An Expressive Auction Design for Online Display Advertising

AUTHORS: Sébastien Lahaie, David C. Parkes, David M. Pennock

Li PU & Tong ZHANG
Motivation

• Online advertisement – allow advertisers to specify what kinds of users they are targeting: geographical, user profiles, behavioral

• Targeting slots → targeting users
Motivation

- **Bidding language:**
  - “Give me a number”
  - “Please tell me your values of all outcomes”

- Expressive about the preferences of advertisers, and succinct as well
Motivation

• Allocation and Pricing algorithms:
  • Computing efficient allocation in combinatorial auctions can be NP-hard
  • Good design of bidding language leads to tractable allocation and pricing algorithms

• Incentive-compatibility:
  • Minimize the incentives of bidders to manipulate
Goals

• Efficient and scalable allocation algorithm
• Unique market-clearing prices that minimize the bidders’ incentives to game
• Efficient, combinatorial algorithm for computing market-clearing prices

• Market-clearing: be cleared of all surplus and shortage (supply = demand)
Overview

- Advertisers are charged by impressions
- At the beginning of a period, all advertisers report their bids for different types of impressions
Definition of Impression

- **attributes** \( A = \{A_1, \ldots, A_s\} \)
  \[ A_1 = \{CA, MA, NY, \ldots, undefined\} \]
  \[ A_s = \{0:00 \sim 1:00, \ldots, undefined\} \]

- **values**

- **impressions** \( \langle a_1, \ldots, a_s \rangle \in M \)
  \( \langle CA, \ldots, 7:00 \sim 8:00 \rangle \)

- **number of all impressions**

\[
m = |M| = \prod_{t=1}^{s} |A_t| \]
Impressions and Prices

- **bundle of impressions**
  \[ x_i = [x_i(1) \ x_i(2) \ ... \ x_i(m)] \in \mathbb{Z}_+^M \]
  \[ x_i(j) \in \{0, 1, 2, \ldots\} \] number of times an impression was displayed

- **price for each impression**
  \[ p = [p(1) \ p(2) \ ... \ p(m)] \]

- **total price of bidder** \( i \) :
  \[ p \cdot x_i = \sum_{j \in M} p(j)x_i(j) \]

- **utility of bidder** \( i \) :
  \[ u_i(x_i) = v_i(x_i) - p \cdot x_i \]
Allocation of Impressions

- supply of impressions $z = [z(1) \ z(2) \ ... \ z(m)]$

- feasible: allocated $\leq$ supply
  \[
  \sum_{i=1}^{n} x_i(j) \leq z(j), \forall j \in M
  \]

- feasible allocation $x = [x_1 \ x_2 \ ... \ x_n] \in \Gamma$

- efficient: $x \in \arg \max_{y \in \Gamma} \sum_{i=1}^{n} v_i(y_i)$

- why efficient? long term goals
Market-clearing Prices

- market-clearing prices are also called competitive equilibrium (CE) prices

- demand set: \( D_i(p) = \arg \max_{x_i} v_i(x_i) - p \cdot x_i \)

\[
CE = \langle x, p \rangle \quad a) \quad x_i \in D_i(p) \quad \forall i \in N
\]

\[
b) \quad p(j) = 0 \text{ if } \sum_{i \in N} x_i(j) < z(j) \quad \forall j \in M
\]

- if \( p \) are CE prices

\[x \text{ is efficient } \iff \langle x, p \rangle \text{ is a CE}\]
Bid Tree

value = $.0 + $.0 + $.5 + $.1 = $.6

each node of the tree is a subset $T_k \subseteq M$, $\mathcal{T}_i = \{T_k\}$
Properties of Bid Tree

- Each node of the tree is a subset $T_k \subseteq M$, $\mathcal{T}_i = \{T_k\}$
- $\mathcal{T}_i$ is **laminar**: $\forall T, T' \in \mathcal{T}_i$, either $T \subseteq T'$, $T' \subseteq T$, or $T \cap T' = \emptyset$

$$v_{iT}(r) = \begin{cases} b_{iT}r, & \text{if } 0 \leq r \leq c_{iT} \\ -\infty, & \text{otherwise} \end{cases}$$

$$v_i(x_i) = \sum_{T \in \mathcal{T}_i} v_{iT} \left( \sum_{j \in T} x_i(j) \right)$$

$\langle CA, auto \rangle$
$\langle CA, sports, 7:00 \sim 8:00 \rangle$
$\langle CA, sports, blog, 7:00 \sim 8:00 \rangle$
Example of Bidder Value

Bidding tree $\mathcal{T}_i$ of bidder $i$

$\langle CA, auto, night \rangle : 500$
$500 \times \$.5 = 250$

$\langle CA, sports, news, night \rangle : 1000$
$1000 \times \$.5 + 1000 \times \$ - .1 = 400$

$\langle FL, auto, news \rangle : 40$
$40 \times \$.2 = 8$

Total value of bidder $i : v_i(x_i) = 250 + 400 + 8 = 658$
Queries: Value and Demand

• **Value**: input – allocation and bidding tree \( x_i, T_i \)
output – value of the bidder \( v_i(x_i, T_i) \)

• **Demand**: input – prices of each type of impression and bidding tree \( p, T_i \)
output – allocation to maximize utility \( x_i \)

**Demand Query Algorithm:**

1. Compute marginal surplus \( \pi_i(j) = \sum_{T \in T_i : j \in T} b_{iT} - p(j) \)
2. Pick out the impressions that have positive surplus (profit) and sort them in descending order
3. Assign a number as large as possible to the current most profitable impression, update the bidding tree and repeat
Example of Demand Query

\[
\pi_i(j) = \sum_{T \in \tau_i | j \in T} b_{iT} - p(j)
\]

\[
\pi_i = \begin{bmatrix}
$.1 + $.4 - $.1 \\
$.1 - $.05 \\
$.1 - $.05 \\
$.2 - $.1 \\
$.2 - $.05 \\
\end{bmatrix} = \begin{bmatrix}
$.4 \\
$.05 \\
$.05 \\
$.1 \\
$.15 \\
\end{bmatrix}
\rightarrow \text{(sort)} \rightarrow M = \begin{bmatrix}
\langle CA, sports \rangle \\
\langle CA, fashion \rangle \\
\langle CA, auto \rangle \\
\langle FL, sports \rangle \\
\langle FL, fashion \rangle \\
\end{bmatrix}
\]

\[
M = \begin{bmatrix}
\langle CA, sports \rangle \\
\langle CA, fashion \rangle \\
\langle CA, auto \rangle \\
\langle FL, sports \rangle \\
\langle FL, fashion \rangle \\
\end{bmatrix}
\]

\[
p = \begin{bmatrix}
$.1 \\
$.05 \\
$.05 \\
$.1 \\
$.05 \\
\end{bmatrix}
\]
Example of Demand Query

$$\begin{bmatrix}
\langle CA, sports \rangle \\
\langle FL, fashion \rangle \\
\langle CA, fashion \rangle \\
\langle CA, auto \rangle 
\end{bmatrix}$$

$$x_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix}
\langle CA, sports \rangle \\
\langle FL, fashion \rangle \\
\langle CA, fashion \rangle \\
\langle CA, auto \rangle 
\end{bmatrix}$$

$$x_i = \begin{bmatrix} 100 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
Example of Demand Query

\[
M = \begin{bmatrix}
\langle CA, sports \rangle \\
\langle FL, fashion \rangle \\
\langle CA, fashion \rangle \\
\langle CA, auto \rangle 
\end{bmatrix}
\]

\[
x_i = \begin{bmatrix}
100 \\
0 \\
0 \\
0 
\end{bmatrix}
\]
Example of Demand Query

\[
M = \begin{bmatrix}
\langle CA, sports \rangle \\
\langle FL, fashion \rangle \\
\langle FL, sports \rangle \\
\langle CA, fashion \rangle \\
\langle CA, auto \rangle
\end{bmatrix}
\]

\[
x_i = \begin{bmatrix}
100 \\
50 \\
0 \\
0 \\
0
\end{bmatrix}
\]
Example of Demand Query

\[ M = \begin{bmatrix} \langle CA, sports \rangle \\ \langle FL, fashion \rangle \\ \langle FL, sports \rangle \\ \langle CA, fashion \rangle \\ \langle CA, auto \rangle \end{bmatrix}, \quad x_i = \begin{bmatrix} 100 \\ 50 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]
Take into account forecasted supply $z(j)$. The allocation problem is a linear program:

$$\max_x \sum_{i \in N} \sum_{T \in T_i} \sum_{j \in T} b_{iT} x_i(j)$$

subject to:

$$\sum_{j \in T} x_i(j) \leq c_{iT} \quad (T \in T_i, i \in N)$$

$$\sum_{i \in N} x_i(j) \leq z(j) \quad (j \in M)$$

$$x_i(j) \geq 0 \quad (i \in N, j \in M)$$

**Proposition 1**: When agents submit their valuations as bid trees, the corresponding allocation LP has an integer optimal solution.
Allocation

- **Dual of the allocation LP:**

\[
\begin{align*}
\min_{\pi, p} & \quad \sum_{i \in N} \sum_{T \in \mathcal{T}_i} \pi_{iT} c_{iT} + \sum_{j \in M} p(j) z(j) \\
\text{s.t.} & \quad \sum_{\{T \in \mathcal{T}_i | j \in T\}} \pi_{iT} \geq \sum_{\{T \in \mathcal{T}_i | j \in T\}} b_{iT} - p(j) \quad (i \in N, j \in M) \\
& \quad \pi_{iT} \geq 0 \quad (i \in N, T \in \mathcal{T}_i) \\
& \quad p(j) \geq 0 \quad (j \in M)
\end{align*}
\]

- **Use subgradient method** on the dual of the allocation LP:

\[
p^{k+1} = p^k + \beta_k g^k
\]

\[
g^k = y^k - \sum_i x_i^k , \text{ where } y^k \text{ is a revenue-maximizing allocation at price } p^k.
\]
Pricing and Incentives

- **Proposition 2**: If $x$ is an optimal primal solution to the allocation LP and $p$ is an optimal dual solution, then $\langle x, p \rangle$ is a competitive equilibrium.

- **Proposition 3**: The set of competitive equilibrium prices is a lattice when valuations are described by bid trees.
  - unique minimal element $p \rightarrow$ most possible surplus
  - unique maximal element $\bar{p} \rightarrow$ most possible revenue

- VCG payment is not feasible because of linear pricing

- Choose $p$ to minimize the incentives of misreporting
  (minimize the improvement of payoff when misreporting)
Proposition 1 & 2

LP solution \( <x,p> \) is a CE

A fractional CE exists

M-concave valuations: impressions are “substitutes”

A fractional CE \( \rightarrow \) an integer CE

Proposition 1

Write down the dual LP

By complementary slackness conditions, \( p(j) > 0 \) implies all supply are allocated \( \rightarrow \) revenue maximized

\( x_i(j) > 0 \) and \( \pi_iT(j) > 0 \) implies \( x \) maximize bidder i’s utility at price \( p \)

Proposition 2
Bidder Feedback

- If advertiser $i$ wants to get at least $d_{iT}$ impressions from sites in $T \subseteq M$, he has to ensure that $c_{iT} \geq d_{iT}$ and increase the bid $b_{iT}$ to get the desired volume.

- Add a new constraint: $\sum_{j \in T} x_i(j) \geq d_{iT}$

Let $\lambda$ be the optimal value of the dual variable corresponding to this constraint.

- Proposition 4: Suppose bidder $i$ increases $b_{iT}$ to $b_{iT} + \lambda$ in its bid tree, and leaves the bids in the other nodes unchanged. Then there exists an efficient allocation, with respect to the new profile of bid trees, in which $i$ receives at least $d_{iT}$ units of impressions from $T \subseteq M$. 

Conclusion

- A bidding language (bid tree) that allows advertisers to specify values for different types of impressions, and ignore the irrelevant attributes
- Scalable allocation and pricing algorithms
- Bidder feedback to help advertisers assess the value of some nodes to get desired number of impressions

- Since the number of impressions may be very large, what attributes the seller should choose?
- Since the mechanism is not incentive-compatible, what is the dynamics of a series of bidding?