Models of Morphogenesis

Radhika Nagpal
Outline

• Cellular Automata
• Reaction-Diffusion Models
• DLAs
• L-Systems
Cellular Automata

- Stylized/Discrete Dynamical Systems
- Ulam and von Neumann, 1940s
  - Self-reproducing machines
Conway’s Game of Life
Models of Natural Systems

- Lattice Gas models
- Social Insect models
- Traffic Models, Epidemic Spread
- Evolutionary models
Chemical Patterns

Reaction Diffusion Models

[Turing, 1952]
\[
\frac{\partial a}{\partial t} = f(a, b) + D_a \left( \frac{\partial^2 a}{\partial x^2} + \frac{\partial^2 a}{\partial y^2} \right)
\]
\[
\frac{\partial b}{\partial t} = g(a, b) + D_b \left( \frac{\partial^2 b}{\partial x^2} + \frac{\partial^2 b}{\partial y^2} \right)
\]

Reaction–diffusion model (Turing)
Gray-Scott Model

Equations:

\[
\frac{\partial u}{\partial t} = r_u \nabla^2 u - uv^2 + f(1 - u)
\]

\[
\frac{\partial v}{\partial t} = r_v \nabla^2 v + uv^2 - (f + k)v
\]

Chemical Reaction:

\[
U + 2V \rightarrow 3V
\]

\[
V \rightarrow P
\]
Parameter Space
Activator-Inhibitor Model
(Grierer and Meinhardt)

Key: Local self-enhancement, Long range Inhibition
Activator–inhibitor patterns (Young’s model)
Examples

• Belousov-Zhabotinsky Reaction (BZR)
• AngleFish Stripe formation
• Symmetry-breaking (Budding Yeast)
Crystal Growth

Diffusion-limited Aggregation (DLA)

A DLA pattern (Witten and Sander)
Snow Crystal Growth and "The No-Two-Alike Conjecture"

- Nucleation around a dust particle
  - Grows to hexagonal prism, since smooth facets grow most slowly
  - Simple plate unstable as crystal grows larger ... corners sprout arms
  - Crystal moves to different temperature ... plates grow on arms
  - Crystal moves through many different temperatures ... each change causes new growth behavior on arms

Complex history → Complex crystal shape
Each arm experiences same history → Symmetry
No two paths similar → No two alike
Branching Patterns

$L$-systems and Grammar Rules
Types of Rules/Structures

• Lineage (Context free)
• Context sensitive (Information flow)
• Environmentally-sensitive
• Phyllotaxis

• Good match for describing the spatial-temporal patterns of plants
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Monopodial branching - raceme
Sources of Images and Movies:

- Reaction Diffusion Movie: Amorphous Computing Website
- Snowflake Images: Rasmussen & Libbrecht Collection (http://snowflake.com/)
- Game of Life: movie from Georgia Tech Altera Page (http://users.ece.gatech.edu/~hamblen/ALTERA/altera.htm)
Extra slides