Problem 1: A professor has $n$ umbrellas. She walks to the office in the morning and walks home in the evening. If it is raining, she will carry an umbrella, as long as she has an umbrella at her current location, and if it is not raining, she will not. Suppose that it is raining on any given journey with probability $p$, independent of past weather. What is the long-run proportion of journeys on which the professor gets wet?

Problem 2: Consider any Markov chain defined on states $\{0, 1, 2, \ldots, N\}$, where each state moves only to its neighbors or itself. That is, for every $i \neq 0, N$, the transition probabilities are $p_{i,i-1}, p_{i,i}, p_{i,i+1} > 0$; for $i = 0$, $p_{i,i}, p_{i,i+1} > 0$; for $i = N$, $p_{i,i}, p_{i,i-1} > 0$. All other $p_{ij} = 0$. Prove that such a Markov chain has a unique stationary distribution, and write down a formula/expression that gives the stationary distribution $\pi_i$ in terms of the $p_{ij}$ for all $i$. (Hint: Study Theorem 7.10 in Mitzenmacher/Upfal.)

Do the following exercises from Mitzenmacher and Upfal:

9.3, 9.9, 9.11.