Auction Theory

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Auctions: A Special Case of Mech. Design

- Allocation problems
  – finite set $\mathcal{G}$ of items to allocate
  – variations possible (e.g. information goods, configurable items)
  – 1:N settings typical, N:M possible.
- Agent models
  – private values vs. common values
  – "no externalities"
  – quasi-linear, i.e. $u_i(S, p) = v_i(S) - p$ for item(s) $S \subseteq \mathcal{G}$ at price p; i.e. risk-neutral
- Mechanism properties
  – Budget-balanced ("trading mechanisms")
  – efficiency (maximize total *value*), or revenue (maximize the *utility* of a single agent)

Private vs. Common Values

- Private values [e.g. *antique collectors, contractors*]
  – independently distributed, according to some prior, $F_i(\theta)$, for agent i; priors common knowledge [iid is special case]
  – models of information asymmetry also possible
- Common values [e.g. *oil drilling rights*]
  – common value, V, info. agent i, $v_i \sim H(V)$, independent draw from a common distribution.
  – learning about someone else's value useful
- Interdependent values
  – e.g. inherent differences in production costs; but some shared "problem difficulty"

Model of agent valuations changes auction prescriptions.

Private Values

- Single-item variations
- Reverse auctions
- Iterative vs. sealed-bid
- Collusion, trust, privacy
- Variations: double auctions, multi-unit auctions, combinatorial auctions, multiattribute auctions, etc.

Single-item: Efficient

[Vickrey 61]

- English/Vickrey (second-price)
- Dutch/FPSB (first-price)

All efficient. (Why does Vickrey not break Green-Laffont imposs?)

Also, all *revenue-equivalent* (if IID, quasi-linear, symmetric). Let $v_{(k)}$ denote the k-th order statistic.

- First-price Sealed-bid/Dutch
  – best-response, $B(v) = E[v_{(2)} | v_{(1)} = v]$; expected revenue, $E[B(v_{(1)})] = E[v_{(2)}]$
- Vickrey/English
  – revenue $E[V_{(2)}]$

Thm. [Rev. Equiv.] In any efficient auction, the expected payoff to every bidder, and the seller is the same.

Optimal Auction Design

[Myerson'81]

Consider a seller with value v_0, and suppose the seller can set a reservation price $r \geq 0$.

Tradeoff: between loss of revenue when $v_0 < v_{(1)} < r$; and gain in revenue when $v_0 < r < v_{(1)}$.

Thm. The revenue-maximizing (optimal) single-item auction is a Vickrey auction with $r = B^{-1}(v_0)$.

i.e., the seller should set $r > v_0$, such that $B(r) = v_0$.

optimal auction $\neq$ efficient auction

eBay proxy agents

- Provide an “upper bid-limit” to the eBay agent, which competes in an English auction until price reached.
- Revelation principle!
  – English $\Rightarrow$ Vickrey
- **Note**: issue of trust.

Closing Rules

[Roth & Ockenfels 01]; eBay vs. Amazon (auctions now dead).

- **eBay** [hard closing rule]

  – industry in “sniping”, favors bidders with better technology

  – empirically, limits information revelation during the auction, many bidders do not use proxy agents [esp. experienced bidders]

  – bidders can implicitly collude and avoid price wars; uses the fact that there is a probability that bids will fail

- **Amazon** [soft closing rule]

  – removes this “arms race” for bidding technology

  – empirically, encourages bidding earlier in the auction

  – now it is hard to enforce implicit collusion

Multi-period Auctions

(e.g. Priceline, eBay, etc.)

You want a single item, and can participate in a sequence of Vickrey auctions. What should you do?

Notice, then that the strategyproofness of Vickrey auctions is quite brittle.

Some recent work considers the design of incentive-compatible *sequential auctions*, in which it is an equilibrium strategy for agents to bid truthfully in the first time-period in which they arrive. (Lavi & Nisan'00; Friedman & Parkes'02; Gallien'02)

Reverse Auctions/Private marketplaces

One buyer, multiple sellers. [e.g. *GM and its suppliers*]

- Descending price [second-price]
  – price starts high, continues to fall until only one supplier is left.
- Ascending price [first-price]
  – price starts low, continues to increase until one supplier accepts.

Iterative vs. Sealed-bid

- Cost of communication
- Cost of delay
- Cost of information revelation
- Common vs. Private values
- Cost of valuation
- Ability to manipulate
- Cost of participation
- Transparency

Collusion

E.g. **Bidder rings.** Group of bidders get together beforehand, and decide that only one will participate in the auction. Share gains afterwards. [Robinson 85]

- problems in reaching an agreement, sharing rewards
- first-price [Dutch, FPSB]
  – collusion is not self-enforcing because the selected bidder must submit a very small bid
- second-price [Vickrey, English]
  – collusion is self-enforcing, because deviators are punished.
- *shills*, "pulling bids off chandelier"
  – a tool for sellers to fight collusion

Trust

- **Vickrey auction**.
  – bidders must trust the auctioneer not to submit a false bid. [without risk]
  – *computational remedies?* [bid verif. mechanism, trusted 3rd party]

- **English auction**.
  – more transparent, although the auctioneer can still use a “shill” to increase the bid price [some risk]
  – how does this compare to setting a reservation price?

Information Revelation

[Rothkopf et al. 90]

- In a contracting example, the Vickrey auction awards a contract to the lowest bidder, but makes payment equal to the second-lowest bid.
  – Political problems? What if the FCC used this rule?
- **Repeated auctions.** In the context of repeated auctions, whenever I reveal my true value for an item, that can be used against me in the future.
  – Business implications, within a supply-chain context?

perhaps English auctions have more desirable properties?
computational remedies?

Double Auctions

Multiple buyers, multiple sellers, each with private information. Suppose bids, $b_1 \geq b_2 \geq \ldots \geq b_m$, and asks, $s_1 \leq s_2 \leq \ldots s_n$. Compute l^*, s.t. $b_{l^*} \geq s_{l^*}$ and $b_{l^*+1} < s_{l^*+1}$.

- strategyproof, efficient and budget-balanced impossible
  (Myserson & Satterthwaite 83)

- McAfee-Double auction
  – compute candidate trading price, $p_0 = 1/2(b_{l^*+1} + s_{l^*+1})$, if $s_{l^*} \leq p_0 \leq b_{l^*}$ clear first l^* bids and asks at this price, clear first l^*-1 bids at price b_{l^*} and first l^*-1 asks at price s_{l^*}.
  – strategy-proof, BB, not EFF.

- k-DA
  – execute first l^* bids and asks; for a uniform price $s_{l^*} + k(b_{l^*} - s_{l^*})$, for some $k \in [0, 1]$.
  – not strategyproof or EFF, but BB and good efficiency in practice, in particular for large markets.

Multi-unit Auctions: Single-item Bids

N units of a homogeneous item. First, consider the special case in which each bidder demands a *single unit*. Let $v_i \geq 0$ denote the value of bidder i.

Def. The VCG auction for this special case sells the items to the N highest bidders, each pays the $N+1$st highest bid price.

$$p_{\mathrm{vick},i} = b_i - \left(\sum_{j \leq N} b_j - \sum_{j \leq N+1, j \neq i} b_j \right) = b_{N+1}$$

Multi-unit Auctions

Single bid, (k_i, b_i), for k_i units, from each agent. Let $x_i \in \{0, 1\}$ define whether bid i is accepted, and p_i denote payment by agent i.

(1) compute x^* to solve (weighted knapsack) problem:

$$V^* = \max_x \sum_i x_i p_i$$

$$\text{s.t.} \sum_i x_i k_i \leq N$$

(2) compute payments, $p_i = b_i - (V^* - V^{-i})$ if $x_i = 1$, with $p_i = 0$ otherwise; where V^{-i} is maximal value over subproblem induced by removing bid from agent i.

Note. exclusive-or bid generalizations easy to define. See Suri et al. approximation to a multi-unti VCG mechanism later in course.

Iterative Multi-Unit Auctions

- Ausubel “clinching” auction, for *decreasing marginal values*

  – maintains a single ask price, but determines final payment of an agent along the path of the auction.

  – terminates with the efficient allocation, and the Vickrey payment; straightforward bidding is an equilibrium strategy.

- eBay “Yankee” auction.

  – maintains a per-unit price, agents submit bids for fixed quantities; auction terminates as soon as there is no overdemand.

  – terminates with Vickrey outcome in special cases; but in general not efficient.

Note: later in course, see a variation on Ausubel’s ascending-price auction that is robust to false-name bids [Iwaski et al.]

Multiple Heterogeneous Items

$\mathcal{G}$ items, values $v_i(S)$ for $S \subseteq \mathcal{G}$. Outcome $S = (S_1, \ldots, S_N)$ is feasible if $S_i \cap S_j = \emptyset$ for all i, j.

efficient: maximize $\sum_i v_i(S_i)$.

Examples: course registration; take-off/landing; logistics; bus routes, etc.

Computational challenges: winner-determination (weighted set-packing), bidding languages, preference elicitation.

Possible Solutions

- Simultaneous ascending price auctions

  – work well if “gross-substitutes” property satisfied

  – in general, lead to exposure problem

- Combinatorial auctions [non-linear prices, contingent bids]

  – sealed-bid auctions, apply VCG mechanism.

- Ascending-price auctions

  – can be vulnerable to the *threshold problem* (coordination across small bidders)

  – revenue-maximizing designs (Milgrom&Ausubel’02)

  – efficient designs (Parkes&Ungar’02)

Fast & Strategyproof Comb. Auctions

Lehmann et al. 99

- single-minded bidders: there is a single set $S \subseteq \mathcal{G}$ demanded by each agent. (still NP-hard).
- greedy, monotonoic allocation rule: sort bids by some criterion, then take bids in order if not in conflict.
  – e.g. scheme with norm $a/|S|^{1/2}$ approximates within factor of $|\mathcal{G}|^{1/2}$.
- strategyproof auction: charge each winner the per-item price of the first unsuccessful bid.

Example: goods A, B. bidders Red (10,A); Green (19,AB); Blue $(8, B)$.

Additional Auction Variations

- **Multiattribute auctions**

  – configure the attributes (e.g. quality, speed, color) of an item in addition to the price

  – optimal auction, two-attributes, continuous attribute levels (Che'93)

  – efficient price-based auction, multiple attributes, discrete attribute levels (Parkes&Kalagnanam'02)

- **Exchanges (combinatorial)**

  – multiple buyers & sellers, all with contingencies

  – important, for example, in the FCC wireless spectrum allocation setting

  – can consider a family of VCG-based payment rules (Parkes,Kalagnamam, Eso'01)

Common Value Settings

[Wilson 77; Kagel & Levin 86; Bazerman & Samuelson 83]

- $\$8$ pennies in a jar; collect sealed bids
  – average bid $\$5.13$, winning bid $\$10.01$
  – winner's curse, all get an unbiased estimate, $f(\cdot)$
  – bids increase in $f(\cdot)$ in equil.
  – winner is one with *most optimistic estimate*, "adverse selection bias"
- Simple model; signal $s_i \sim U(V - \epsilon, V + \epsilon)$
  – should bid $b_i \approx s_i - \epsilon$

Interdependent Values

[Milgrom & Weber 82]

- *Model:* if one agent has a high value, then other agents are more likely to have high values
  – ascending $\succ$ Vickrey; because the winning bidder's surplus is due to private information
  – the more the price is related to the information of other agents, the lower the "information rent" of the winning bidder
- Linkage principle
  – if the seller has any private information, should precommit to releasing the information honestly
  – same argument; better to allow competition across bidders and drive price