Economics of Network Growth

Radcliffe Exploratory Seminar on Dynamic Networks: Behavior, Optimization and Design

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University of Minnesota & Imperial College
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S-CURVES

Proportion of Maximum Extent

Year

0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1800 1820 1840 1860 1880 1900 1920 1940 1960 1980 2000

Canals

Rail

Telegraph

Oil Pipelines

Gas Pipelines

Surfaced Roads
LEGEND

- Interstate
- Undivided Highway
- Divided Highway
- No Construction Log
1980

I-94
I-35E
I-35
I-494
I-35W
U.S. Hwy. 52
St. Hwy. 5
St. Hwy. 13
St. Hwy. 95
U.S. Hwy. 169
I-694
St. Hwy. 12
St. Hwy. 62
St. Hwy. 55

DAKOTA
HENNEPIN
SCOTT
ANOKA
CARVER
WASHINGTON
RAMSEY

U.S. Hwy. 12
St. Hwy. 5
U.S. Hwy. 212
St. Hwy. 5
U.S. Hwy. 63
St. Hwy. 282
St. Hwy. 13
U.S. Hwy. 65
St. Hwy. 5
St. Hwy. 55

CARVER
SCOTT

1980
# The Realm of Network Evolution

<table>
<thead>
<tr>
<th>Decision-making unit</th>
<th>Ownership</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Public</td>
</tr>
<tr>
<td>Centralized</td>
<td>Network-based</td>
</tr>
<tr>
<td>Coalition</td>
<td>Area-based</td>
</tr>
<tr>
<td>Autonomous</td>
<td>Link-based</td>
</tr>
</tbody>
</table>
A Tale of Two Networks

• It was the best of plans, it was the worst of plans ...
Washington, DC

- Network (like city) planned as a whole, by a single (collective) agent in 1950s and 1960s, opened 1976
- Plans largely unchanged over 50 years despite changing circumstances
- 171 km, 564 k pax/day (out of Baltimore Washington metro pop of 8.5 m) [note also small Baltimore Metro]

London, England

- Network “planned” by many independent agents (Metropolitan Line, District Line, Northern Line, etc.) over period from 1863 onwards
- Agents were consolidated over time, first by private sector (Charles Yerkes), then public sector (government)
- Private sector is being reintroduced using “PPP”, private companies now responsible for infrastructure
- 408 km, 2.67 m pax/day (out of 7.5 m pop)
Network Growth Occurs Over Space and Time

• Economics historically (since Classical period anyway) was inured to space and time, assumed perfect information, and dealt with population aggregates.

• Need to take different approaches: agent-based, taking place over space and time, with imperfect information, and heuristics.
- Hierarchy (relative importance)
- Topology (connectedness)
- Morphology (shape)
Iteration $i$

Degeneration criteria

Land use allocation / re-allocation

Shortest path finding

Trip generation

Traffic assignment

Cost model

Revenue model

Investment model

Access cost calculation / re-calculation

End dynamics?

Flowchart

Exogenous Input

Network Degeneration & land use reallocation

Travel Demand Model

Investment Models

Next year

End
Revenue And Cost Models

• Toll is the only source of revenue
• Annual revenue generated by a link is total toll paid by the travelers

\[
\tau_a = \rho_0 l_a^{\rho_1} v_a^{\rho_2}
\]

\[
R_a = \tau_a (365 \cdot f_a)
\]

• Initially assume only one type of cost, function of length, flow, link speed

\[
C_a = \mu \cdot l_a^{\alpha_1} \cdot f_a^{\alpha_2} \cdot v_a^{\alpha_3}
\]
Network Investment Model (1)

- A link based model
- Speed of a link improves if revenue is more than cost of maintenance, drops otherwise

\[ v_{a}^{t+1} = v_{a}^{t} \left( \frac{R_{a}}{C_{a}} \right)^{\beta} \]

Where:

- \( v_{a}^{t} \) is speed of link \( a \) at time step \( t \),
- \( \beta \) is speed reduction coefficient.
- No revenue sharing between links: Revenue from a link is used in its own investment
Initial Assumptions

- **Base case**
  - Network - speed $\sim U(1, 1)$
  - Land use $\sim U(10, 10)$
  - Friction factor $w=0.01$
  - Travel cost, Revenue $d_a \{\lambda = 1.0, \rho_o = 1.0, \rho_1 = 1.0, \rho_2 = 0.0\}$
  - Infrastructure Cost $\{\mu = 365, \alpha_1 = 1.0, \alpha_2 = 0.75, \alpha_3 = 0.75\}$
  - Investment model $\{\beta = 1.0\}$
  - Speeds on links running in opposite direction between same nodes are averaged
SONG: Simulator of Network Growth 1.0

0. Network Type: 20X20 Grid Network

1. Speed Distribution: Uniform
   Speed Multiplier: 100%

2. Land use Distribution: Uniform
   Land Use Multiplier: 100%

3. Travel Demand Model
   3.1 Value of time: 1.0
   3.2 Friction factor: 0.01

4. Revenue Model
   4.1 Toll rate: 1.0
   4.2 Coeff. of length: 1.0
   4.3 Coeff. of speed: 0.0

5. Cost Model
   5.1 Coeff. of length: 1.0
   5.2 Coeff. of flow: 0.75
   5.3 Coeff. of speed: 0.75

6. Investment Model
   6.1 Speed impr coeff.: 1.0

End of Network Dynamics (Equilibrium Not Reached).
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20X20 Grid Network

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</tr>
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<td>Uniform</td>
</tr>
<tr>
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</tr>
<tr>
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<td></td>
</tr>
<tr>
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<td>1.0</td>
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<td>0.01</td>
</tr>
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<td></td>
</tr>
<tr>
<td>Toll rate</td>
<td>1.0</td>
</tr>
<tr>
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</tr>
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<tr>
<td>Investment Model</td>
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   6.1 Speed impr coeff.: 1.0

End of Network Dynamics (Equilibrium Not Reached).

Restore Evolve Absolute Speed << < < > > > Year 18 >>
Hierarchy

- Succeeded in growing transportation networks from undifferentiated links to differentiated structure
- Sufficiency of simple link based revenue and investment rules in mimicking a hierarchical network structure
- Hierarchical structure of transportation networks is a property of boundaries & asymmetries not entirely a design
- Investment policy can drive shape of hierarchy
- Model scales to metropolitan area
• Existing connectivity measures
  - Basic topological measures: the number of edges (e), the number of vertices (v), the number of maximally connected components, number of circuits
  - Alpha, Beta, Gamma indexes
• Need measures to identify and quantify connection patterns of networks

**Connectivity**

• Measures of effectiveness (MOEs)
  - Cumulative change of social welfare
  - Network mobility measures (TTT, VKT)
  - Accessibility

![Ring](image)
![Web](image)
![Star](image)
![Hub-and-spoke](image)
Measures of connection patterns

• Branch link vs. Circuit link
• Branch, web, ring
• Circuitness ($\phi_{\text{circuit}}$) and treeness ($\phi_{\text{tree}}$)
• Beltness ($\phi_{\text{belt}}$)
Weakest Link Model

• Begin with ultra-connected network
• Each iteration, kill the weakest link
• Allow rest of links to grow or shrink as the investment rules dictate
• Repeat
• (stop when obtain minimally connected network, I.e. tree)
<table>
<thead>
<tr>
<th></th>
<th>Iteration #</th>
<th>50</th>
<th>150</th>
<th>200</th>
<th>250</th>
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</thead>
<tbody>
<tr>
<td><strong>The Whole Network</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H$ Speed entropy</td>
<td></td>
<td>3.041</td>
<td>2.801</td>
<td>2.927</td>
<td>3.263</td>
</tr>
<tr>
<td>$\gamma$ The gamma index</td>
<td></td>
<td>0.863</td>
<td>0.569</td>
<td>0.451</td>
<td>0.334</td>
</tr>
<tr>
<td>$D$ Density</td>
<td></td>
<td>3.677</td>
<td>2.269</td>
<td>1.518</td>
<td>1.003</td>
</tr>
<tr>
<td>$e$ Number of edges</td>
<td></td>
<td>80</td>
<td>80</td>
<td>200</td>
<td>240</td>
</tr>
<tr>
<td>$n$ Number of vertices</td>
<td></td>
<td>45</td>
<td>41</td>
<td>97</td>
<td>120</td>
</tr>
<tr>
<td>$u$ Number of circuits</td>
<td></td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$z$ Number of subgraphs</td>
<td></td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\phi_{prim}$ The mileage ratio of the primary subgraph</td>
<td></td>
<td>86.4%</td>
<td>91.7%</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>$N_{ring}$ Number of rings</td>
<td></td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$N_{web}$ Number of webs</td>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\phi_{tree}$ The mileage ratio of branching roads</td>
<td></td>
<td>100.0%</td>
<td>41.4%</td>
<td>55.4%</td>
<td>70.7%</td>
</tr>
<tr>
<td>$\phi_{ring}$ The mileage ratio of rings</td>
<td></td>
<td>0.0%</td>
<td>58.6%</td>
<td>0.0%</td>
<td>29.3%</td>
</tr>
<tr>
<td>$\phi_{web}$ The mileage ratio of webs</td>
<td></td>
<td>0.0%</td>
<td>0.0%</td>
<td>44.6%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

**Experiment C2**
Figure 4. The cumulative change of total welfare over time for the asymmetric case
Topology

• Topologies of road networks can be measured, compared, and traced over time.
• The measures of entropy and connection patterns complement existing topological measures of hierarchy and connectivity.
• Improved measurement of network topologies can disclose collective features of road network spatially and temporally and thus guide long-term transportation plans and designs.
EXOGENOUS VARIABLES
Land use, Demographics, Socio-Economic Changes

NETWORK MODEL

Behavior: Travel Demand Estimation
Zone-based, Gravity distribution, Single Mode, User equilibrium traffic assignment

Technology:
- Maintenance Cost
- Construction Cost

Policy:
- Pricing Policy
- Investment Policy
- Ownership

MEASURES OF EFFECTIVENESS
Mobility, Accessibility, Social Welfare, Financial Indicators

Testing
Pricing
And
Ownership
Centralized Ownership

Pricing Policy
- Fuel taxes, Registration fees, Distance-based tolls
  - Average cost pricing

Investment Strategy
- Construction budget = Revenue - Maintenance Cost
- Priority: Links with the highest benefit/cost ratios
  \[ \text{Solve } \max_{\Delta \text{Capacity}} BCRatio(\Delta \text{Capacity}) \]
  for each link
- At the end of each fiscal year: Budget = Expenditure
Decentralized Ownership: Pricing

A Dynamic Pricing Game among All Roads

- Uncertainty and incomplete information

Profit-Maximizing Pricing through Adaptive Learning

1. Estimate a demand curve based on (price, flow) data in previous years for each link;
2. Solve \( \text{Maximize Profit(price)} \);
3. Case A: \( P_{\text{Low}} < \text{Price}^* < P_{\text{High}} \)
   New Price = \( \text{Price}^* \)
   Case B: \( \text{Price}^* < P_{\text{Low}} \) or \( P_{\text{High}} > \text{Price}^* \)
   New Price = \( P_{\text{Low}} (1 - j) \)
   or = \( P_{\text{High}} (1 + j) \)
Decentralized Ownership: Investment

Rate of Return
- \((\text{Lifecycle Revenue} - \text{Lifecycle Cost})/\text{Capital Cost}\)
- Links can borrow or earn interest through a “Bank” agent
- If Rate of return > interest rate \(\Rightarrow\) Borrow & Build capacity
  Otherwise \(\Rightarrow\) Pay off loan or Save

Profit Estimation for Capacity Expansion
- Consider two sources of additional profit after expansion
  1. Ability to charge higher tolls
  2. Ability to attract more users
Fixed Demand: Equilibrium Capacity

Equilibrium Capacity on a 10x10 Grid Network

Capacity (veh/hr)
- 0 ~ 1000
- 1000 ~ 2000
- 2000 ~ 4000
- 4000 ~ 8000
- > 8000

Socially Optimal

Centralized Ownership

Decentralized Ownership
Fixed Demand: Equilibrium Tolls

Equilibrium Toll

Toll ($)

- Blue: 0 ~ 0.5
- Green: 0.5 ~ 1
- Yellow: 1 ~ 4
- Orange: 4 ~ 8
- Red: > 8

Socially Optimal

Centralized Ownership

Decentralized Ownership
Decentralized Ownership with Regulation

Determination of the Optimal Ceiling Price

Million $
**Evolution of Benefit**

Net Social Benefit (M$)

- **Socially Optimal**
- **Decentralized: Profit-Maximizing**
- **Centralized: Average Cost Pricing**
- **Decentralized: Price Ceiling**

Year

Net Social Benefit (M$)
E V O L U T I O N  O F  T O L L S

Average Toll ($)

- Socially Optimal
- Decentralized: Profit-Maximizing
- Centralized: Average Cost Pricing
- Decentralized: Price Ceiling

Year

Average Toll ($) vs. Year
Evolution of Capacity

Cumulative Number of Capacity Expansion Projects

- Socially Optimal
- Decentralized: Profit-Maximizing
- Centralized: Average Cost Pricing
- Decentralized: Price Ceiling

Year

0 10 20 30 40 50 60 70 80 90
Ownership

Nothing is perfect

<table>
<thead>
<tr>
<th></th>
<th>Pros</th>
<th>Cons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Centralized</strong></td>
<td>Status quo</td>
<td>Low tolls, Ineffective</td>
</tr>
<tr>
<td></td>
<td>Cost recovery</td>
<td>Sub-optimal capacity</td>
</tr>
<tr>
<td><strong>Decentralized</strong></td>
<td>Responsiveness</td>
<td>High tolls</td>
</tr>
<tr>
<td></td>
<td>Market-oriented</td>
<td>Risk of over-investment</td>
</tr>
</tbody>
</table>

When appropriate regulation is imposed (e.g. $P_{max}$) and/or travel demand is steadily increasing (e.g. 3%) Results are in favor of decentralized market-oriented approach
Conclusions

• This framework allows understanding of changing spatial structure of networks.

• Model of transportation network (with flows moving across space and time as function of costs) along with changing capacity and price of connections (nodes and links) is a model of the economy in miniature.

• Future research could extend this platform to more realistically understand the dynamic relationships between firms (links) and markets (nodes).


Zhang, Lei and David Levinson. (2004a) An Agent-Based Approach to Travel Demand Modeling: An Exploratory Analysis. Transportation Research Record: Journal of the Transportation Research Board #1898 pp. 28-38


¿QUESTIONS?

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