Auction Mechanisms

- Auctions are allocation mechanisms that elicit information about bidders’ valuations and determine prices dynamically.

Common formats:
- Dutch
- English
- FPSB
- SPSB

Variations: eBay, procurement, FCC auctions, sponsored-search auctions; forward vs. reverse

Valuation Models

- Private value
  - value \( v_i \) is known at time of bidding
  - values \( v \sim F \) (independent or correlated)
  - knowledge of other bidders’ values does not affect \( v_i \)
  - “affiliated” \( \approx \) positively correlated

- Interdependent value
  - receive a signal \( \omega_i \in \Omega_i \) before auction
  - estimated value \( v_i(\omega_1, \ldots, \omega_n) \) depends on joint information (may be independent or correlated)
  - signals \( \omega \sim F \) (perhaps correlated)
  - may know private value after receiving good

- Common value
  - \textit{ex post} value is the same to all bidders
  - special case of interdependent

\textit{Standard model:} private value, \( v_i \) i.i.d. sampled from some \( F_i \)
Revenue vs. Efficiency

- Efficiency. Goal is to maximize total allocative value subject to (a) incentive compatibility, (b) individual rationality, (c) no deficit constraints.
- Revenue (= “optimal” auction design.) Goal is to maximize the revenue of the seller subject to (a) incentive compatibility, (b) individual rationality (= “participation”) constraints.

Efficient (private-value) auctions known (VCG).
Optimal auction design challenging; e.g. unknown for 3 item combinatorial auction.

Revenue Equivalence

Theorem. If two auctions implement the same allocation rule and losers pay zero then auctions have the same expected revenue.

Proof idea: focus on IC mechanisms by revelation principle, and show that the payment rule is uniquely defined (within a constant) by the allocation rule.

Optimal Mechanism Design

Myerson’81

Goal: maximize expected profit to a seller.
Input: distribution function $F_i(v_i)$ for each buyer, value $v_0$ of seller
Allocation rule $x(v) \in [0, 1]^n$ and payment rule $p(v) \in \mathbb{R}^n$. Let $U_i(x, p, v_i)$ denote expected utility to agent $i$, $V_0(x)$ denote expected value to seller.

$$\max_{x,p} V_0(x) + E_v[\sum_i p_i(v_i)]$$

s.t. $x$ is feasible
$U_i(x, p, v_i) \geq 0, \ \forall i, \forall v_i$
$U_i(x, p, v_i) \geq U_i(x, p, v_i|w_i), \ \forall i, \forall w_i, \forall v_i$

Myerson’s Solution

Define virtual valuation $\phi_i(v_i) = v_i - \frac{1 - E_i(v_i)}{F_i(v_i)}$ and virtual surplus given allocation $x(v)$ as $\sum_i \phi_i(v_i)x_i(v) + v_0(1 - \sum_j x_j(v))$.

Theorem. The expected profit of any truthful mechanism is equal to its expected virtual surplus.

⇒ reduce the problem of optimal auction design to one of designing an efficient auction with respect to virtual surplus!

Solved... if the virtual-surplus maximizing allocation rule happens to be monotone.
Monotone hazard rate

Def. The hazard rate of a distribution function $F$ is $\frac{f(v)}{1-F(v)}$.

Sufficient condition for virtual valuation $\phi_i(v_i)$ to be monotone non-decreasing in $v_i$ is monotone non-decreasing hazard rate. Satisfied by uniform, exponential and other distributions.

Lemma. When the virtual valuations are monotone non-decreasing then the allocation rule that maximizes virtual surplus given reported bids is monotone.

Optimal auction

- Collect bids $v = (v_1, \ldots, v_n)$
- Sell to bidder $i$ with $\max_j [\phi_j(v_j), v_0]$
- Collect minimal bid price $v'_i$ such that $\phi_i(v'_i) = \max_j [\phi_j(v_j), v_0]$.

Special case of symmetric $F_1 = F_2 = \ldots = F_n$. Then run a Vickrey auction with bids and reserve price of $\phi^{-1}(v_0)$.

E.g., $v_i \sim U[0,1]$ then reserve price of 0.5. Remark: The optimal auction is not efficient!

Ironing

- What to do if the MHR condition fails?
- Myerson proposes to “iron” the virtual valuation functions $\phi$ to find the “closest” monotone approximation $\hat{\phi}$ and then define the optimal auction with respect to these ironed functions.

Procedure:
(a) normalize $\phi$ so that the input runs from 0 to 1 and corresponds to the percentile of distribution $F$;
(b) integrate the normalized $\phi$ with respect to $v_i$;
(c) fit the best convex approximation;
(d) differentiate this convex approximation to generate an ironed virtual valuation.

Other issues

- Proxy agents [eBay, Google]
- Closing rules [Amazon vs. eBay auctions]
- Collusion [second-price worse. Why?]
- Trust [what is assumed?]
- Common value [winner’s curse]
- Interdependent value [informational externalities]
eBay proxy agents

- Provide an “upper bid-limit” to the eBay agent, which competes in an English auction until price reached.
- Revelation principle!
  - English ⇒ Vickrey
- Note: issue of trust.

Closing Rules

[Roth & Ockenfels 01]

- eBay [hard closing rule]
  - industry in “sniping”, favors bidders with better technology
  - empirically, limits information revelation during the auction, many bidders do not use proxy agents [esp. experienced bidders]
- Amazon [soft closing rule]
  - removes this “arms race” for bidding technology
  - empirically, encourages bidding earlier in the auction

Little details matter!

Collusion

E.g. Bidder rings. Group of bidders get together beforehand, and decide that only one will participate in the auction. Share gains afterwards. [Robinson 85]

- problems in reaching an agreement, sharing rewards
- first-price [Dutch, FPSB]
  - collusion is not self-enforcing because the selected bidder must submit a very small bid
- second-price [Vickrey, English]
  - collusion is self-enforcing, because deviators are punished.

Trust

- Vickrey auction.
  - bidders must trust the auctioneer not to submit a false bid. [without risk]
  - computational remedies? [bid verif. mechanism, trusted 3rd party]
- English auction.
  - more transparent, although the auctioneer can still use a “shill” to increase the bid price [some risk]
  - how does a shill compare to setting a reservation price in an auction?
Common Value Settings

- $8 pennies in a jar; collect sealed bids
  - average bid $5.13, winning bid $10.01
  - winner’s curse, all get an unbiased estimate, $f(·)
  - bids increase in $f(·)$ in equil.
  - winner is one with most optimistic estimate, “adverse selection bias”
- Simple model; signal $s_i \sim U(V - \epsilon, V + \epsilon)$
  - should bid $b_i \approx s_i - \epsilon$

[Wilson 77; Kagel & Levin 86; Bazerman & Samuelson 83]

Interdependent Values

- Bidder $i$’s value depends on signals $\omega \in \Omega^n$ of all bidders
- Theorem: efficiency with interdependent value auctions requires the single-crossing condition, at any $\omega$ s.t. $v_i(\omega) = v_j(\omega)$ and agent $i$’s value is the maximum then
  \[
  \frac{\partial v_i}{\partial \omega_i}(\omega) \geq \frac{\partial v_j}{\partial \omega_i}(\omega)
  \]
- Given this property, then can run a “generalized” second-price auction: (a) collect signals, (b) sell to agent with highest value at the value $v_i(\omega_i', \omega_{-i})$ at which agent $i$’s signal $\omega_i$ would allow it to just win the auction.

Remark: SCC not required if a two-step mechanism can be used (Mezzetti’04).

Next Class

- Dynamic auctions
- Sponsored search and the Generalized Second Price Auction


We will have our first student presentations of papers on Thursday 10/21.