Representing Agents’ Beliefs, Preferences and Decision-Making Processes

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The Goal

• A knowledge representation language for representing the way agents think in games
  – beliefs about the game structure
  – different preferences
  – strategies followed
  – strategic reasoning about other agents
Classical Game Theory

• Assumes common knowledge of rationality
• Assumes common knowledge of game structure
• But real agents have
  – different opinions about game structure
  – different beliefs about preferences
  – follow heuristics or social conventions
Bayesian Games

• Each player has set of types
• Type associated with
  – utility function
  – probability distribution over other agents’ types
• Highly expressive
  – different preferences
  – different opinions about game structure
Bayesian Games are not a Natural Solution

- Bayesian games are deficient from the point of view of knowledge representation
  - The whole story of the game must be folded into the utility functions
  - May be exponentially larger than necessary
    - probability distribution over types
    - $n$ decisions with 2 preferences for each decision
  - Equilibrium requires best response
Networks of Influence Diagrams

• Network in which each node represents a mental model used to make decisions
• Each model is a multi-agent influence diagram
• Agents have beliefs about which models other agents use
• Models linked by beliefs: “In model U, agent X believes that Y uses model V”
• Agents may have probability distribution over which model used
Nodes are Mental Models

• Models may differ in
  – Conditional probabilities of variables
  – Influence diagram structure
  – Utility functions
  – Decision variables in one model may become chance variables in another model
    • corresponding to agents following prescribed patterns of play
  – Beliefs about which models other agents use
Network of Influence Diagrams
Example: Main Model

Seismic Structure

Test

Test Results

Oil

Drill

U[Test]

U[Drill]
Network of Influence Diagrams
Example: Model 1

Seismic Structure

Test

Test Results

Drill

Oil

U[Test]

U[Drill]

Different Conditional Probability Table
Network of Influence Diagrams
Example: Model 2

Seismic Structure

Test

Test Results

Oil

Drill

U[Test]

U[Drill]

Different Utility Function
Network of Influence Diagrams
Example: Model 3

Seismic Structure -> Test
Seismic Structure -> Test Results
Test Results -> Oil
Test Results -> Drill
Oil -> Test
Oil -> U[Test]
Drill -> Test
Drill -> U[Drill]

Decision is made automatically
Network of Influence Diagrams
Example: Model 4

Test

Seismic Structure

Test Results

Oil

Drill

U[Test]

U[Drill]

Mod[Drill]

Testers believe that drillers use Model 3 to make their decision.
Network of Influence Diagrams
Example: Main Model Revisited

Seismic Structure → Test
Test → Test Results
Test Results → Mod[Test]
Test Results → Mod[Drill]
Test Results → Oil
Oil → Drill
Drill → U[Drill]

U[Test] → Test
U[Test] → Seismic Structure
U[Drill] → Mod[Drill]
Network of Influence Diagrams
Example: NID Structure

```
  Test
 /   \
|     |
|     |
Test  Test  Drill
/     /     /
|     |     |
Model 1 Model 4 Model 2
      /
     Drill
     /
    Model 3
```
Equilibrium Conditions

- Two kinds of strategy in each mental model
  - Best response strategy
  - Actually played strategy

- These are not necessarily the same!

- Equilibrium conditions:
  - best response is best response with respect to agent’s beliefs about actually played strategies
  - actually played strategies are expectations over strategies played in each of the models agent can use
Comparison to Bayesian Games

- Every Bayesian game can be represented by a NID
- Not every NID can be represented by a Bayesian game
- NIDs can be exponentially smaller
Our Learning Approach

• Model the type of a player by a *social utility function*
  – incorporates different *social preferences* of a player

• Learn a mixture model over types
Social Utility Function

- Individual benefit (IB) \( PO_D - NN_D \)
- Aggregate utility (AU) \( PO_D + PO_A \)
- Advantage of outcome (AO) \( PO_D - PO_A \)
- Advantage of trade (AT) 
  \[
  (PO_D - NN_D) - (PO_A - NN_A)
  \]

Utility = \( w_1 IB + w_2 AU + w_3 AO + w_4 AT \)
From Utilities to Actions

• Normally, a utility function determines an action
• This results in a discontinuous estimate of the action as a function of social preferences – does not allow for noise in play
• To get probabilistic actions, we pass the utility function through a sigmoid

\[ P(\text{accept} \mid x, t) = \frac{1}{1 + e^{-u^t(x)}} \]
Implementing a Player

• Choose proposal that maximizes expected utility
• Sum over different possible types of deliberator
• Learned model provides probability of acceptance for each type

\[
EU(x) = \sum_t P(t)[P(\text{accept} \mid x,t)PO_A(x) + (1 - P(\text{accept} \mid x,t))NN_A] 
\]
Experimental Setup

• Data gathering phase
  – 192 games played by people

• Evaluation phase
  – 21 sets of games between human responder and each of four proposers
    • our model
    • Nash equilibrium
    • Nash bargaining solution
    • human proposer
## Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Score</th>
<th>Proposals accepted</th>
<th>Proposals declined</th>
<th>No offer made</th>
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<tbody>
<tr>
<td>Our model</td>
<td>2880</td>
<td>16</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Nash equilibrium</td>
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<td>Nash bargain</td>
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<td>2</td>
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<td>Human</td>
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<td>4</td>
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