Overview

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Historical Perspective

Milgrom

- Nash (1950): General definition of equilibrium for a large class of games, proof of existence. [Nobel 1994]
  – *"the analytical structure or studying all situations of conflict and cooperation"* (Myerson'99)
- Vickrey (61), birth of *auction theory* [Nobel 1996]
  – later, Myerson applied game theory to study *optimal* auction design.
- FCC Spectrum Auction ($100B by end of 2001), mandate was to put license "in hands of those that value it the most"
- in grad micro textbooks from the early '90s; now used by companies (MarketDesign, CombineNet, FreeMarkets, Frictionless) to design market places; within CS to design and study networked systems.

Aside: FCC Auction

- 51 Major Trading Areas (MTAs), 30 MHz spectrum per MTA. 492 Basic Trading Areas (BTAs), each with one 30 Mhz and four 10 MHz blocks.
  – $51 \times 1 + 492 \times (1 + 4) = 2511$ items
- Clear efficiencies to aggregating licenses.
  – fixed cost of infrastructure, marketing, roaming synergies, etc.
- Simultaneous ascending-price auction (Milgrom/Wilson design).
  – prices on items
  – complex *activity*, and *stopping* rules, *click-box* bidding.
- 1994–2001, more than $\$$10 billion.

Auction 31: Combinatorial (??)

- 700 MHz auction, 12 licenses (6 regions, 10 MHz and 20 MHz in each)
- Diverse preferences (30 MHz for high-speed data service; “fill holes”; build a “national footprint”).
- Limited number of bundles, XOR-across rounds, OR within round, stopping rules, activity rules.
- Still debated, and still not happened.

$\Rightarrow$ future points to a “combinatorial exchange”.

Mechanism Design

- constructive; highly mathematical theory; “inverse game theory”
- can control the *rules of the game* (strategy space, mapping from strategies to outcomes), but not the *behavior* of agents.
- central challenge is the “information problem”, need to elicit information about the preferences of agents to implement a desirable system-wide outcome.
  – e.g. course-registration; location of a new school; share of bandwidth; etc.
- rich theory, with both positive and negative results
  – perhaps too focused on the “revelation principle”

Example: Median Choice Mechanism

Decisions fall on a single dimension $D \subset \mathbb{R}$.

Def. Agents have *single-peaked* preferences over D if for each i and $\theta_i \in \Theta_i$ there exists $p(\theta_i) \in D$ called the "peak" of i's preferences, such that $p(\theta_i) \geq d > d'$ or $d' > d \geq p(\theta_i)$ imply $v_i(d, \theta_i) > v_i(d', \theta_i)$.

Mechanism. Have each agent declare their peak and select the [mean/median?].

Properties? Truthful? Pareto Efficient?

Moulin class: place $n-1$ phantom peaks, take XXX over $2n-1$ peaks. Completely describes class of strategy-proof decision rules in single-peaked setting.

Auction Theory

- Subset of mechanism design; focuses on *resource allocation problems*

  – e.g. allocation of excess seat capacity; sale of digital goods

- **Variations:**

  – market structure

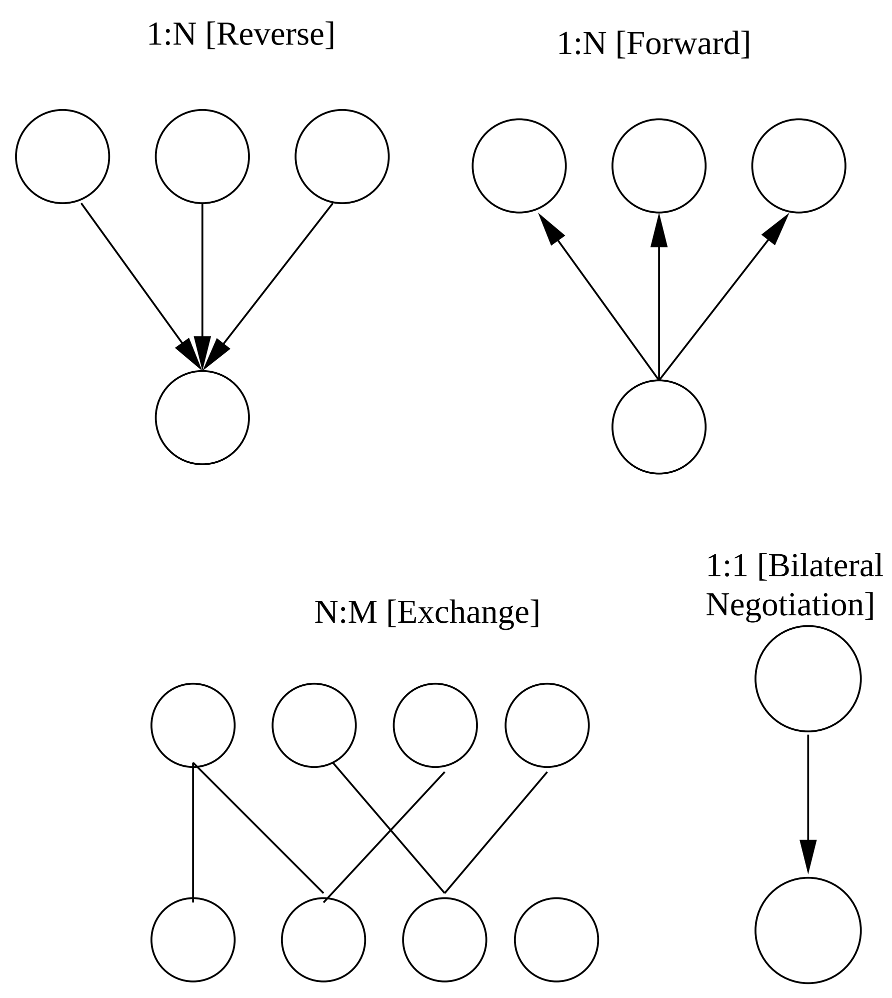
  – valuation model

  – iterative vs. sealed

  – design objective

  – resource allocation problem

Market Structure



Valuation Models

- private
  – each agent knows value; knowledge of other agent's value would not affect value. most plausible when value derived from consumption/use. e.g. bandwidth.
- interdependent
  – bidder has an estimate of value; other bidders may also possess useful information. suited when object can be resold in future (e.g. an arts auction).
- common
  – value unknown at time of bidding, agent has an estimate. actual value same for all bidders. most appropriate when value defined from a market price. (e.g. an auction for oil drilling rights.)

Iterative vs. Sealed

- Sealed-bid auction:

  Vickrey (second-price sealed-bid auction) – First-price sealed-bid

- Iterative auction:

  –English (ascending-price auction)

  – Dutch auction (descending-price auction)

equivalences? tradeoffs? What about combinatorial variations?

Aside: Wellman et al. “Auction protocols for decentralized scheduling”

- Consider a job scheduling problem; agents need a number of units on a shared machine before a particular deadline
- Propose a *simultaneous ascending-price auction*, in which agents can submit bids on individual slots as prices change.
- Efficient in “single unit-demand” special case (also “gross-substitutes” special case)
  – otherwise suffers from *exposure problem*
- Analysis uses *competitive equilibrium prices*/linear-programming theory to demonstrate the efficiency of the auction.

Design Objective

- **Efficiency.** Maximize total value across all possible allocations.

  – object ends up in hands of person who values it the most *ex post*

- **Optimal.** Maximize the payoff to a single agent (e.g. the seller).

  – e.g. maximize expected revenue to the seller.

Note: with resale opportunitues, the two goals become consistent.

Variations: Multi-Unit

K identical items for sale, N agents. Each agent has a value $v_i(n)$ for $n \geq 0$ items.

outcome: $(k_1, \ldots, k_N)$ where $\sum_i k_i \leq N$.

efficient: maximize $\sum_i v_i(k_i)$.

Example 1. Auction for excess seats on an airplane.

Example 2. Reverse auction; IBM buying memory chips.

Computational challenges: winner-determination problem (weighted-knapsack), bidding languages.

Variations: Combinatorial

$\mathcal{G}$ items, values $v_i(S)$ for $S \subseteq \mathcal{G}$. Outcome $S = (S_1, \ldots, S_N)$ is feasible if $S_i \cap S_j = \emptyset$ for all i, j.

efficient: maximize $\sum_i v_i(S_i)$.

Examples: course registration; take-off/landing; logistics; bus routes, etc.

Computational challenges: winner-determination (weighted set-packing), bidding languages, preference elicitation.

Combinatorial Auction Examples

- **Take-off/landing slot auction** [RSB82].
  – Sealed bid, one-round, single-item prices, *secondary market* to clean up the outcome.
- **Chicago GSB course registration** [GSS93].
  – Multiple-round, combinatorial auction (limited expressivity). Computes linear item prices in each round. Fixed number of rounds.
- **Collaborative planning application**. Agents submit bids to perform combinations of sub-tasks, that are composed into an overall plan [HG00].
- **San Francisco Housing Auction**. Sealed-bid, constrained bids [Wired 2000].

The Generalized Allocation Problem

Goods $\mathcal{G} = \{1, \ldots, m\}$, Agents $\mathcal{I} = \{1, \ldots, n\}$.

Let $\lambda_i = (\lambda_i(1), \lambda_i(2), \ldots, \lambda_i(m))$, for $\lambda_i(j) \in \mathbb{Z}$, indicate a transfer of $\lambda_i(j) > 0$ items $j \in \mathcal{G}$ to agent i; alternatively *from* agent i if $\lambda_j < 0$.

Agent i has a valuation function, $v_i(\lambda_i) \in \mathbb{R}$, for every trade $\lambda_i \in \Lambda$.

The *efficient trade* solves:

$$\max_{(\lambda_1, \ldots, \lambda_n)} \sum_{i=1}^{n} v_i(\lambda_i)$$
$$\text{s.t. } \textit{feasible}(\lambda)$$

Many defs. of *feasible*; depends on aggregation, divisibility, free-disposal, ex-ante constraints (e.g. on market concentration), etc.

Example

Goods $\mathcal{G} = \{1, 2\}$, Agents $\mathcal{I} = \{1, 2, 3, 4\}$.

Valuation functions:

agent 1	$v_1([-1, 0]) = -10$,	$v_1([-1, -1]) = -\infty$,	$, \ldots$
agent 2	$v_2([0, -1]) = -5$,	$v_2([-1, -1]) = -\infty$,	$, \ldots$
agent 3	$v_3([1, 1]) = 51$,	$v_3([1, 0]) = 0$,	$, \ldots$
agent 4	$v_4([1, 1]) = 41$,	$v_4([1, 0]) = 0$,	$, \ldots$

and completed to satisfy *free-disposal*.

Efficient allocation:

$$\lambda^* = ([-1, 0], [0, -1], [1, 1], [0, 0])$$

for value $\sum_i v_i(\lambda_i^*) = 36$.

Variations: Multiattribute

Attribute-vector $x \in \mathbb{R}^M$; buyer $v^B(x)$; sellers $c_i(x)$.

Efficient: (x^*, i^*) to maximize $v^B(x^*) - c_{i^*}(x^*)$.

Special case: preferential-independence.
$v^B(x) = \sum_j w_j v_j^B(x_j)$

Example: choosing a meal (wine, cheese, entree, desert, ...); choosing a combination of processor speed and memory; etc.

Issues: extensions to one-many outcomes, dynamic introduction of terms, complex preference space, etc.

Computational Challenges

(A) bidding languages/ winner-determination

(B) communication complexity/ preference-elicitation costs

(C) simple bidding strategies

(D) maintaining incentive properties as introduce approximations

Example: Winner-Determination

Set packing problem:

$$\begin{aligned} \max \quad & \sum_{S \subseteq \mathcal{G}} b(S) x_S \\ \text{s.t.} \quad & \sum_{S \ni j} x_S \leq 1, \quad \forall j \in \mathcal{G} \\ & x_S = \{0, 1\}, \quad \forall S \subseteq \mathcal{G} \end{aligned}$$

Given a ground set $\mathcal{G}$ and a collection $\mathcal{V}$ of subsets find the largest weight collection of subsets that are pairwise disjoint. NP-hard.

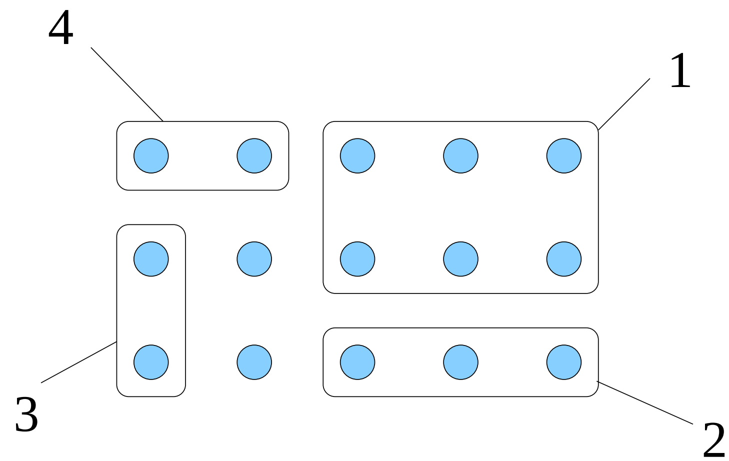
Solvable instances:

– bidding on a circle (consecutive items); (e.g. parcels on the shore of a lake)

– gross-substitues preferences (including single-item demand special case); can solve as a linear-program relaxation.

Example: Minimal Pref. Elicit.

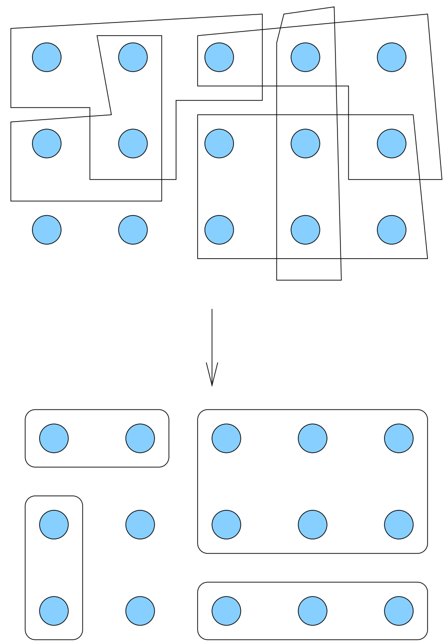
- *Single item:* values $v_1 = 4, v_2 = 8, v_3 = 12$.
  – info. $v_1 \leq 8$; $v_2 = 8$; $v_3 \geq 8$.
- *Combinatorial auction:* agents have non-overlapping optimal bundles.



. . . can compute and verify the efficient allocation in both cases.

Untangling Best-Response

[Parkes & Ungar, 00]



⇒ proposed an ascending-price combinatorial auction, agents follow *myopic best-response* strategies, terminates with efficient allocation.

Incentive/GVA

(Vickrey 61, Groves 71, Clarke 73)

1. Agents $i \in \mathcal{I}$ submit *bids* $\hat{v}_i(S)$ for all bundles $S \subseteq G$ of routes.
2. Compute allocation $\hat{\mathbf{S}}$ to maximize reported value:

$$\hat{V}(\mathcal{I}) = \max_{(S_1,\ldots,S_I)} \sum_i \hat{v}_i(S_i)$$

3. Compute *best allocation without each agent* i:

$$\hat{V}(\mathcal{I} \setminus j) = \max_{(S_1,\ldots,S_I)} \sum_{j \neq i} \hat{v}_j(S_j)$$

4. Implement $\hat{\mathbf{S}}$, and compute Payments:

$$p_{\text{gva}}(i) = \hat{v}_i(\hat{S}_i) - \left(\hat{V}(\mathcal{I}) - \hat{V}(\mathcal{I} \setminus j)\right)$$

This is **efficient** and **strategy-proof**.

Introducing Approximations

Theorem. A VCG-Based Mechanism is strategyproof if and only if it is implemented with an outcome rule that is *maximal in range*.

Definition. Outcome rule $f(\theta)$ is maximal in range if it implements

$$\max_{k \in \overline{\mathcal{K}}} \sum_i w_i v_i(k, \theta_i), \quad \forall i, \forall \theta$$

where $\overline{\mathcal{K}}$ is the *range*, i.e. the set of outcomes reachable with $f(\theta)$ for some $\theta \in \Theta$; weights $w_i \geq 0$.

$\Rightarrow$ must be very careful when introducing approximations.

Fast & Strategyproof Auctions

Lehmann et al. 99

- single-minded bidders: there is a single set $S \subseteq \mathcal{G}$ demanded by each agent. (still NP-hard).
- greedy, monotonoic allocation rule: sort bids by some criterion, then take bids in order if not in conflict.
  – e.g. scheme with norm $a/|S|^{1/2}$ approximates within factor of $|\mathcal{G}|^{1/2}$.
- strategyproof auction: charge each winner the per-item price of the first unsuccessful bid.

Example: goods A, B. bidders Red (10,A); Green (19,AB); Blue $(8, B)$.

False-name Bids (a teaser)

(Yokoo et al.)

- Economic solutions, such as the Vickrey auction, are strategyproof.
  – but, agents can manipulate multi-unit auctions through false-name bids.
  – can we design auctions that are strategyproof *and* robust to false-name bids? (what about EFFICIENT, SP, and FNBP?)

	A	B	AB
1	7	0	7
2	0	0	12
3	0	7	7

	A	B	AB
1	7	7	14
2	0	0	12
3	0	0	0

Next class

- Game theory
  – Normal form; incomplete information games
  – Dominant, Nash, Bayesian-Nash, *ex post* Nash equilibrium
  – Dominated strategies
  – Computational issues