

Efficient Elicitation w/ Costly Queries

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Outline

- **Efficient Elicitation** (joint work w/ Sebastien Lahaie)
 - Ask queries of agents to determine an efficient allocation with a “small” number of (natural) queries.
- **Elicitation w/ Costly Queries**
 - Ask queries of agents to determine a bounded-optimal allocation given a limited number of (costly) queries

Efficient Elicitation: Combinatorial Allocation Problem

- m goods, n agents.
- Quasi-linear utility functions:

$$u_i(S, p) = v_i(S) - p$$

on bundle $S \subseteq G$

- Complements and Substitutes
- Find an allocation (S_1, \dots, S_n) that solves:

$$\max_{(S_1, \dots, S_n)} \sum_{i=1}^n v_i(S_i)$$

Efficient Elicitation Problem

- Suppose some valuation class V .
- Suppose access to:
 - value queries
 - best-response queries
- Can the efficient allocation be determined by asking a number of queries that are “not much larger” than n , m , and $size(v_1, \dots, v_n)$?
- Can we stop elicitation early, before we know complete valuations?

Frameworks

Learning

- Function Class C
 - Monotone Boolean functions
- Representation Class C
 - Monotone DNF formulae
- Target function $f: X \rightarrow Y$
 - Boolean domain X
 - m -dimensional
 - Boolean or real-valued range Y

Elicitation

- Valuation Classes V_1, \dots, V_n
 - Free-disposal
- Bidding Languages V_1, \dots, V_n
 - XOR bids
- True valuations $v_i: X \rightarrow Y$
 - Domain of bundles X
 - m goods
 - Range Y of non-negative real values

Queries (1)

Learning

- Membership query
- Present an input x .
- Oracle returns the truth-value $f(x)$.

Elicitation

- Value query
- Present a bundle x .
- Agent returns the exact value $v_i(x)$.

Queries (2)

Learning

- Equivalence query
- Maintain manifest hypothesis \tilde{f}
- Present manifest hypothesis to the oracle
- Oracle replies 'Yes' if
$$\tilde{f}(x) = f(x), \quad \forall x \in X$$
- Else presents some input x' such that:
$$\tilde{f}(x') \neq f(x')$$

Elicitation

- Demand query
- Maintain manifest valuations $\tilde{v}_1, \dots, \tilde{v}_n$
- Present allocation (x_1, \dots, x_n) and Lindahl prices $p_i(x)$
- Agent i replies 'Yes' if
$$x_i \in \arg \max_{x \in X} v_i(x) - p_i(x)$$
- Else presents a bundle x'_i such that:
$$v_i(x'_i) - p_i(x'_i) > v_i(x_i) - p_i(x_i)$$

Objectives

Learning

- Determine target function *exactly*.
- Use only membership and equivalence queries.
- Run-time is polynomial in m and $size(f)$.

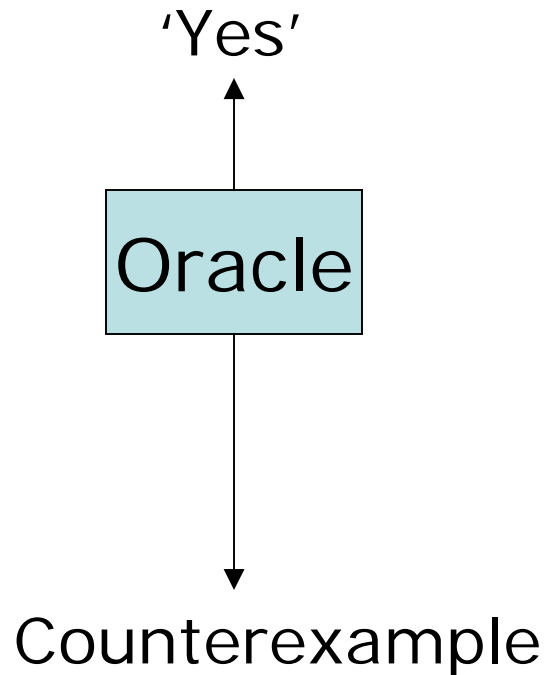
Elicitation

- Determine *optimal* allocation to the agents.
- Use only value and demand queries.
- Communication is polynomial in n , m , and $size(v_1, \dots, v_n)$.

Simulation

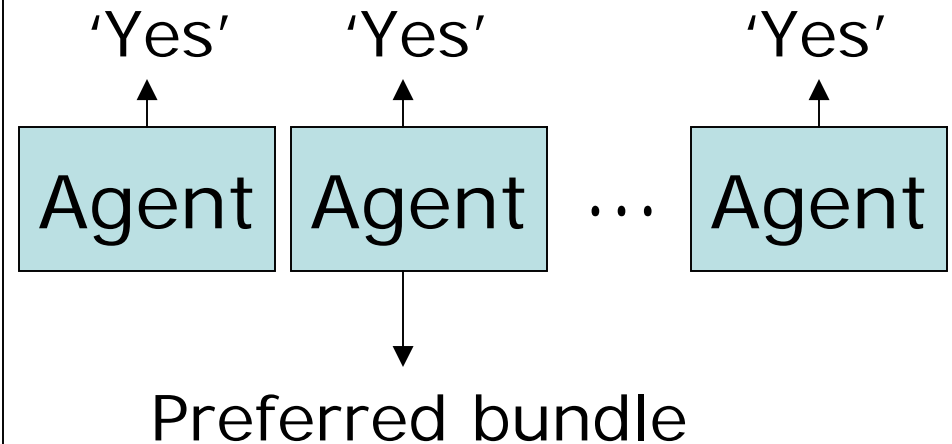
Equivalence

(Learning is solved)



Demand

(Elicitation is solved, by Duality)



Then

- Preferred bundle, or
 - Proposed bundle
- Is a counterexample.

Main Results

Theorem 1. Valuations can be *elicited* with value queries quadratic in $\text{size}(v)$ and demand queries linear in $\text{size}(v)$, if they can be *exactly learned* with membership and equivalence queries.

Theorem 2. Valuations can be *elicited* with *polynomial communication* with value and demand queries if they can ...

Example: XOR Bids

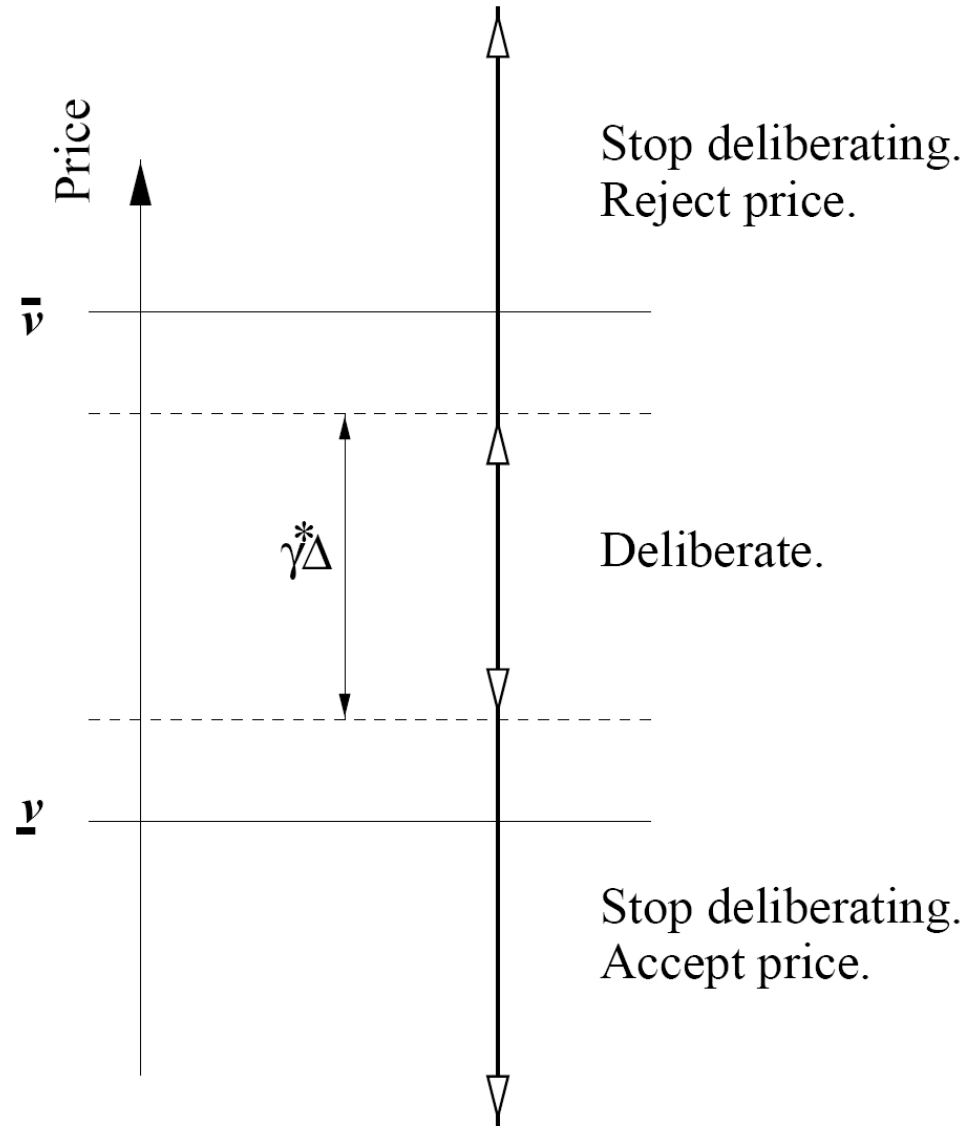
- Choose an “XOR valuation class” V , which allows enumeration of “interesting” bundles, e.g.
(ABC, \$100) xor (GHI, \$150) xor (BH, \$50)
- XOR bids can be **efficiently learned**, generalizing a learning algorithm for “monotone Disjunctive Normal Form” functions.
 - ⇒ valuations can be **efficiently elicited**, with no more queries than the size of the XOR representation

Elicitation w/ Costly Queries

- Talked so far about “efficient elicitation”
 - i.e. determining enough information to make optimal decision
- But, what if this is still too costly? What else can we do?
 - interesting to compare the performance of different market institutions in this setting.

Simple Model

- Valuations $[\underline{v}_i, \bar{v}_i]$
- Refine $\Delta_i \rightarrow \alpha \Delta_i$
- Cost C



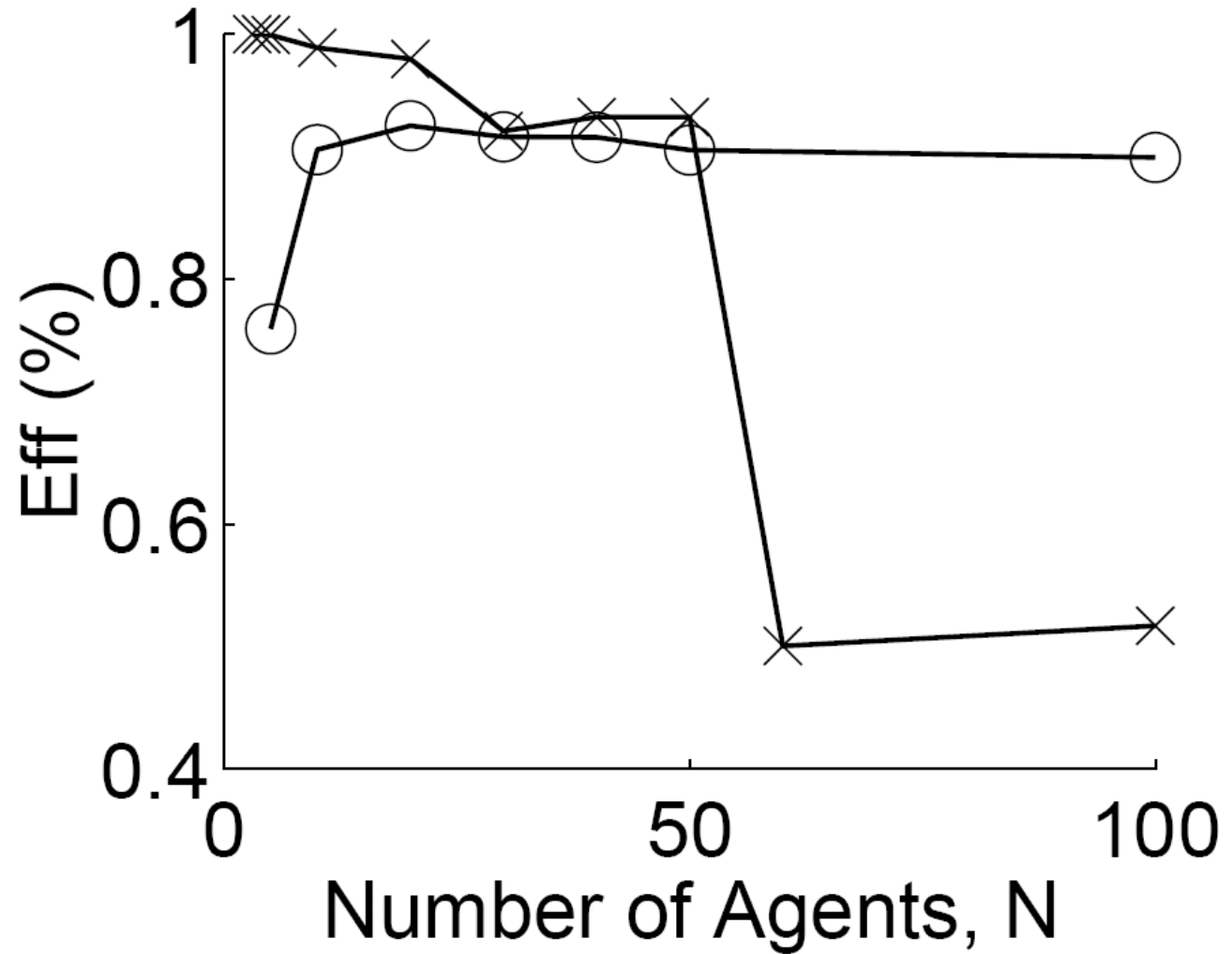
- Follow Russell & Wefald (1991) model of meta-deliberation
 - deliberate when “expected gain in utility” > “cost”
 - consider a *sequence* of refinements
 - myopic
- Compare allocation efficiency for different auction institutions

Sealed-Bid vs. Posted-Price

$\alpha=0.7$

$C=0.05$

value $U(0,10)$

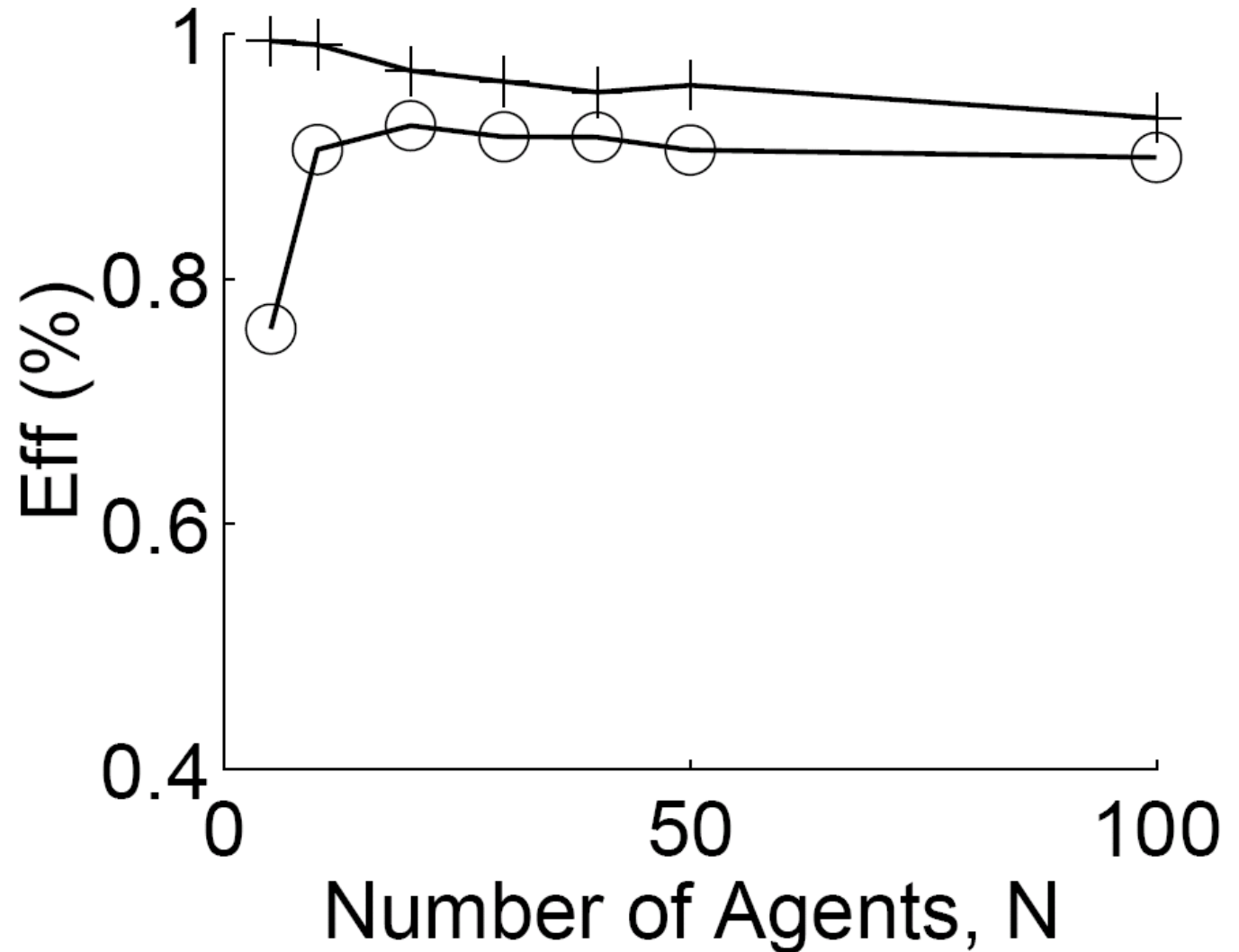


Posted-Price vs. Ascending-Price

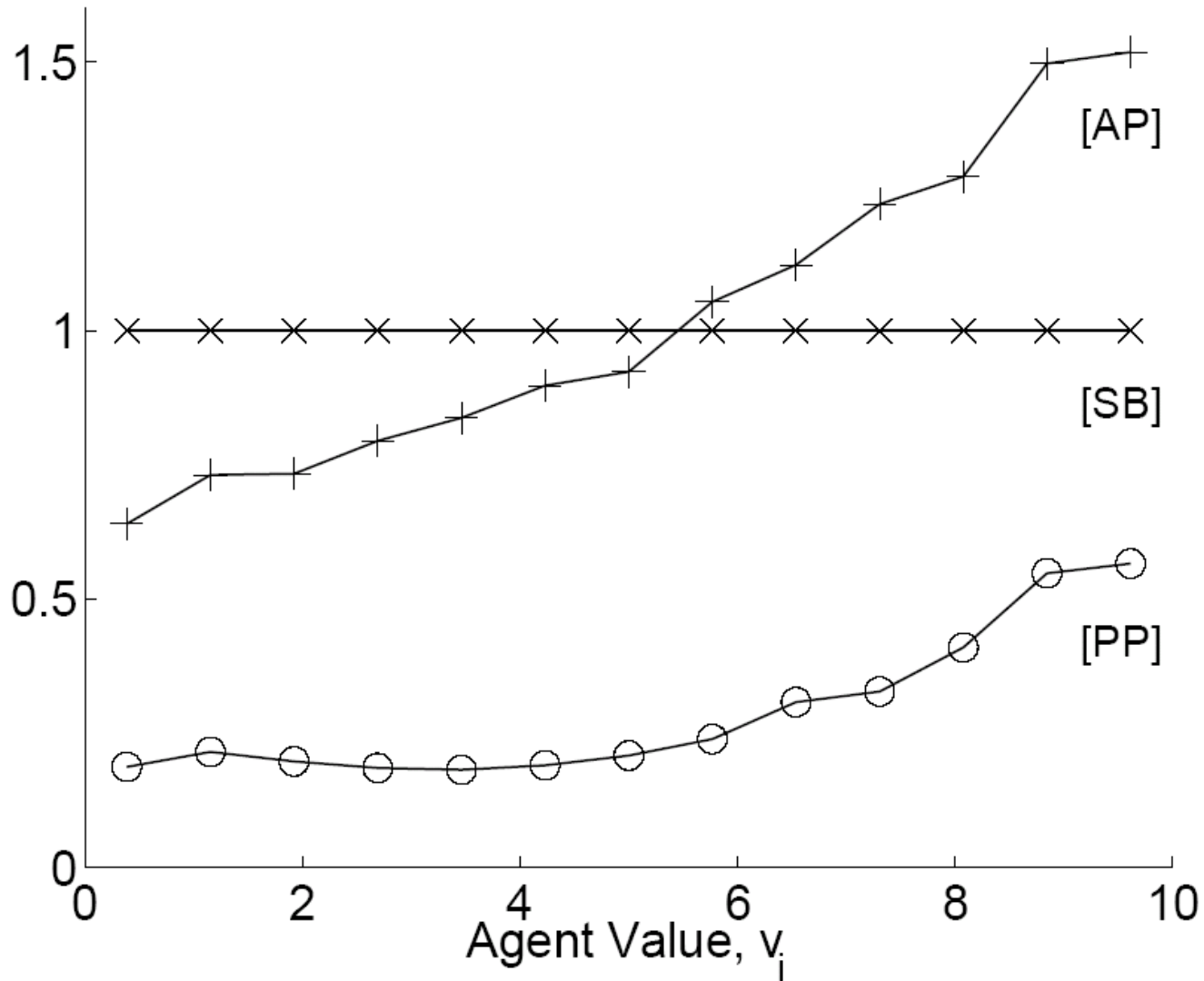
$\alpha=0.7$

$C=0.05$

value $U(0,10)$



Deliberation Steps vs. Value



Summary

- **Computational learning theory**
 - method to structure elicitation so that worst-case number of queries “not much more” than minimal representation in a valuation class
- **Models of Bounded-rationality**
 - method to think about costly elicitation