Today's letter comes from T-Rex. "Oh no!" he writes, "I have to make a decision and am uncertain of the consequences!"

Since you need to predict the future, T-Rex, why not use a prediction market? They can predict the weather or who will be the next president!

That sounds good, but how does it work?

In a prediction market, a set of experts make a series of predictions over some set of outcomes, like whether it's sunny, cloudy or rainy on a particular day.

<table>
<thead>
<tr>
<th>tomorrow’s weather!</th>
<th>start</th>
<th>expert 1</th>
<th>expert 2</th>
<th>expert 3</th>
<th>…</th>
</tr>
</thead>
</table>
| sunny               |      | 33%      | 42%      | 65%      | 60% ...
| cloudy             |      | 33%      | 30%      | 20%      | 20% ...
| rainy              |      | 33%      | 28%      | 15%      | 20% ...

Experts attempt to correct past predictions, and are scored by how well they improve the accuracy of the market using strictly proper scoring rules, like the logarithmic scoring rule. These rules mean experts expect to maximize their score by predicting what they really believe! If we let outcomes be a set $O$ and an expert believes $q$, then for any other prediction $p$

$$
\sum_{o \in O} q_o \log(q_o) > \sum_{o \in O} q_o \log(p_o).
$$

And experts are paid for improving the market’s accuracy, so they get the difference of their and the last expert’s score. Expert 2, for example, expects to be paid

$$
\sum_{o \in O} q_o[\log(p_o^2) - \log(p_o^1)].
$$

This is great, I’ll just run a prediction market to decide what to do. Tell me, oh market, what is more awesome, ballooning or terror stomping?

But wait, T-Rex! That’s not how prediction markets work. A prediction market requires you actually observe what you ask people to predict, and you can’t go both ballooning and terror stomping, you can only pick one. Look at this example:

<table>
<thead>
<tr>
<th>terror stomping</th>
<th>current</th>
<th>beliefs</th>
<th>profitable lie</th>
</tr>
</thead>
<tbody>
<tr>
<td>awesome?</td>
<td>30%</td>
<td>60%</td>
<td>60%</td>
</tr>
</tbody>
</table>

Since you pick the more awesome action, only that market will be scored. When you pick ballooning an expert with the above beliefs will make no money if they’re honest, but if they convince you to pick terror stomping you’ll reward them!

But that’s the less awesome option!

Exactly, but you’ll never know that! Experts have an incentive to be accurate, not to help you get what you want.

Damn, experts!

Instead, you need to run a decision market. Running two prediction markets means you have to do both things, but a decision market knows you can only do one. It reviews the market and assigns a probability to each action, then uses a decision scoring rule to normalize experts’ scores.

**Decision Market**

<table>
<thead>
<tr>
<th>Probability Action Taken</th>
<th>Expected Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>terror stomping</td>
<td>$\frac{1}{d_s} \log(p_o) = \log(p_i)$</td>
</tr>
<tr>
<td>ballooning</td>
<td>$\frac{1}{d_b} \log(p_o) = \log(p_i)$</td>
</tr>
</tbody>
</table>

Great! So experts’ expected scores are the same as in a set of prediction markets. Finally a way to make decisions! But wait, this means I might have to take an un-awesome action because those probabilities must be positive! And according to this paper* every myopic incentive compatible decision market has this problem! Oh darn! Terror stomping again!


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