Dynamics of Bid Optimization in Online Advertisement Auctions

C. Borgs, J. Chayes, O. Etessami, N. Immorlica, K. Jain, M. Mahdian

By Luděk Cigler & Thomas Léauté

October 21, 2008
Outline

1. Problem description
2. ROI heuristic
3. Dynamics of the system
4. Discussion
Ad-auctions, multiple bidders with limited budgets

Goal for the advertisers: spread the bids on each word so that the profit is maximized and budget not exceeded

Goal for the mechanism: Maximize the revenue of the auction
Model of the auction

- Multiple keywords
- Single slot per keyword
- Different advertisers can have different click-through rates for the same slot
- 1-day budget, bids updated once per day (day = unit of time, not necessarily 24 hours)

Formal description of the problem:

- \( b_{ij} \): bid of advertiser \( i \) on keyword \( j \)
- \( U_j(b_{ij}) \): day-long net utility of bidding \( b_{ij} \) on keyword \( j \)
- \( P_j(b_{ij}) \): day-long charge for bidding \( b_{ij} \) on \( j \)
ROI heuristic I: Rationale

Maximize

\[ \sum_j U_j(b_{ij}) \]

subject to

\[ \sum_j P_j(b_{ij}) \leq B_i \]

Lagrangian relaxation:

\[ \forall j : \frac{\partial U_j}{\partial P_j} \bigg|_{b_{ij}} = b_{ij}^* = \lambda \]

→ marginal return-on-investment (ROI) should be constant for all words
ROI heuristic II: Approximation

- Marginal ROI difficult to estimate/undefined in most cases: non-continuous utilities & payments
- Approximated ROI for bid $b$ and keyword $j$:

$$\text{ROI}_j(b) = \frac{U_j(b)}{P_j(b)}$$
ROI heuristic III: Algorithm

- Bids of an advertiser $i$ are adjusted by parameter $R_i$
- Start with arbitrary $R_i \in (0, 1]$
- On the next day:
  $$R_i := \begin{cases} R_i e^{-\epsilon} & \text{If the advertiser ran out of budget day before} \\ \min(R_i e^{\epsilon}, 1) & \text{otherwise} \end{cases}$$
- Set bid for keyword $j$
  $$b_{ij} := R_i u_{ij}$$
Dynamics of the system: example

- Suppose two players bidding according to ROI heuristics
- Both with budgets $500, utility $1 for each impression, single keyword, 1000 impressions per day
- First price auction
- Initial bids: A $0.5e^\epsilon$, B $0.5$
- 1st day: Bidder A gets all clicks, runs out of budget, revenue $500$
- 2nd day: Bidders switch their bids, roles, revenue $500$

$\Rightarrow$ If all players play the ROI heuristic, in 1st price (and 2nd price) auctions this can lead to cycling
Fighting the cycles: Perturbations

- Problem in the example: One bidder grabs all impressions, runs out of budget
- What if they split the impressions?

Perturbate the bids:

$$b'_{ij} := b_{ij} \exp(-\eta)$$

where $\eta \sim U(0, \delta)$ is randomly generated for each bidder and query

$\Rightarrow$ In the previous example, introducing perturbations leads to a gradual increase of bids to almost $1$, revenue almost $1000$. 

C. Borgs, J. Chayes, O. Etessami, N. Immorlica, K. Jain, M. Mahdian – By Luděk Cigler & Thomas Léauté

Dynamics of Bid Optimization in Online Advertisement Auctions
Theoretical results for the first price auction, if all advertisers use the ROI heuristic:

"Given a precision bound $\gamma$, there exists a fixed number of iterations of the perturbed heuristics after which the agents:"

- run out of budget late in the day
- spend most of their budget, or bid most of their total utility."

"Price vector converges to market equilibrium for the first price auction."
Convergence results: Experiments 1

Figure: Change in bids, Example 1
Convergence results: Experiments II

Figure: Change in revenue, Example 1
Convergence results: Experiments III

Figure: Change in efficiency, Random instance
Convergence results: Experiments IV

Figure: Change in revenue, Random instance
Discussion

- How reasonable is it to assume that all bidders play the same strategy?
- > 100 *days* to converge seems unrealistic (agents change, budgets change,...)
- Cycle prevention: Perturbations vs. random selection of bidders to play best response
- Trust issues involved with perturbations ("*what I bid is not what I wanted to bid*")
- How to explain the perturbed mech. to users