Is it Feasible to Reconstruct Body Segment 3-D Position and Orientation Using Accelerometric Data?

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***Abstract*—The analysis of the mechanics of the musculo-skeletal system during the execution of a motor task requires the determination of the instantaneous position and orientation of the body segments involved in relation to an inertial system of reference. By using adequately assembled uniaxial accelerometric sensors, an easy-to-manage measurement system can be obtained that estimates the three-dimensional position and orientation (P&O) of a body segment through an appropriate analytical model. However, the extent to which experimental errors, in particular accelerometers (ACs) assembly inaccuracies, affect such estimation has never been systematically investigated.**

This paper systematically analyzes the sensitivity of analytical models of body segment P&O reconstruction through a six-AC system and a nine-AC system to different sources of experimental error. We simulated and statistically assessed the performance of these models in the case of body segment motions typical of movements under muscular control.

The results obtained indicated that the inaccuracy in the orientation of the individual AC's active axes and the offset error in the AC responses were the major sources of P&O estimation errors. In particular, no accurate estimation of position was possible with the analytical models analyzed. Under the motion conditions simulated in this study, no substantial advantages were found in using a nine-AC system rather than a six-AC system.

Considering that the magnitudes of the simulated experimental errors were quite low ($\leq$0.1 deg: AC's orientation; $\leq 10^{-4}$ m: uncertainty of the distance between two ACs; $\leq 10^{-2}$ ms^{-2}: random error; $0.5 \cdot 10^{-2}$ ms^{-2}: offset error), the results indicate that none of the two ACs systems analyzed is suitable for body segment P&O estimation in routine biomechanical applications.

***Index Terms*—accelerometer, accuracy, human movement analysis, simulation, 3-D rigid body position and orientation.**

I. Introduction

THE ANALYSIS of the mechanics of the musculo-skeletal system during the execution of a physical exercise requires the determination of the instantaneous position and orientation (P&O) of the body segments involved, generally modeled as rigid bodies, relative to an inertial system of reference. This result is most frequently achieved by tracking the movement of clusters of light-emitting or -reflecting point-markers associated with the body segments of interest, using a plurality of optoelectronic sensors [4]. Similarly, ultrasound-emitting point-markers and microphonic sensors may be used [1]. Knowledge of the position of at least three noncollinear points allows for the reconstruction of the P&O of the rigid body segment [3]. A different technique consists in using a magnetic sensing system, made of a triaxial assembly of miniature coils. This three-dimensional (3-D) marker is rigidly connected with a body segment and, when immersed in an *ad hoc* generated magnetic field, produces six signals that describe its P&O [15].

The above techniques exhibit different characteristics, but they share the drawback of having a definite measurement volume within which the movement must take place. In addition, point-marker emissions may not be detected by the relevant sensors because of the interposition of other body segments. These techniques constrain the movements they intend to investigate. When, instead, magnetic markers are used, the magnetic field is influenced by metallic objects within or near the measurement volume, which amongst other inconveniences, renders the calibration of the system awkward. Since all the above techniques provide information on displacement, acceleration must be estimated for it is necessary for the description of relevant dynamics. This estimation entails double time-differentiation of noisy displacement data, which is a very critical process, as emphasized by ample literature [5], [10], [16].

An alternative approach to the problem of P&O estimation consists in measuring acceleration with a 3-D marker made of adequately assembled accelerometric sensors, and estimating the relevant P&O through integration algorithms. As opposed to the techniques previously illustrated, this technique sets no constraints to the measurement volume, nor does it suffer from any shadowing effect. In addition, accelerometers (ACs) are small and light enough to allow the construction of 3-D markers that can be easily connected with a body segment without hindering the execution of the motor tasks.

In the literature, ACs were used to monitor the level of human physical activity [8], to assess tremors [7] or to measure the deformation wave that travels along the skeletal system when the foot impacts the floor during locomotion [9]. For these applications, however, the accurate estimation of body segment P&O is irrelevant. Other studies used AC measurements to calculate the relative angle between two joints, but did not provide information about absolute displacements. An example of the latter type of application is the study by Willemsen [17], who used an analytical model receiving as input the measurements of twelve properly arranged uniaxial ACs.

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Morris [13] introduced an analytical model aimed at reconstructing the 3-D P&O of a body segment using the signals provided by six uniaxial piezoresistive ACs. This model was applied for the reconstruction of absolute shank motion, regarded as a mono-dimensional link, during level walking. Padgaonkar [14] used a model based on the measurements of nine uniaxial ACs. They analyzed the sensitivity of both their model and Morris' to systematic errors in acceleration measurements, and compared the performance of the two models using experimental data recorded during simple oscillations and during an impact test. The results indicated that the six-AC method was intrinsically unstable, as opposed to the nine-AC method, which was found to be stable and relatively insensitive to small systematic errors. Liu [11] provided analytical support to the empirical findings by Padgaonkar [14] concerning the instability of the six-AC model. Nevertheless, as instabilities take time to build up, both the studies by Liu and Padgaonkar *et al.* envisage the applicability of the six-AC system over short intervals of time.

From this brief literature review, it appears that no systematic analysis is available on the propagation of the measurement errors that may affect AC sensors when estimating the 3-D P&O of a rigid body segment. As a matter of fact, at present, the burden of proof of model stability rests with the six-AC user, who must determine, case by case, the time beyond which P&O estimation errors become intolerably large [14]. Therefore, there is no structured information by which we can possibly evaluate the feasibility of using accelerometric data to reconstruct the voluntary movement of a given body segment.

This paper systematically analyzes the sensitivity to different sources of experimental error of analytical model of body segment P&O estimation using a six-AC system and a nine-AC system. Among all the potential experimental error sources, particular attention was devoted to the assembly of the ACs. The case of voluntary body segment motions was analyzed specifically for locomotor acts.

II. Methods

A. Mathematical Models

Let us consider a local reference frame (O, xyz) solid with a rigid body. The absolute P&O of this rigid body be defined by the origin position vector $\mathbf{o}$ and by the orientation matrix $\mathbf{R}$, respectively. A uniaxial AC be attached to the rigid body in point P. Its local position is defined by the position vector $\mathbf{p}$ and the orientation of its sensitive axis by the unit vector α. During a roto-translational movement, the AC would ideally measure the projection a of the absolute acceleration of point P onto its sensitive axis

$$a = \alpha \cdot [\mathbf{a}_{\mathrm{o}} + [\dot{\omega} \times \mathbf{p} + \omega \times (\omega \times \mathbf{p})] + \mathbf{g}] \tag{1}$$

where $\boldsymbol{\omega}$ denotes the angular velocity of the local frame, $\boldsymbol{a}_0$ the local frame origin acceleration, and $\mathbf{g}$ the gravitational acceleration.

Let us now consider a 3-D accelerometric marker made of six uniaxial ACs, arranged in three couples as indicated in Fig. 1(a). This assembly is characterized by the fact that the ACs of a couple have parallel active axes located at a distance r from each other. Furthermore, for the sake of simplicity, the arrangement may be such that the active axes of each couple are aligned with a different local axis. Note that, as opposed to Padgaonkar [14], this assembly exempts us from placing two or more ACs at the same point, which is practically not feasible.

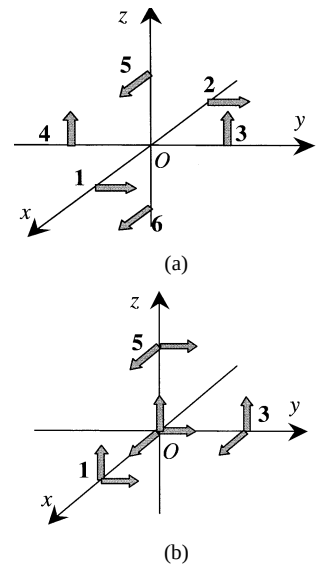

Fig. 1. Mounting configuration of the (a) six-AC and (b) nine-AC assemblies. Arrows indicate P&O of the active axis of each AC.

By subtracting the measurements taken by the ACs of each couple [see (1)] and performing simple calculations, the following scalar equations are obtained:

$$\begin{aligned}\dot{\omega}_x &= \frac{a_{3z} - a_{4z}}{r} - \omega_y \omega_z \\ \dot{\omega}_y &= \frac{a_{5x} - a_{6x}}{r} - \omega_x \omega_z \\ \dot{\omega}_z &= \frac{a_{1y} - a_{2y}}{r} - \omega_x \omega_y,\end{aligned} \tag{2}$$

where a_{ij} is the acceleration component measured by the AC located in the ith point and aligned with the jth local axis, and ω_j are the angular velocity components in the local frame. The fact that this equation system exhibits a simple structure and is relatively easy to integrate is due to the specific AC's arrangement.

The nine monoaxial AC's arrangement proposed in [14] was used [Fig. 1(b)]. Note that in this assembly, owing to the higher number of ACs, more than one uniaxial AC are assumed to be mounted in the same physical point. Although this assumption entails an approximation, it simplifies the related mathematical model. The distance between two ACs of a parallel sensitive axis couple is again r.

By subtracting the measurements taken by the AC couples and performing simple substitutions, the following linear uncoupled equations are obtained:

$$\begin{aligned}\dot{\omega}_x &= \frac{a_{3z} - a_{0z}}{2r} - \frac{a_{5y} - a_{0y}}{2r} \\ \dot{\omega}_y &= \frac{a_{5x} - a_{0x}}{2r} - \frac{a_{1z} - a_{0z}}{2r} \\ \dot{\omega}_z &= \frac{a_{1y} - a_{0y}}{2r} - \frac{a_{3x} - a_{0x}}{2r}.\end{aligned} \tag{3}$$

Thus, the knowledge of either six or nine acceleration measurements allows, through the integration of (2) or (3), respectively, the calculation of the local components of angular velocity. These can be employed to estimate, through further integration, the orientation matrix $\mathbf{R}$ of the local frame with respect to an inertial frame.

For readability purposes, rigid body orientation will be also expressed in the following using the nautical angles roll $\boldsymbol{\psi}$, pitch $\boldsymbol{\theta}$, and yaw $\boldsymbol{\phi}$, readily available from the orientation matrix elements [18]. The definition of the nautical angles is worth remembering here: roll, pitch, and yaw are the angular displacements of a mobile body about its longitudinal axis, about a horizontal axis perpendicular to the longitudinal one, and about an axis perpendicular to the previous two, respectively. In the notation used in this paper, the above axes coincide with the local frame axes $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$, respectively.

The acceleration of the local frame's origin is given by

$$\boldsymbol{a}_{\mathrm{o}} = \mathbf{R} \cdot \boldsymbol{a}_{\mathrm{os}} - \mathbf{g} \tag{4}$$

where $\boldsymbol{a}_{\mathrm{os}}$ is equal to

$$\begin{bmatrix} \dfrac{a_{5x} + a_{6x}}{2} \\ \dfrac{a_{1y} + a_{2y}}{2} \\ \dfrac{a_{3z} + a_{4z}}{2} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} a_{0x} \\ a_{0y} \\ a_{0z} \end{bmatrix} \tag{5}$$

depending on whether the six- or the nine-AC's assembly is used. The position vector oof the local frame with respect to the inertial frame can hereafter be estimated through double integration of a_{o}.

B. Error Sources and Their Propagation to the Measured Accelerations

The following error sources have been taken into account for each AC: the errors $\mathbf{p}_e$ and α_e, as shown in Fig. 2, are equal to the difference between actual and nominal position ($\mathbf{p}_a$ and $\mathbf{p}$) and orientation (α_a and α), respectively; the offset error $\mathbf{e}_o$ and the sensitivity error $\boldsymbol{\gamma}$ (these are the errors that affect the calibration parameters and that may be temperature dependent); the random error $\mathbf{e}_{\mathrm{r}}$.

These errors propagate to the acceleration $\boldsymbol{a}_{\mathrm{m}}$ measured by each AC of the assembly according to the following equation derived from (1):

$$\begin{aligned} a_e &= a_m - a \\ &= \{\alpha_e \cdot [\dot{\omega} \times (\boldsymbol{p} + \boldsymbol{p}_e) + \omega \times [\omega \times (\boldsymbol{p} + \boldsymbol{p}_e)]] \\ &\qquad + \alpha \cdot [\dot{\omega} \times \boldsymbol{p}_e + \omega \times (\omega \times \boldsymbol{p}_e)]\} \\ &\quad + \alpha_e \cdot a_o + \alpha_e \cdot \boldsymbol{g} + \gamma \cdot a_o + \boldsymbol{e}_o + \boldsymbol{e}_r. \end{aligned} \tag{6}$$

The relationship between the above experimental errors and the estimated P&O of the body may be obtained by substituting (6) in the systems of (2) or (3), for the six- or nine-AC assemblies respectively, using the system of (4), and by double integrating the result. However, the complication of the mathematics involved does not allow for an analytical sensitivity analysis. Thus, the most effective way to acquire relevant information is by simulating the effect of possible experimental error sources in selected motions. Nevertheless, some general indications on error propagation can be drawn from (6), which may be useful in designing the above-mentioned simulation.

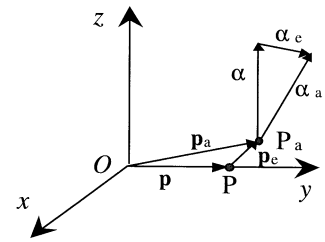

Fig. 2. Systematic errors affecting position ($\mathbf{p}_e$) and orientation (α_e) of each AC. $\mathbf{p}_a$, $\mathbf{p}$ and α_a, α are the actual and nominal values of P&O, respectively, in the local frame, of each AC of the assembly. α_a and α are unit vectors.

In (6), six additive error terms may be identified. The first term depends on the rotational component of the movement; the second and the fourth, on its translational component. Therefore, they are equal to zero when there is neither rotation nor translation. The third term depends on gravity in the presence of an orientation error. The second and third error terms are only affected by the orientation error α_e. The third term is different from zero when a gravitational field is present, and may also cause an error when the body is stationary, which produces an apparent movement. It is evident that this term becomes negligible when the translational acceleration equals several gs, as during impacts. It is also worthwhile noticing that, since both the second and third error terms take on different values for the different ACs of an assembly, besides an apparent linear displacement, they may also cause an apparent rotation [see (2) and (3)]. The fourth and fifth terms account for sensitivity and offset errors, respectively, in the AC's measurements. The sixth term accounts for the random error with a zero mean.

III. Simulation

Experimental error propagation was investigated for a number of paradigmatic motions of the rigid body and for different error sources and values. Motions were selected that represented, both in terms of amplitude and velocity, those typical of body segments during voluntary locomotion movements. In addition, the fact that rotations are normally quasiplanar was taken into consideration.

A. Paradigmatic Motions

The paradigmatic movements investigated (Table I) were the following.

1) Stationary body in the absence of the gravitational field (trials S1 and S2). This motion condition highlighted the propagation of the sixth and fifth error terms in (6), which depend on the random error and the offset error, respectively.
2) Stationary body in the presence of the gravitational field (trial S3). This highlighted the propagation of the third error term associated with AC orientation errors.
3) Translation along a specified axis of the local frame according to a sinusoidal time function, in the absence of the

TABLE I
PARADIGMATIC MOTION CONDITIONS FOR EACH SIMULATED 100-TRIAL SERIES. EACH SERIES IS IDENTIFIED BY A SERIAL CODE; $|\mathrm{p_e}|$ IS THE MAGNITUDE OF THE SIMULATED AC'S POSITIONING ERROR; $|\alpha_e|$ IS THE MAGNITUDE OF THE SIMULATED AC'S AXES ORIENTATION ERROR

Motion	Trial series code	Gravity	$\lvert\mathbf{p}_e\rvert$ (m *10^{-4})	$\lvert\hat{\alpha}_e\rvert$ (deg)	Sensitivity error	Offset error (ms^{-2} *10^{-2})	Random error (m s^{-2} *10^{-2})	Oscillation amplitude (m) or (deg)	Axis direction cosines
Stationary	**S1**	0	0	0	0	0	1	-	-
	S2	0	0	0	0	0.5	0	-	-
	S3	g	0	0.1	0	0	0	-	-
Translation	**T1**	0	0	0.1	0	0	0	0.1	1, 0, 0
	T2	0	0	0	2*10^{-4}	0	0	0.1	1, 0, 0
	T3	0	0	0.1	0	0	0	0.5	1, 0, 0
	T4	0	0	0	2*10^{-4}	0	0	0.5	1, 0, 0
Rotation	**R1**	0	1	0	0	0	0	30	1, 0, 0
	R2	0	0	0.1	0	0	0	30	1, 0, 0
	R3	0	1	0	0	0	0	60	1, 0, 0
	R4	0	0	0.1	0	0	0	60	1, 0, 0
Rototranslation	**RT1**	g	1	0.1	2*10^{-4}	0.5	1	0.1/30	1, 0, 0
	RT2	g	1	0.1	2*10^{-4}	0.5	1	0.1/30	0, 1, 0
	RT3	g	1	0.1	2*10^{-4}	0.5	1	0.1/30	0, 0, 1

gravitational field. The propagation of the second error term is highlighted in trials T1 and T3, and that of the fourth error term in trials T2 and T4. Since the AC's geometrical arrangement is symmetrical with respect to the local frame axes, the results of the translation along a frame axis can be considered valid for the other two axes as well.

4) Oscillation about an axis of the local frame (rotation) in the absence of the gravitation field, according to a sinusoidal time function. This condition highlighted systematic error propagation associated with the first error term. The effects of P&O errors have been investigated separately (trials R1, R3 and R2, R4). Also, in this case, are the results of the rotations about one axis valid for the other two axes.
5) Helical motion along and about an axis of the local frame (rototranslation). In this way, realistic cases were simulated. Because of $\boldsymbol{g}$, different results are to be expected for rototranslations along and about the three local frame axes. Thus, a simulation for each of them was carried out (trials RT1–RT3).

Each paradigmatic motion was analyzed for a time interval of 4 s, which was chosen as a typical minimum time of analysis for movements under muscular control. The frequency of the linear and angular oscillations was 1 Hz, and their amplitudes were set at 0.1 m and 30 degrees, and 0.5 m and 60 degrees, respectively (in two separate tests). At those values, the velocity and acceleration ranges encountered in most of the motor actions were spanned. The distance r between the two ACs of a couple was assumed to be 3 cm for both the six- and nine-AC assemblies, a value compatible with the usual dimensions of a transducer.

B. Experimental Error Values

Preliminary simulation trials indicated that, when the above errors are not accounted for and only truncation errors are present, inaccuracies in P&O estimation were negligible.

The instrumental noise and the quantization error were the two main components of the random error. For the instrumental noise, reference was made to the AC model 3031 of ICSensors, which is one of the best candidates for the present application because of its geometrical and low-noise characteristics. In a bench test, the noise was $8 \cdot 10^{-3}$ ms^{-2} pp, when the signal was low-pass filtered at 50 Hz, a value compatible with the maximum frequency content of human movement accelerations. The quantization error for a ± 2 g acceleration range and a 12-bit A/D converter was estimated to be $2.9 \cdot 10^{-3}$ ms^{-2}. Thus, the total random error was set at $1 \cdot 10^{-2}$ ms^{-2} pp with mean value zero.

Offset and sensitivity errors can be reduced to a minimum through accurate calibration and temperature compensation. The values considered herein, and reported in Table I, may constitute the residual errors after the accomplishment of these procedures.

Other systematic errors are represented by the positioning and orientation inaccuracy both within the case of individual ACs and within the six- or nine-AC assembly. Manufacturers seldom give the values of the former errors. Reference was made to the dual-axis AC, ADXL 202, of Analog Devices, that has an alignment error of ± 1 degree. An additional AC error source that affects the AC system response in a manner similar to the orientation error—and can, thus, be included in this one—is the cross-axis sensitivity, that is the undesired sensitivity to accelerations normal to the active axis. The sensor model just cited is affected by a cross-axis sensitivity error of $\pm 2\%$ of the sensi-

tivity along the active axis. The values assumed in Table I for positioning and orientation errors are, therefore, lower than those achievable in practice. With these low values the effects on the AC system response may be in the order of magnitude of those achievable after accurate calibration procedures carried out on the whole system.

C. Calculation of Error Propagation

For each of the motion conditions reported in Table I, the nominal position vector $\mathbf{o}$ and the orientation matrix $\mathbf{R}$ were represented versus time. In this way, nominal $\boldsymbol{\omega}$ and $\boldsymbol{a}_0$ could be calculated. Then, by using the information about the nominal geometry of the six- or nine-AC assemblies (vectors $\mathbf{p}$ and α), and setting values for the relevant errors (Table I), the acceleration that each AC would measure was computed in each sampled instant of time using (1) and (6). The systems of (2) and (3) for the six- and nine-AC assemblies, respectively, allowed the estimation of the orientation matrix $\mathbf{R}_e$ of the rigid body. The matrix of the orientation error $\mathbf{E}_r$ between the nominal and the estimated orientation of the local frame was calculated for each instant of time by means of the equation $\mathbf{E}_r = \mathbf{R}^T\mathbf{R}_e$. A representation in terms of nautical angles of the orientation error was then extracted.

With (4) we estimated the position vector and then, by comparison with the nominal position vector, we computed the position errors.

The whole simulation algorithm was implemented in Matlab R12. The differential equation system, for the calculation of the orientation matrix, was solved by means of the ordinary differential equation system solver tool, which is based on the Runge–Kutta method. The function *quadl* we used for the integration is based on the adaptative Lobatto quadrature algorithm, with absolute tolerance error in the integral set to 10^{-9}.

The sampling rate was 100 samples/s, well beyond the maximum frequency (1 Hz) of the simulated motions. The simulation algorithm was tested for numerical errors by running the previously described helical motions (RT1, RT2, RT3), for both six and nine-AC-assemblies, with the error sources set at zero. Maximum errors were found to be similar for the two assemblies and always lower than $0.5 \cdot 10^{-2}$ degrees and 10^{-5} m for P&O, respectively.

One-hundred trial series were run for each of the conditions investigated. In each trial, a random value was assigned to each error source in the range indicated in Table I. The number one hundred was chosen since a pilot study showed that further trials did not modify the results. For each trial series, the orientation error and the translation error have been described by their maximum absolute values during simulated motions of 4 s.

IV. Results

During each simulated motion trial, the angular errors in the estimation of the AC systems orientation are proportional to the time of observation, and vary in a way dependent on the time history of the considered motion. The two AC systems behave in a similar manner; just as an example, Fig. 3 reports the angular errors versus time found in a trial of the RT2 series for the six-AC system.

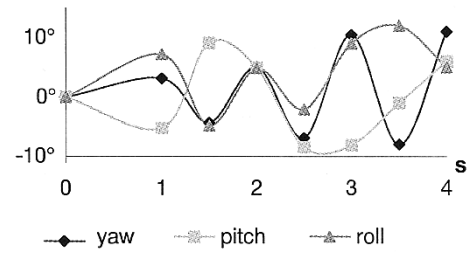

Fig. 3. Six-AC system. Angular errors versus time in one trial in rototranslation motion RT2.

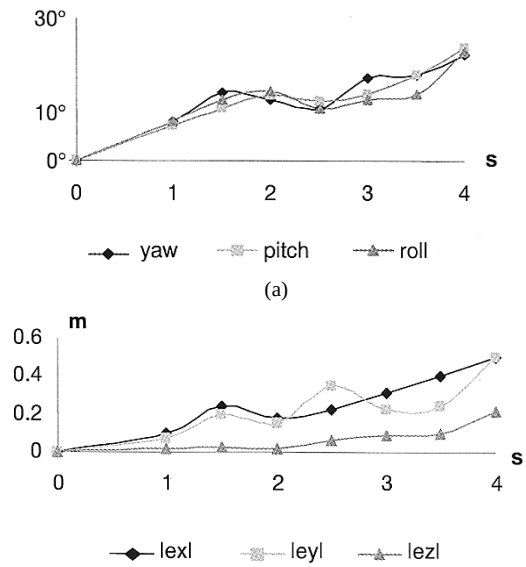

Fig. 4. Six-AC system. Maximum absolute angular (a) and translation (b) errors encountered, in each instant of time, in the rototranslation motion RT2 trial series.

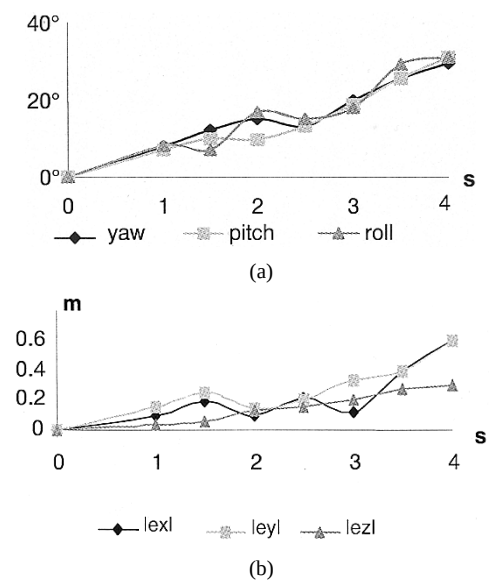

Fig. 5. Nine-AC system maximum absolute angular (a) and translation (b) errors encountered, in each instant of time, in the rototranslation motion RT2 trial series.

Figs. 4 and 5 report the maximum absolute angular and translation errors encountered, in each instant of time, in the RT2 series, for the six- and nine-AC systems, respectively. There are

no isolated peaks, therefore, the maximum values within 4 s can be regarded as descriptive of the estimation error involved. The maximum values of orientation and translation errors are reported in Table II for the six-AC system and in Table III for the nine-AC system.

A. Inaccuracy of the AC System Due to Errors in the Response of Each AC Sensor

In the absence of gravity, the effect of random noise, $\mathbf{e}_{\mathrm{r},}$, is the same along the three axes, and is not the major source of inaccuracies. The offset error, $\mathbf{e}_{\mathrm{o}}$ that affects the system response in a similar manner, has a considerably higher effect.

The effect of the sensitivity error, γ, is seen in translation motions, and is similar in the two systems; the displacement error has only one component along the direction of the movement. Here, it should be borne in mind that the sensitivity error is likely to have been underestimated.

B. Inaccuracy of the AC System Due to Alignment and Positioning Errors of Each AC Sensor Within the Assembly

The sensor alignment error affects system response under different motion conditions, whether stationary (S2), translational (T1, T3), or rotational (R2, R4).

Under stationary conditions, system inaccuracy is originated by the errors in the estimation of the $\boldsymbol{g}$ fraction to be subtracted from the response of each AC sensor. These errors are integrated over time through (2) or (3), thus producing an apparent rotation of the system and, hence, an apparent acceleration estimated by means of (4). The apparent displacement along the $\mathbf{z}$ axis is much shorter than the displacements along the horizontal axes. This is made readily clear by considering, for instance, that an error in the pitch angle, $\mathbf{e}_\theta$ results in errors in the measurement of the acceleration components, respectively along the $\mathbf{x}(\mathbf{e}_{\mathrm{ax}})$ and $\mathbf{z}(\mathbf{e}_{\mathrm{az}})$ axes, given by

$$\mathbf{e}_{\mathrm{ax}} = \boldsymbol{g}\sin(\mathbf{e}_\theta) \qquad \mathbf{e}_{\mathrm{az}} = \boldsymbol{g}[1-\cos(\mathbf{e}_\theta)]. \tag{7}$$

For low values of $\mathbf{e}_\theta$, $\mathbf{e}_{\mathrm{az}}$ is much lower than $\mathbf{e}_{\mathrm{ax}}$.

Under the translational condition, sensor axis misalignment leads to a displacement error along the motion axis that is lower than the errors along the other two axes. This can be easily understood by looking at (2) and (3), and considering that the possible difference in response between two ACs identically oriented within the assembly is minimum, if the acceleration is parallel to the ideal sensor active axes.

Under the rotational condition, system inaccuracy has the same magnitude along the three axes. Both upon translation and rotation the errors are small, but it has to be taken into account that sensor misalignments are likely to have been underestimated. Nevertheless, even the considered low misalignment leads to high system orientation and translation errors under the stationary condition.

The sensor positioning error affects system response upon rotation (R1, R3), and has equal effects along the three axes.

C. Inaccuracy of the AC System in Simulated Rototranslation

When combining rotation plus translation with gravity acceleration and all the errors in the response of each AC sensor, system P&O errors rapidly increase in comparison with pure translation and rotation movements. These errors, however, are not much higher than those under stationary condition. The displacement error along the vertical axis remains lower than those along the horizontal axes.

TABLE II
SIX-AC SYSTEM MAXIMUM ANGULAR AND POSITION ERRORS DURING 4-s SIMULATED MOTIONS FOR THE PARADIGMATIC CONDITIONS DESCRIBED IN TABLE I

Motion	Trial series code	$\lvert e_\psi \rvert$ (deg)	$\lvert e_\theta \rvert$ (deg)	$\lvert e_\phi \rvert$ (deg)	$\lvert eX \rvert$ ($m*10^{-2}$)	$\lvert eY \rvert$ ($m*10^{-2}$)	$\lvert eZ \rvert$ ($m*10^{-2}$)
Stationary	**S1**	1.2	1.2	1.2	2.1	2.2	2.2
	S2	7.2	7.2	7.2	15	15	15
	S3	13	15	15	29	30	0.8
Translation	**T1**	0.0	0.0	0.0	0.1	0.6	0.6
	T2	0.0	0.0	0.0	0.1	0.0	0.0
	T3	0.3	0.0	0.3	0.5	3.5	3.5
	T4	0.0	0.2	0.0	3.8	0.0	0.0
Rotation	**R1**	1.4	1.4	1.4	3.0	3.0	3.0
	R2	0.3	0.3	0.3	0.3	0.3	0.3
	R3	2.9	3.0	2.9	4.1	4.1	4.1
	R4	0.6	0.5	0.6	0.7	0.7	0.8
Rototranslation	**RT1**	22	24	23	50	51	22
	RT2	22	24	23	50	50	22
	RT3	22	24	23	50	50	21

TABLE III
NINE-AC SYSTEM MAXIMUM ANGULAR AND POSITION ERRORS DURING 4-s SIMULATED MOTIONS FOR THE PARADIGMATIC CONDITIONS DESCRIBED IN TABLE I

Motion	Trial series code	$\lvert e_\psi \rvert$ (deg)	$\lvert e_\theta \rvert$ (deg)	$\lvert e_\phi \rvert$ (deg)	$\lvert eX \rvert$ ($m*10^{-2}$)	$\lvert eY \rvert$ ($m*10^{-2}$)	$\lvert eZ \rvert$ ($m*10^{-2}$)
Stationary	**S1**	1.3	1.3	1.3	2.3	2.3	2.3
	S2	9.1	9.1	9.1	23	23	23
	S3	18	20	20	31	31	0.9
Translation	**T1**	0.1	0.0	0.0	0.1	0.7	0.7
	T2	0.0	0.0	0.0	0.2	0.0	0.0
	T3	0.5	0.3	0.0	0.6	3.8	3.8
	T4	0.0	0.2	0.3	4.5	0.0	0.0
Rotation	**R1**	1.6	1.7	1.6	3.1	3.1	3.2
	R2	0.4	0.4	0.4	0.4	0.3	0.4
	R3	4.1	4.0	4.1	4.8	4.8	4.9
	R4	0.8	0.8	0.8	0.8	0.9	0.9
Rototranslation	**RT1**	30	31	31	60	60	30
	RT2	30	31	31	60	59	30
	RT3	30	31	31	60	60	30

V. Discussion and Conclusion

Prior to the present study, only Padgaonkar [14] seems to have posed the problem of AC's orientation and positioning inaccuracies, when talking of transverse sensitivity. However, they specified neither the causes nor the relative importance of the above inaccuracies for the estimation of a body segment P&O.

Although system P&O inaccuracies, due to response errors of the AC sensors and to their own P&O errors, depend on different variables, a comparison among these error sources is useful as it may supply relevant indications to AC's manufacturers.

On this ground, the present study indicates, for the motions considered, that the errors in the orientation of the AC's active axes (including cross-axis sensitivity) and the offset error sensor response are the major causes of inaccuracies when estimating system P&O. Most importantly, these inaccuracies are not-negligible even for the short periods of analysis (4 s) that we adopted in the present study.

The above results will be better appreciated if one considers that the low values we assigned to the error sources are hardly achievable, especially for the systematic errors, even after accurate calibration procedures.

As expected, the greater the acceleration, the greater the error.

For the six-AC system, the accumulation of errors with time was due to the dependence, in (2), of angular acceleration on the previous estimates of angular velocity, as on the whole, mentioned in [14]. The results we obtained for the six-AC model are compatible with those reported by Padgaonkar (where after 0.1 s the estimations of the model were wholly unreliable). This error accumulation was responsible for the fact that no accurate estimation of position was possible with the analytical models used. In fact, the errors in the estimation of orientation are propagated to the estimation of position because of the gravitational field.

The analysis of Tables II and III show that all the considered errors affect both models in a similar fashion, but have a stronger effect on the nine-AC model. It should be considered that the construction of the nine-AC model is more critical, owing to the necessity of positioning several substantially coincident monoaxial ACs. In conclusion, the six-AC model, in spite of its reported instability [11], is preferable to the nine-AC assembly.

Considering that the experimental errors given as input to the simulation were assigned very low magnitudes, these results indicate that neither AC system presented is suitable for body segment P&O estimation in a clinical setting.

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