CS 700 Computational Mechanism Design
Homework 3: Mechanism Design and Auction Theory

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Due: Tuesday October 14, 2008, at the beginning of class. You may use any sources that you want, but you must cite the sources that you use. You can also work in pairs– just list the person you work with. *Work hard on making the proofs clear, concise, and easy to read.*
Total points: 160

1. (20 pts) Consider a sealed-bid auction that is defined in terms of:

   (A1) an agent-independent price-function so that for every v_{-i}, every allocation $x_i \in X$ to agent i, and for all v_i, the payment $p_i(v_i, v_{-i}) = p(x_i, v_{-i})$ and depends only on the allocation and the valuations of other agents.

   (A2) an allocation rule $x_i(v) \in X$ that selects an allocation $x_i(v) \in \arg\max_{x \in X}\{v_i(x) - p(x_i, v_{-i})\}$, for every agent i.

   Assume that X contains a "null" allocation, for which $v_i(x) = 0$ for all possible valuations v_i.

   (a) (10 pts) Prove formally that an auction that satisfies A1 and A2 is truthful in a dominant-strategy equilibrium.

   (b) (10 pts) Define the second-price Vickrey auction in these terms (i.e. exhibit an agent-independent price function (A1) and demonstrate that the winner determination rule satisfies (A2).)

   (c) (**extra credit**) Define the Vickrey-Clarke-Groves mechanism for a combinatorial allocation problem in these terms.

2. (10 pts) Consider a single-parameter domain with known interesting set $W_i \subseteq A$ for each agent i and private value $v_i \in \mathbb{R}$. Fixing v_{-i}, show that the WMON condition implies that if the alternative $f(v_i, v_{-i}) \in W_i$ then $f(v'_i, v_{-i}) \in W_i$ for $v'_i \geq v_i$. (See the second set of lecture notes, slides 23–25).

3. (25 pts) Consider a private-values auction for a single item. Let $x_i(v_1, \ldots, v_n)$ denote the probability that agent i is allocated the item given bids $v \in [0, h]^n$, where v_i is agent i's value for the item. Agent i's expected payment is $p_i(v) \in \mathbb{R}_{\geq 0}$. It is know that mechanism $M = (x, p)$ is incentive compatible (in expectation with respect to the randomization internal to the mechanism) if and only if $x_i(v)$ is monotone non-decreasing and the (normalized) payment is

$$p_i(v) = v_i x_i(v) - \int_{z=0}^{v_i} x_i(z, v_{-i}) dz \qquad (1)$$

   (a) Draw pictures to provide graphical intuition why monotone non-decreasing x and payment p_i is sufficient for incentive compatibility. Fix v_{-i} and consider both a misreport $v'_i > v_i$ and a misreport $v'_i < v_i$.

   (b) Derive the payment rule implied by Eq. (1) for the special case of a deterministic allocation rule. Name a mechanism in which you have seen something like this before.

4. (25 pts) Consider an open-out cry ascending-price auction that proceeds as follows: an ask price is maintained, initialized to \$1. In each round, one or more agents can "shout" a new bid price (at least the current ask price), and becomes the current winner and the ask price is set to \$1 greater than this bid price. Ties are broken at random.

   The auction closes when there are no new bids, with the final winner paying its bid price. The "myopic best-response" strategy is to bid at the new ask price while (i) losing, and (ii) the ask price is less than the agent's value.

   (a) Consider an auction with two bidders, 1 and 2, with values \$50 and \$10. Construct an example of a strategy profile that shows that myopic best-response is not a dominant strategy for bidder 1.

   (b) In what (incomplete infomation game) equilibrium concept is myopic best-response an equilibrium?

   (c) Consider an English "clock" auction, in which no jump bids are allowed (bidders can only respond at the current ask price) and once a losing bidder stops bidding it cannot start bidding again. This can be thought about as having every bidder raise his or her arm while the price is acceptable, lowering their arm at the point at which they are no longer willing to accept the price and never raising it again. Provide an *intuitive argument* for why, or why not, myopic best-response is a dominant strategy equilibrium. (No need to give a formal proof.)

5. (25 pts) Consider the Myerson optimal auction for two agents, agent 1's value is unifom $v_1 \sim U(0,1)$ and agent 2's value is uniform $v_2 \sim U(0,2)$. Assume that the seller's value for the item is $v_0 = 0$.

   (a) Define the virtual valuation function for each agent. Are they monotone non-decreasing?

   (b) Provide some intuition for the asymmetry in the virtual valuation functions: why does this seem a reasonable way to improve revenue in this auction? (Limit your response to a few sentences.)

6. (30 pts) Consider the Myerson optimal auction for agents that each have valuations v_i drawn from distribution function F with density function

$$f(v_i) = \begin{cases} 1/2 & \text{, if } v_i \in [0,1) \\ 1/4 & \text{, if } v_i \in [1,3] \end{cases}$$

   (a) Verify graphically the monotone hazard rate condition fails and that the virtual valuations are not monotone non-decreasing.

   (b) Myerson proposes to "iron" non-monotonic virtual valuation functions $\phi(v_i) = v_i - \frac{1-F(v_i)}{f(v_i)}$, by

   (i) Defining normalized $\gamma(q) = \phi(F^{-1}(q))$ for $q \in [0,1]$ where $F^{-1}(q)$ is the inverse distribution function so that $w = F^{-1}(q)$ is the value such that $F(w) = q$.

   (ii) Integrating $\gamma(q)$ with respect to q to obtain $\beta(q)$.

   (iii) Constructing the convex lower hull of $\beta(q)$ (this will be a curve that follows $\beta(q)$ but has a straight-line tangent where $\beta(q)$ is concave.)

   (iv) Differentiating the curve representing the convex lower hull to obtain an ironed $\hat{\gamma}(q)$ and thus an ironed $\hat{\phi}(v_i)$.

   Remarkably, Myerson proved that running his method on these ironed virtual valuations provides the revenue-optimal auction.

   By graphing the curves, perform the ironing process of Myerson to generate the ironed virtual valuation functions $\hat{\phi}(v_i)$.

7. (25 pts) Melange.

   (a) Explain (informally) why the second-price sealed bid auction is worse with respect to collusion than the first-price sealed bid auction. [Hint: consider the collusive strategy and how stable it is against deviations from members of the coalition.]

(b) True or False: A single eBay auction is (strongly) strategically equivalent to a second-price sealed-bid auction. Explain in a sentence or two. (Strong strategic equivalence requires that the set of equilibria are identical.)

(c) Given an argument for why "sniping" (i.e., waiting until an auction is about to close) may be a rational strategy on eBay. [There are many arguments.]

(d) You want to buy one LCD screen and face a sequence of second-price sealed bid auctions, one on Monday and one on Tuesday. Is your dominant strategy to bid truthfully on Monday? If yes, explain why. If no, provide a counter example.