for evidence of the strategy studied here. In particular, once a trader crosses the 5 percent threshold, how often does he reverse his trades, and what is the profit on such trades?

Finally, as noted in the introduction, even for individuals who are subject to the short-swing profit rule, short-term buy-sell strategies are possible if undertaken by two individuals. If the disclosure of a trade by insider A moves the stock price, and if insider B knows that insider A's trade was not information based, then insider B knows of a profitable trading opportunity. It would be interesting to examine the Section 16(a) filings to see if there is evidence that multiple insiders engage in the sort of trade reversals discussed here.

Appendix

Proof of Proposition 1. It was established in the text that if the insider's strategy satisfies Equations (2) and (5), then date-1 bid and ask prices are given by Equations (3) and (4), and for equilibrium date-1 order flows, $x_{b1} + x_{s1} = N + 1$, date-2 bid and ask prices are given by Equations (8) and (9). It remains to establish that the insider's optimal strategy satisfies Equations (2) and (5). To do this, date-2 prices for out-of-equilibrium order flows, $x_{b1} + x_{s1} = N$, must also be specified. Suppose that for an out-of-equilibrium order flow at date 1, the market maker's beliefs are given by $g(\theta_H, \theta_L | x_{b1}, x_{s1}, \emptyset) = \frac{g(\theta_H, \theta_L | x_{b1}, x_{s1}, \emptyset)}{2}$ and $g(\theta_H, \emptyset | x_{b1}, x_{s1}, \emptyset) = g(\theta_L, \emptyset | x_{b1}, x_{s1}, \emptyset) = \frac{(1 - q)}{2}$. And suppose that if the insider does not trade at date 1, then his date-2 strategy is the same as Equation (2). Then, given the beliefs, out-of-equilibrium date-2 bid and ask prices given by Equations (3) and (4) yield the market maker a zero expected profit on the bid and ask sides of the market. Given the out-of-equilibrium prices, the insider's optimal date-2 strategy satisfies Equation (2). This is because if the insider is informed, he maximizes his payoff by buying (selling) if $\theta = \theta_H (\theta = \theta_L)$, and if the insider is uninformed, Inequality (1) implies that he will trade $-z_0$ shares. Thus, out-of-equilibrium date-2 bid and ask prices are given by Equations (3) and (4).
Now consider an informed insider who observes $\theta = \theta_H$. Since $\theta_H \geq A_2(x_{b1}, x_{s1}, \emptyset) \geq B_2(x_{b1}, x_{s1}, \emptyset)$, it is optimal for the insider to buy at date 2. Now consider the insider's date-1 trade. Let $y$ denote the number of outsiders who buy at date 1. Given that it is optimal to buy at date 2, the insider's expected utility is

$$E[U(u_b + (z_0 + 2)\theta_H - A_1 - A_2(\bar{y} + 1, N - \bar{y}, \emptyset))]$$

if he buys at date 1, 

$$U(u_b + (z_0 + 1)\theta_H - A_1)$$

if he does not trade at date 1, and

$$E[U(u_b + z_0\theta_H + B_1 - A_2(\bar{y}, N + 1 - \bar{y}, \emptyset))]$$

if he sells at date 1. Since $\theta_H \geq A_2(y + 1, N - y, \emptyset)$, (25) exceeds (26). Since $2\theta_H - A_1 - A_2(y + 1, N - y, \emptyset) \geq \theta_H - A_1 = B_1 - \theta_L \geq B_1 - A_2(y, N + 1 - y, \emptyset)$, (25) exceeds (27). Thus, it is optimal for the insider to buy at date 1. Similar reasoning implies that if the insider is informed and $\theta = \theta_L$, then it is optimal for the insider to sell at both dates 1 and 2. Thus, an informed insider's optimal strategy satisfies Equations (2) and (5).

Now consider an uninformed insider with $z_0 = -1$. If he buys at date 1 and $y$ outsiders buy at date 1, then his expected utility equals

$$u_{bb}(y) = E[U(u_b + \bar{\theta} - A_1 - A_2(y + 1, N - y, \emptyset))],$$

$$u_{bs}(y) = E[U(u_b - \bar{\theta} - A_1 + B_2(y + 1, N - y, \emptyset))],$$

or

$$u_{b0}(y) = U(u_b - A_1),$$

depending on whether he buys, sells, or does not trade at date 2, respectively. Thus, his expected utility if he and $y$ outsiders buy at date 1 and he trades optimally at date 2 equals

$$u_b(y) = \max\{u_{bb}(y), u_{bs}(y), u_{b0}(y)\}. \quad (28)$$

If he sells at date 1 and $y$ outsiders buy at date 1, then his expected utility equals

$$u_{sb}(y) = E[U(u_b - \bar{\theta} + B_1 - A_2(y, N + 1 - y, \emptyset))],$$

$$u_{ss}(y) = E[U(u_b - 3\bar{\theta} + B_1 + B_2(y, N + 1 - y, \emptyset))],$$

or

$$u_{s0}(y) = E[U(u_b - 2\bar{\theta} + B_1)],$$

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depending on whether he buys, sells, or does not trade at date 2, respectively. Thus, his expected utility if he sells and \( y \) outsiders buy at date 1 and he trades optimally at date 2 equals

\[
 u_s(y) = \max\{u_{sb}(y), u_{ss}(y), u_{so}(y)\}. \tag{29}
\]

If he does not trade at date 1, then Inequality (1) and the fact that

\[
 A_2(x_{b1}, x_{s1}, \emptyset) = A_1
\]

for \( x_{b1} + x_{s1} = N \) imply that it is optimal to buy at date 2, and thus his expected utility equals

\[
 U(u_b - A_1). \tag{30}
\]

It is immediate that Equation (28) is at least as high as (30); at date 1, buying is preferred to not trading. Comparing the expected utility of buying at date 1 to selling at date 1,

\[
 E[u_b(y)] - E[u_s(y)] = \sum_{y=0}^{N} u_b(y) \text{Prob}(y) - \sum_{y=0}^{N} u_s(y) \text{Prob}(y)
\]

\[
 = \sum_{y=0}^{N} u_b(y) \text{Prob}(y)
\]

\[
 - \sum_{y=0}^{N} u_s(N - y) \text{Prob}(N - y)
\]

\[
 = \sum_{y=0}^{N} (u_b(y) - u_s(N - y)) \text{Prob}(y),
\]

where the last equality follows since \( \text{Prob}(y) = \text{Prob}(N - y) \). We will show that \( u_b(y) \geq u_s(N - y) \) for all \( y \). To do so, we use two facts, which follow from Equations (3), (4), (8), and (9):

\[
 \bar{\theta} - B_1 = A_1 - \bar{\theta} \tag{31}
\]

and

\[
 \bar{\theta} - B_2(y + 1, N - y, \emptyset) = A_2(N - y, y + 1, \emptyset) - \bar{\theta} \tag{32}
\]

By Equations (31) and (32), \(-A_1 + B_2(y + 1, N - y, \emptyset) = B_1 - A_2(N - y, y + 1, \emptyset)\). Therefore,

\[
 u_{bs}(y) = u_{sb}(N - y). \tag{33}
\]
By Equations (31), (32), and concavity of \( U(\cdot) \),

\[
\begin{align*}
  u_{bb}(y) &= E[U(u_b + \tilde{\theta} - A_1 - A_2(y + 1, N - y, \emptyset))] \\
  &= E[U(u_b + \tilde{\theta} + 4(\tilde{\theta} - \tilde{\theta}) - A_1 - A_2(y + 1, N - y, \emptyset))] \\
  &= E[U(u_b - 3\tilde{\theta} + B_1 + B_2(N - y, y + 1, \emptyset))] \\
  &= u_{ss}(N - y). \tag{34}
\end{align*}
\]

By Equation (31) and concavity of \( U(\cdot) \),

\[
\begin{align*}
  u_{b0}(y) &= U(u_b - A_1) \\
  &= U(u_b + B_1 - 2\tilde{\theta}) \\
  &> E[U(u_b - 2\tilde{\theta} + B_1)] \\
  &= u_{s0}(N - y). \tag{35}
\end{align*}
\]

That \( u_b(y) \geq u_s(N - y) \) follows from (33), (34), and (35). Thus, the optimal date-1 strategy for an uninformed insider with \( z_0 = -1 \) satisfies Equation (2). Given that he buys at date 1, his date-2 position is \( u_0 = w + \tilde{\theta} - A_1 \) and \( z_0' = 0 \). That his optimal date-2 strategy satisfies Equation (5) follows from the definitions of \( A_R \) and \( B_R \). Similar reasoning implies that the optimal strategy for an uninformed insider with \( z_0 = 1 \) also satisfies Equations (2) and (5).

\[\blacksquare\]

**Proof of Proposition 2.** It was established in the text that if the insider's strategy satisfies Equations (2) and (5), then the date-1 bid and ask prices are given by Equations (3) and (4), and for equilibrium disclosures, \( d = 1 \) and \( d = -1 \), the date-2 bid and ask prices are given by Equations (15) and (16). It remains to establish that given such bid and ask prices, the insider's optimal strategy satisfies Equations (2) and (5). To do this, date-2 prices for an out-of-equilibrium disclosure, \( d = 0 \), must also be specified. As shown in the proof of Proposition 1, for this out-of-equilibrium move, there are beliefs such that the resulting date-2 prices are given by Equations (3) and (4).

Now consider an informed insider who observes \( \theta = \theta_H \). Since \( \theta_H \geq A_2(x_{b1}, x_{s1}, d) \geq B_2(x_{b1}, x_{s1}, d) \), it is optimal for the insider to buy at date 2. Now consider the insider's date-1 trade. Given that it is optimal to buy at date 2, the insider's utility equals

\[
U(u_b + (z_0 + 2)\theta_H - A_1 - A_2(x_{b1}, x_{s1}, 1)) \tag{36}
\]

if he buys at date 1,

\[
U(u_b + (z_0 + 1)\theta_H - A_1) \tag{37}
\]
if he does not trade at date 1, and
\[ U(u_b + z_0 \theta_H + B_1 - A_2(x_{b1}, x_{s1}, -1)) \]  (38)
if he sells at date 1 (since the date-2 price only depends on the insider's disclosure, the insider faces no uncertainty regarding the date-2 price). Since \( \theta_H \geq A_2(x_{b1}, x_{s1}, 1) \), (36) exceeds (37), and since \( 2\theta_H - A_1 - A_2(x_{b1}, x_{s1}, 1) \geq \theta_H - A_1 = B_1 - \theta_L \geq B_1 - A_2(x_{b1}, x_{s1}, -1) \), (36) exceeds (38). Thus, it is optimal for the insider to buy at date 1. Similar reasoning implies that if the insider is informed and \( \theta = \theta_L \), then it is optimal for the insider to sell at both dates 1 and 2. Thus, an informed insider's optimal strategy satisfies Equations (2) and (5).

Now consider an uninformed insider with \( z_0 = -1 \). If he buys at date 1, then his expected utility equals
\[ v_{bb} = E[U(u_b + \bar{\theta} - A_1 - A_2(x_{b1}, x_{s1}, 1))], \]
\[ v_{bs} = E[U(u_b - \bar{\theta} - A_1 + B_2(x_{b1}, x_{s1}, 1))], \]
or
\[ v_{b0} = U(u_b - A_1), \]
depending on whether he buys, sells, or does not trade at date 2, respectively. Thus, his expected utility if he buys at date 1 and trades optimally at date 2 equals
\[ v_b = \max\{v_{bb}, v_{bs}, v_{b0}\}. \]  (39)
If he sells at date 1, then his expected utility equals
\[ v_{sb} = E[U(u_b - \bar{\theta} + B_1 - A_2(x_{b1}, x_{s1}, -1))], \]
\[ v_{ss} = E[U(u_b - 3\bar{\theta} + B_1 + B_2(x_{b1}, x_{s1}, -1))], \]
or
\[ v_{s0} = E[U(u_b - 2\bar{\theta} + B_1)], \]
depending on whether he buys, sells, or does not trade at date 2, respectively. Thus, his expected utility if he sells at date 1 and trades optimally at date 2 equals
\[ v_s = \max\{v_{sb}, v_{ss}, v_{s0}\}. \]  (40)
If he does not trade at date 1, then Inequality (1) and the out-of-equilibrium ask, \( A_2(x_{b1}, x_{s1}, 0) = A_1 \), imply that it is optimal to buy at date 2, and thus his utility equals
\[ U(u_b - A_1). \]  (41)

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It is immediate that Equation (39) is at least as high as (41); at date 1, buying is preferred to not trading. To compare the expected utility of buying at date 1 to selling at date 1 we use the following fact, which follows from Equations (15) and (16). For \( d \in [-1, 1] \),

\[
\tilde{\theta} - B_2(x_{b_1}, x_{s_1}, d) = A_2(x_{b_1}, x_{s_1}, -d) - \bar{\theta}.
\] (42)

By Equations (31) and (42), \(-A_1 + B_2(x_{b_1}, x_{s_1}, 1) = B_1 - A_2(x_{b_1}, x_{s_1}, -1).\) Therefore,

\[
v_{bs} = v_{sb}.
\] (43)

By Equations (31), (42), and concavity of \( U(\cdot) \),

\[
v_{bb} = E[U(u_0 + \tilde{\theta} - A_1 - A_2(x_{b_1}, x_{s_1}, 1))]
= E[U(u_0 + \tilde{\theta} - 4\tilde{\theta} + B_1 + B_2(x_{b_1}, x_{s_1}, -1))]
> E[U(u_0 - 3\tilde{\theta} + B_1 + B_2(x_{b_1}, x_{s_1}, -1))]
= v_{ss}.
\] (44)

By Equation (31) and concavity of \( U(\cdot) \),

\[
v_{b0} = U(u_0 - A_1)
= U(u_0 - 2\tilde{\theta} + B_1)
> E[U(u_0 - 2\tilde{\theta} + B_1)]
= v_{s0}.
\] (45)

That \( v_b \geq v_s \) follows from (43), (44), and (45). Thus, the optimal date-1 strategy for an uninformed insider with \( z_0 = -1 \) satisfies Equation (2). Given that he buys at date 1, his date-2 position is \( u'_0 = w + \tilde{\theta} - A_1 \) and \( z'_0 = 0 \). That his optimal date-2 strategy satisfies Equation (5) follows from the definitions of \( A_R \) and \( B_R \). Similar reasoning implies that the optimal strategy for an uninformed insider with \( z_0 = 1 \) also satisfies Equations (2) and (5).

**Proof of Proposition 4.**

1. Suppose \( q \leq 1/(N + 3) \).

Suppose the insider’s trading strategy satisfies Equations (2) and (5) and \( D(u'_0, z'_0, x_1, x_{b_1}, x_{s_1}, I) = x_1 \). By Proposition 2, the date-1 bid and ask prices are given by Equations (3) and (4), and for equilibrium order flows and disclosures, \( x_{b_1} + x_{s_1} = N + 1 \) and \( d \in [-1, 1] \), the date-2 bid and ask prices are given by Equations (15) and (16). It remains to verify that given such bid and ask prices, the insider’s optimal strategy satisfies Equations (2) and (5), and \( D(u'_0, z'_0, x_1, x_{b_1}, x_{s_1}, I) = x_1 \). To
do so, date-2 prices for out-of-equilibrium order flows and disclosures, \( x_{b1} + x_{s1} = N \) and \( d \in [0, \emptyset] \), must also be specified.

(i) Consider an equilibrium order flow and an out-of-equilibrium disclosure, \( x_{b1} + x_{s1} = N + 1 \) and \( d = \emptyset \). Suppose that the market maker's out-of-equilibrium beliefs are given by \( g(\theta_H, \theta_L \mid x_{b1}, x_{s1}, \emptyset) = g(\theta_L, \theta_L \mid x_{b1}, x_{s1}, \emptyset) = 1/2 \) and \( g(\theta_H, \emptyset \mid x_{b1}, x_{s1}, \emptyset) = g(\theta_L, \emptyset \mid x_{b1}, x_{s1}, \emptyset) = 0 \). And suppose that if the insider trades at date 1 but does not disclose his trade, then his date-2 trading strategy satisfies Equation (5). Given these beliefs and this date-2 insider trading strategy, the market maker's expected profit on the bid and ask sides of the market equals

\[
\frac{N}{2}(\theta_H - b_2) + \frac{(N/2 + 1)(\theta_L - b_2)}{2} \quad \text{and} \quad \frac{(N/2 + 1)(a_2 - \theta_H) + N/2(a_2 - \theta_L)}{2}
\]

respectively. Thus, for \( x_{b1} + x_{s1} = N + 1 \), out-of-equilibrium zero expected profit bid and ask prices equal

\[
B_2(x_{b1}, x_{s1}, \emptyset) = \frac{\theta_H N + \theta_L (N + 2)}{2N + 2}
\]

and

\[
A_2(x_{b1}, x_{s1}, \emptyset) = \frac{\theta_H (N + 2) + \theta_L N}{2N + 2}
\]  

(46)

respectively. That the insider’s optimal date-2 trading strategy, given the out-of-equilibrium prices, satisfies Equation (5) follows from \( \theta_H > A_2(x_{b1}, x_{s1}, \emptyset) > B_2(x_{b1}, x_{s1}, \emptyset) > \theta_L \), which implies that an informed insider prefers to buy (sell) at date-2 if \( \theta = \theta_H \) (\( \theta = \theta_L \)), and from the definitions of \( A_R \) and \( B_R \), which imply that an uninform ed insider prefers to buy (sell) at date-2 if \( a_2 < A_R \) (\( b_2 > B_R \)).

(ii) Consider an out-of-equilibrium order flow and an out-of-equilibrium disclosure, \( x_{b1} + x_{s1} = N \) and \( d = 0 \) or \( d = \emptyset \). As shown in the proof of Proposition 1, for this out-of-equilibrium move, there are beliefs such that the resulting date-2 prices are given by Equations (3) and (4).

Thus, out-of-equilibrium date-2 prices satisfy Equation (46) if \( x_{b1} + x_{s1} = N + 1 \) and \( d = \emptyset \), and satisfy Equations (3) and (4) if \( x_{b1} + x_{s1} = N \).

Now consider the insider's disclosure decision. Suppose the insider buys at date 1. If the insider discloses his date-1 buy, then the date-2 bid and ask equal \( B_2(x_{b1}, x_{s1}, 1) \) and \( A_2(x_{b1}, x_{s1}, 1) \) respectively. By
Equation (15) and the fact that $q \leq 1/(N + 3)$,
\[
A_2(x_{b1}, x_{s1}, 1) = A_2^{NT}(N + 1)
\]
\[
= \frac{\theta_H \left( \left( \frac{N}{2} + 1 \right) q + \frac{N}{4} (1 - q) \right) + \theta_L \frac{N}{4} (1 - q)}{\frac{N}{2} + q}
\]
\[
\leq \frac{\theta_H \left( \left( \frac{N}{2} + 1 \right) \frac{1}{N+3} + \frac{N}{4} \left(1 - \frac{1}{N+3}\right) \right) + \theta_L \frac{N}{4} \left(1 - \frac{1}{N+3}\right)}{\frac{N}{2} + \frac{1}{N+3}}
\]
\[
= \frac{\theta_H (N + 2) + \theta_L N}{2N + 2} = A_2(x_{b1}, x_{s1}, \emptyset).
\]

Further, $B_2(x_{b1}, x_{s1}, 1) > \tilde{\theta} > B_2(x_{b1}, x_{s1}, \emptyset)$. Therefore, the insider prefers to disclose his date-1 buy; whether he buys or sells at date 2, he trades at a more favorable price. Similar reasoning implies that the insider would also prefer to disclose a date-1 sell. Thus
\[
D(u_0', z_0', x_1, x_{b1}, x_{s1}, I) = x_1
\]
is optimal; the insider always discloses his date-1 trade. Finally, since the insider faces the same date-1 and date-2 prices as with mandatory disclosure, the insider's optimal trading strategy satisfies Equations (2) and (5) (by Proposition 2).

2. Suppose there is an equilibrium in which the insider's trading strategies satisfy Equations (2) and (5), $D(u_0', z_0', x_1, x_{b1}, x_{s1}, I) = x_1$, the date-1 bid and ask pricing functions satisfy Equations (3) and (4), and for equilibrium date-1 order flows and disclosures, the date-2 bid and ask pricing functions are given by Equations (15) and (16).

In this equilibrium, an informed insider who observes $\theta = \theta_H (\theta = \theta_L)$ buys (sells) at dates 1 and 2. For him to disclose his date-1 trade requires
\[
A_2(x_{b1}, x_{s1}, 1) \leq A_2(x_{b1}, x_{s1}, \emptyset)
\]
and
\[
B_2(x_{b1}, x_{s1}, -1) \geq B_2(x_{b1}, x_{s1}, \emptyset)
\]
for all $x_{b1}$ and $x_{s1}$. For a given $x_{b1}$ and $x_{s1}$, denote the out-of-equilibrium beliefs in response to nondisclosure, $d = \emptyset$, as $g(\theta_H, \theta_L | x_{b1}, x_{s1}, \emptyset) = \alpha_H, g(\theta_H, \theta_L | x_{b1}, x_{s1}, \emptyset) = \alpha_L, g(\theta_H, \emptyset | x_{b1}, x_{s1}, \emptyset) = \beta_H$, and $g(\theta_L, \emptyset | x_{b1}, x_{s1}, \emptyset) = \beta_L$, where $\alpha_H, \alpha_L, \beta_H, \beta_L \geq 0$ and $\alpha_H + \alpha_L + \beta_H + \beta_L = 1$ (dependence on $x_{b1}$ and $x_{s1}$ is suppressed).

The prices, $A_2(x_{b1}, x_{s1}, \emptyset)$ and $B_2(x_{b1}, x_{s1}, \emptyset)$ can be bounded. The bid-ask spread is maximized by maximizing the insider's expected trading profit. Since $A_2(x_{b1}, x_{s1}, 1) \geq \tilde{\theta} \geq B_2(x_{b1}, x_{s1}, -1)$, Inequal-
ity (47) implies \( A_2(x_{b1}, x_{s1}, \emptyset) \geq \tilde{\theta} \geq B_2(x_{b1}, x_{s1}, \emptyset) \). At such prices, the insider’s date-2 expected profit is maximized with a date-2 strategy of only trading if informed, buying (selling) if \( \theta = \theta_H (\theta = \theta_L) \). Thus, the date-2 bid-ask spread is maximized with this date-2 insider strategy. Given this strategy and the above beliefs, the market maker’s expected profit on the bid and ask sides of the market equals

\[
(\alpha_H + \beta_H) \frac{N}{2} (\theta_H - b_2) + \left( \alpha_L \left( \frac{N}{2} + 1 \right) + \beta_L \frac{N}{2} \right) (\theta_L - b_2)
\]

and

\[
\left( \alpha_H \left( \frac{N}{2} + 1 \right) + \beta_H \frac{N}{2} \right) (a_2 - \theta_H) + (\alpha_L + \beta_L) \frac{N}{2} (a_2 - \theta_L)
\]

respectively. Given these expected profits, the zero expected profit bid and ask prices equal

\[
\tilde{b}_2 = \frac{\theta_H \frac{N}{2} (\alpha_H + \beta_H) + \theta_L \left( \left( \frac{N}{2} + 1 \right) \alpha_L + \frac{N}{2} \beta_L \right)}{\frac{N}{2} + \alpha_L}
\]

and

\[
\tilde{a}_2 = \frac{\theta_H \left( \left( \frac{N}{2} + 1 \right) \alpha_H + \frac{N}{2} \beta_H \right) + \theta_L \frac{N}{2} (\alpha_L + \beta_L)}{\frac{N}{2} + \alpha_H}
\]

respectively. The out-of-equilibrium prices must satisfy \( A_2(x_{b1}, x_{s1}, \emptyset) \leq \tilde{a}_2 \), and \( B_2(x_{b1}, x_{s1}, \emptyset) \geq \tilde{b}_2 \).

These bounds and Inequality (47) imply that \( A_2(x_{b1}, x_{s1}, 1) \leq \tilde{a}_2 \) and \( B_2(x_{b1}, x_{s1}, -1) \geq \tilde{b}_2 \), which is equivalent to

\[
\frac{\theta_H \left( \left( \frac{N}{2} + 1 \right) q + \frac{N}{4} (1 - q) \right) + \theta_L \frac{N}{4} (1 - q)}{\frac{N}{2} + q} \leq \frac{\theta_H \left( \left( \frac{N}{2} + 1 \right) \alpha_H + \frac{N}{2} \beta_H \right) + \theta_L \frac{N}{2} (\alpha_L + \beta_L)}{\frac{N}{2} + \alpha_H}
\]

and

\[
\frac{\theta_H \frac{N}{4} (1 - q) + \theta_L \left( \left( \frac{N}{2} + 1 \right) q + \frac{N}{4} (1 - q) \right)}{\frac{N}{2} + q} \geq \frac{\theta_H \frac{N}{2} (\alpha_H + \beta_H) + \theta_L \left( \left( \frac{N}{2} + 1 \right) \alpha_L + \frac{N}{2} \beta_L \right)}{\frac{N}{2} + \alpha_L}
\]

These inequalities involve weighted averages of \( \theta_H \) and \( \theta_L \), and are
equivelent to

\[
\frac{N}{4} (1 - q) \geq \frac{N}{2} \left( \frac{\alpha_L + \beta_L}{\frac{N}{2} + \alpha_H} \right)
\]

and

\[
\frac{N}{4} (1 - q) \geq \frac{N}{2} \left( \frac{\alpha_H + \beta_H}{\frac{N}{2} + \alpha_L} \right)
\]

or

\[
\frac{1 - q}{\frac{N}{2} + q} \geq \max \left\{ \frac{\alpha_L + \beta_L}{\frac{N}{2} + \alpha_H}, \frac{\alpha_H + \beta_H}{\frac{N}{2} + \alpha_L} \right\}.
\]

Since

\[
\max \left\{ \frac{\alpha_L + \beta_L}{\frac{N}{2} + \alpha_H}, \frac{\alpha_H + \beta_H}{\frac{N}{2} + \alpha_L} \right\} \geq \max \left\{ \frac{\alpha_L + \beta_L}{\frac{N}{2} + \alpha_H + \beta_H}, \frac{\alpha_H + \beta_H}{\frac{N}{2} + \alpha_L + \beta_L} \right\}
\]

\[
\geq \frac{1}{\frac{N}{2} + \frac{1}{2}}
\]

Inequality (48) implies

\[
\frac{1 - q}{\frac{N}{2} + q} \geq \frac{1}{\frac{N}{2} + \frac{1}{2}}
\]

which simplifies to

\[
q \leq \frac{1}{N + 3}.
\]

References


