Hilbert Curves

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1 Results

1.1 Correlation of All Points

The first experiment looks at the general correlation between Euclidean distance and Hilbert distance. Hilbert distance is the distance between two vectors after they have been mapped to 1-dimensional Hilbert space. Euclidean distance is the square-root of the sum of the squares of the differences for each component of the vector.

We ran an experiment where each dimension ranged from 0 to 1000 on a side and we assigned each vector random coordinates. We ran experiments with two, three, and four dimensions.

The results, shown in Figure 1, confirm our intuition about what the distribution of distances should be. For example, look at the Figure for two dimensions. The Hilbert line on a unit square heads up from $(0, 0)$ to $(1, 0)$, across to $(1, 1)$, and down to $(0, 1)$. We see the maximum Euclidean distance reached between $(0, 0)$ and $(1000, 1000)$. This experiment serves to emphasize that there is *not* a direct correlation between Euclidean distance and Hilbert distance between arbitrary vectors.

1.2 Nearest Point Correlation

This experiment looks at the correlation between the ranks of closest points in Euclidean and Hilbert space. We assigned 1000 points randomly along axes of length 1000, as before. For each point, we found the nearest point to it in Euclidean space. We then found the Hilbert distance of this point to all other points and determined the rank of this nearest Euclidean point. We did the same process for the nearest point on the Hilbert line.

The results are shown in Figure 2. They suggest that the nearest neighbor in one space tends to be the nearest neighbor in the other space for two- and three-dimensions, but not for four- and five-dimensions. The next step of this will be to look for a more general clustering description.

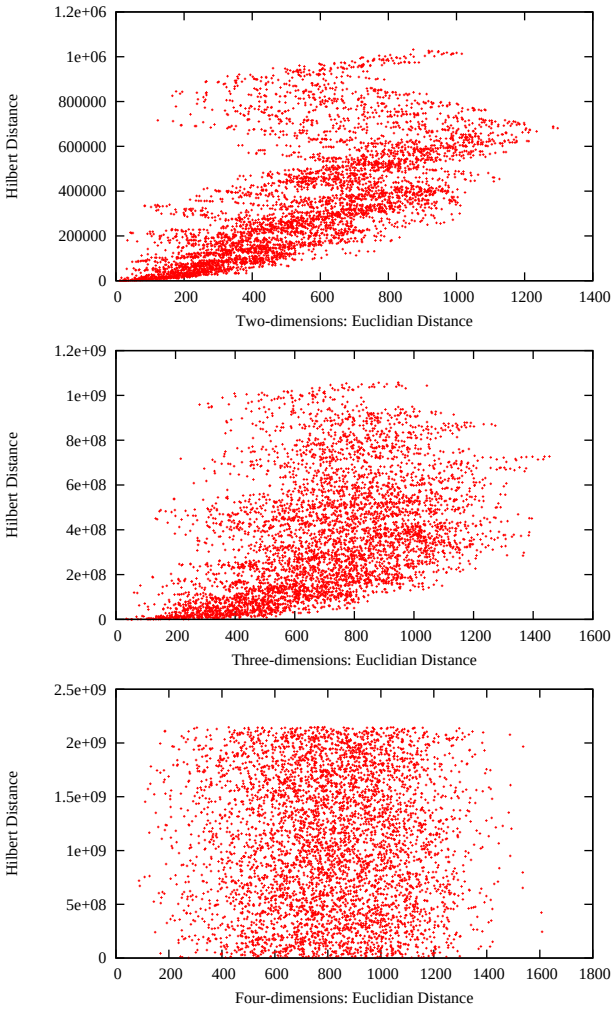

Figure 1: Correlation between Euclidean distance and Hilbert distance.

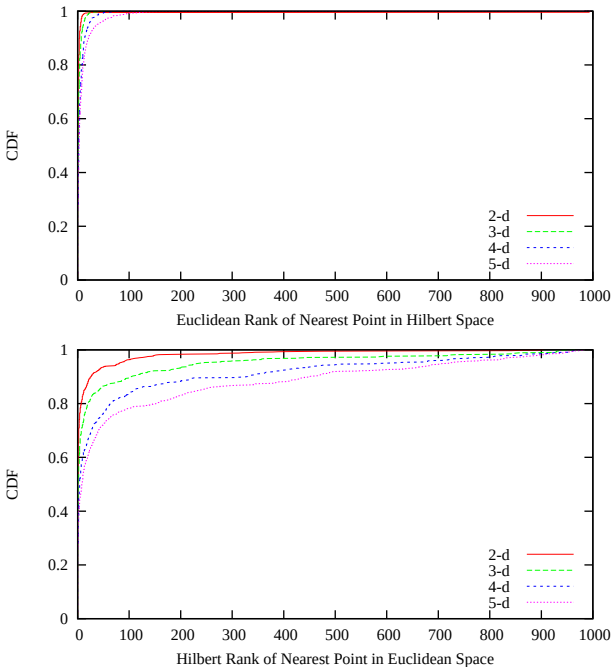

Figure 2: CDFs showing how near in Euclidean space the nearest Hilbert neighbor is and how near in Hilbert space the nearest Euclidean neighbor is.