Improved Low-Density Parity-Check Codes Using Irregular Graphs and Belief Propagation

Michael G. Luby[1]
International Computer Science Institute
Berkeley, CA
Email luby@icsi.berkeley.edu

Michael Mitzenmacher
Digital Systems Research Center
Palo Alto, CA
Email michaelm@pa.dec.com

M. Amin Shokrollahi
International Computer Science Institute
Berkeley, CA
Email amin@icsi.berkeley.edu

Daniel A. Spielman
Department of Mathematics, M.I.T.
Cambridge, MA
Email spielman@math.mit.edu

Abstract — **We construct new families of low-density parity-check codes, which we call *irregular codes*. When decoded using belief propagation, our codes can correct more errors than previously known low-density codes. Our improved performance comes from using codes based on irregular random bipartite graphs, based on the work of [2]. Previously studied low-density codes have been derived from regular bipartite graphs. Initial experimental results for our irregular codes suggest that, with improvements, irregular codes may be able to match turbo code performance.**

I. Introduction

We have constructed new families of low-density codes, which we call *irregular codes*. The terminology comes from the fact that the parity-check matrices of our codes yield highly irregular bipartite graphs. These codes have significantly improved performance over previously known codes of this type. Using belief propagation, they can correct a substantially higher number of errors, albeit at the expense of a slightly slower running time. Further information can be found in an extended version of this paper, available as TR-97-044 of the International Computer Science Institute or TN-98-009 of Digital Systems Research Center.

II. Irregular Graphs: Intuition and Example

We offer some intuition as to why using irregular graphs should improve performance. Consider trying to build a regular low-density code that transmits at a fixed rate. From the point of view of a message node, it is best to have high degree, since the more information it gets from its check nodes, the more accurately it can judge what its correct value should be. In contrast, from the point of view of a check node, it is best to have low degree, since the lower the degree of a check node, the more valuable the information it can transmit to its neighbors. These two competing requirements must be appropriately balanced. Previous work has shown that for regular graphs, low degree graphs yield the best performance [1]. If one allows irregular graphs, however, there is significantly more flexibility in balancing these competing requirements. Message nodes with high degree will tend to their correct value quickly. These nodes then provide good information to the check nodes, which subsequently provide better information to lower degree message nodes. Irregular graph constructions thus lead to a wave effect, where high degree message nodes tend to get corrected first, and then message nodes with slightly smaller degree, and so on down the line.

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Code Rate 1/2	
Left Degrees	$\lambda_3 = 0.44506$, $\lambda_5 = 0.26704$, $\lambda_9 = 0.14835$, $\lambda_{17} = 0.07854$, $\lambda_{33} = 0.04046$, $\lambda_{65} = 0.02055$
Right Degrees	$\rho_7 = 0.35282$, $\rho_8 = 0.29548$, $\rho_{19} = 0.10225$, $\rho_{20} = 0.18321$, $\rho_{84} = 0.04179$, $\rho_{85} = 0.02445$

Table 1: Sample code parameters.

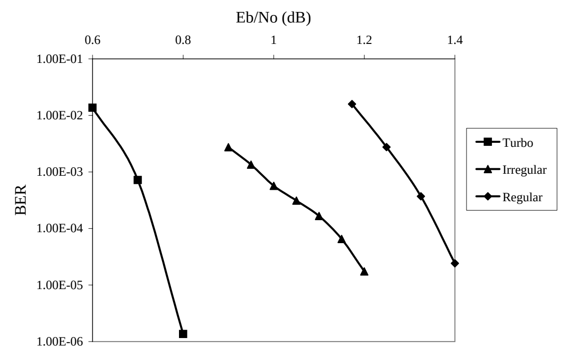

Figure 1: From left to right, rate 1/2 turbo codes, irregular codes, and regular codes.

This intuition (which we observe in our experiments) unfortunately does not provide clues as to how to construct appropriate irregular graphs. Moreover, because belief propagation is not yet well understood mathematically, creating the proper irregular graphs appears a daunting challenge. We meet this challenge by using irregular graphs that have been proven to be effective for erasure codes that function in a similar manner [2]. In the area of erasure codes, the mathematical framework has been established to both design irregular graphs and prove their effectiveness.

We provide an example of the irregular graphs used in Table 1. In the table, λ_i (ρ_i) denotes the fraction of nodes of degree i on the left (right) hand side of the graph. Note that given a vector λ and ρ one can construct a graph with (approximately) the correct node fractions for any number of nodes.

References

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[2] M. Luby, M. Mitzenmacher, M. A. Shokrollahi, D. A. Spielman, and V. Stemann, "Practical Loss-Resilient Codes," *Proc. 29th Symp. on Theory of Computing*, 1997, pp. 150–159.