A Projective Framework for Radiometric Image Analysis

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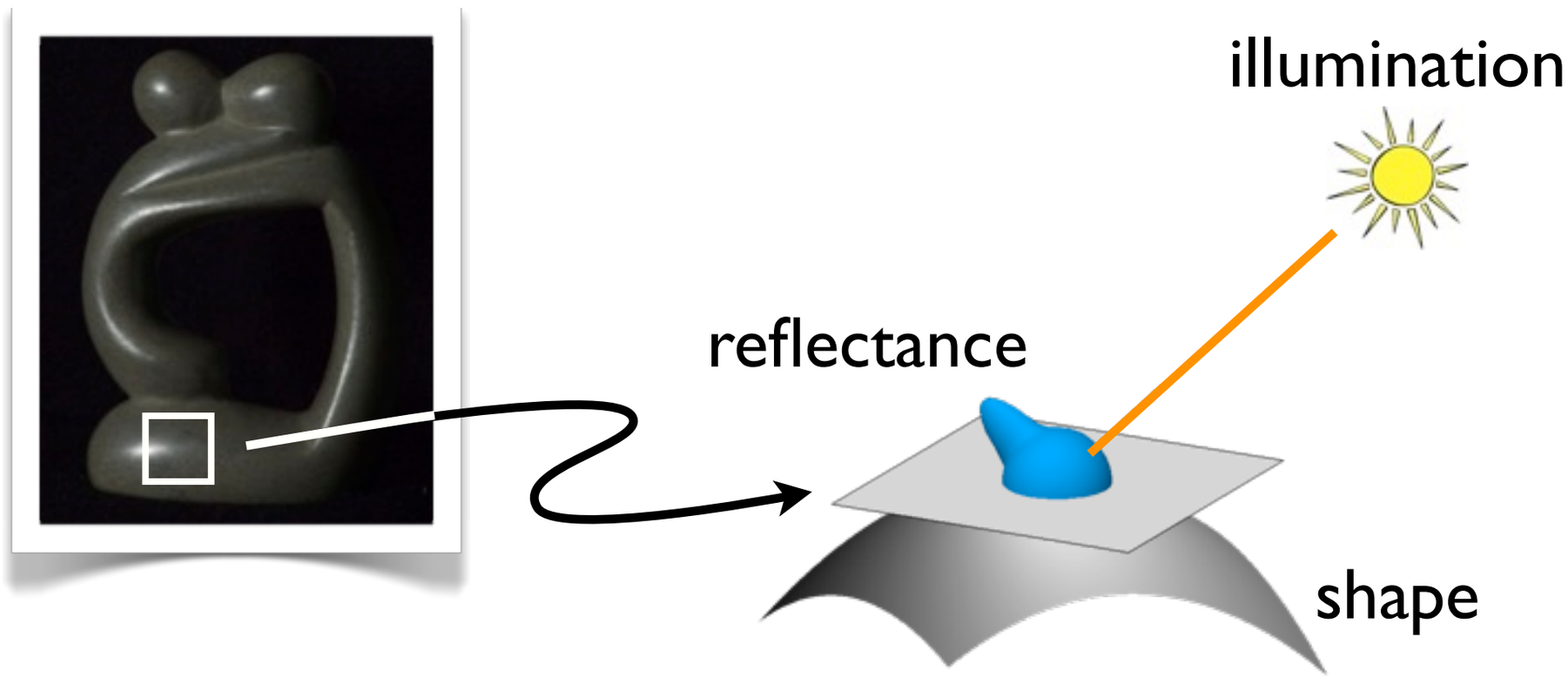
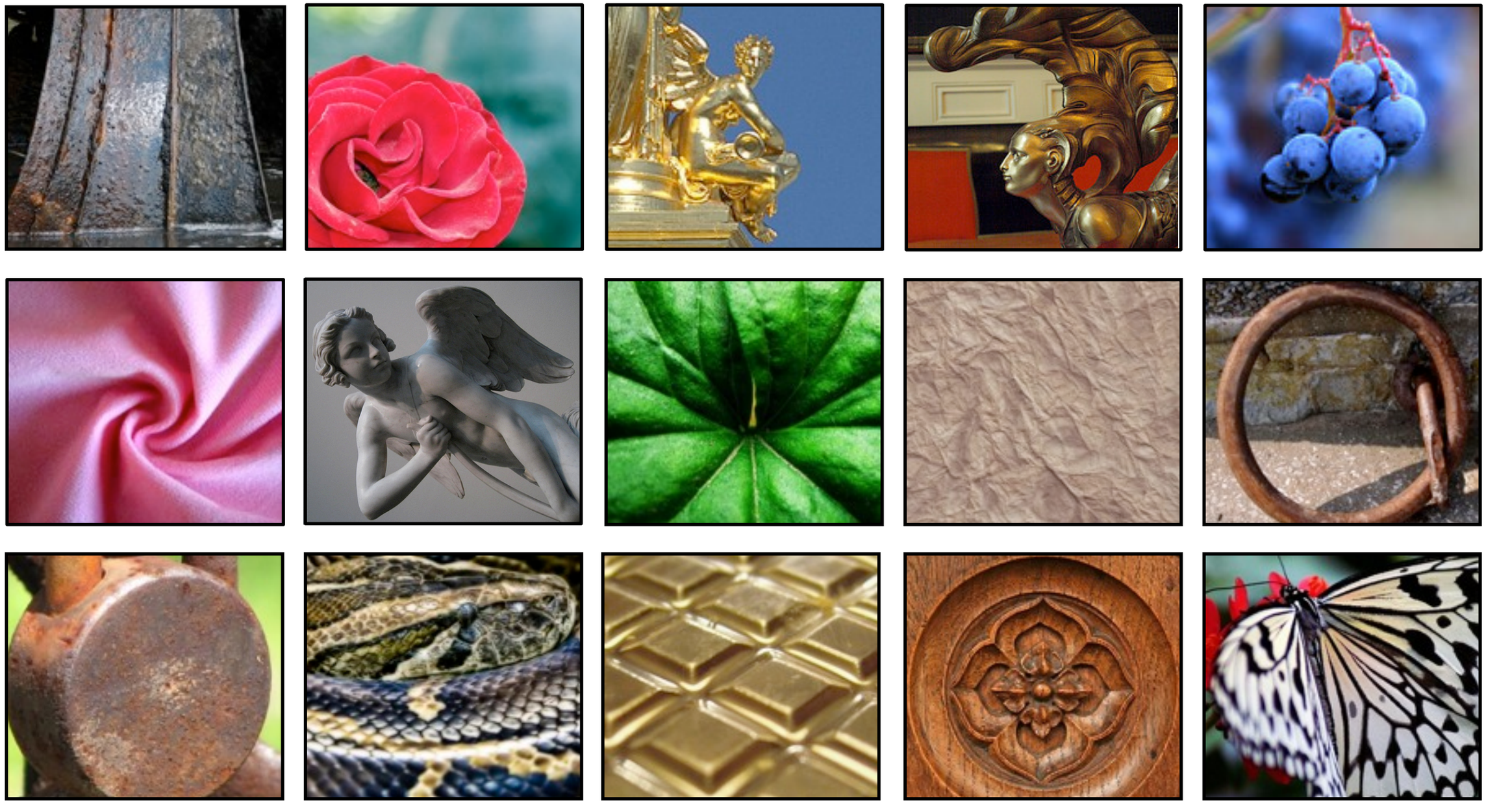
Radiometric Image Analysis

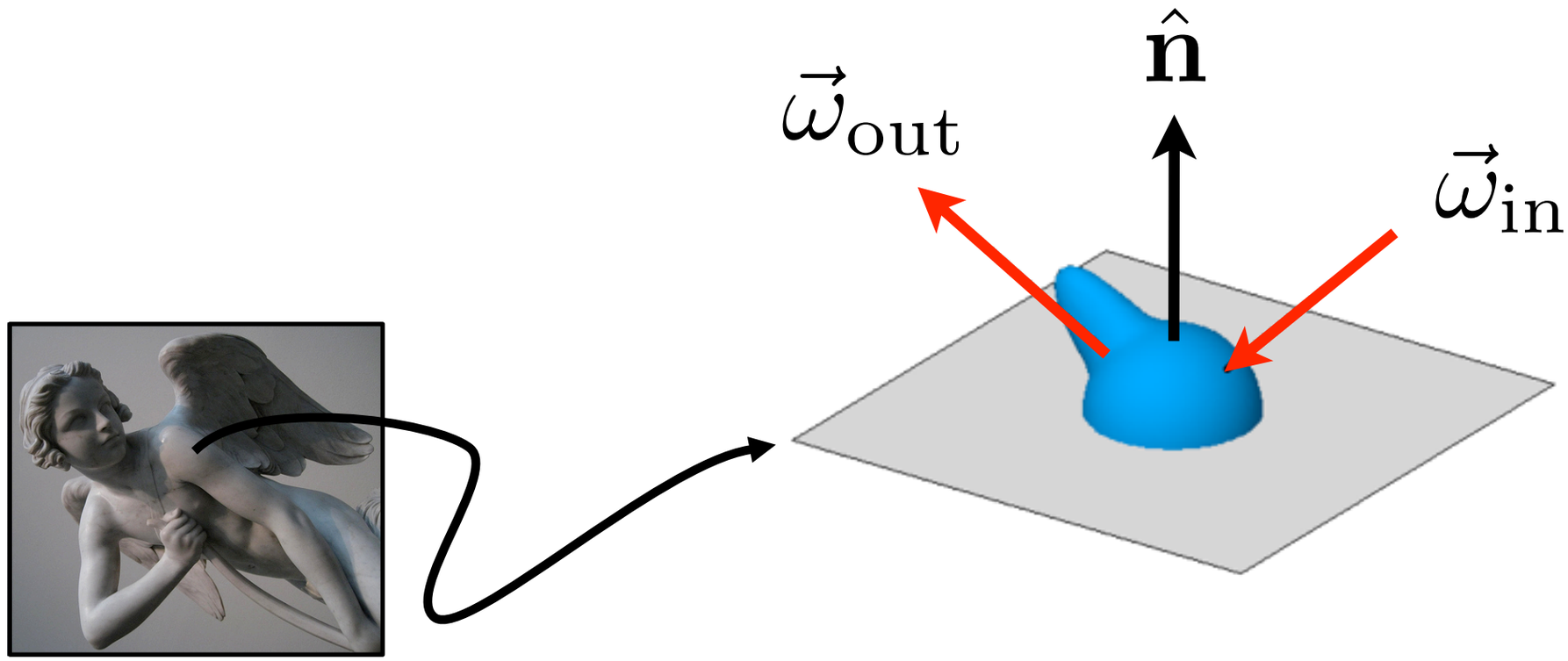


$$I \Longrightarrow \text{shape, illumination, reflectance}$$

Reflectance



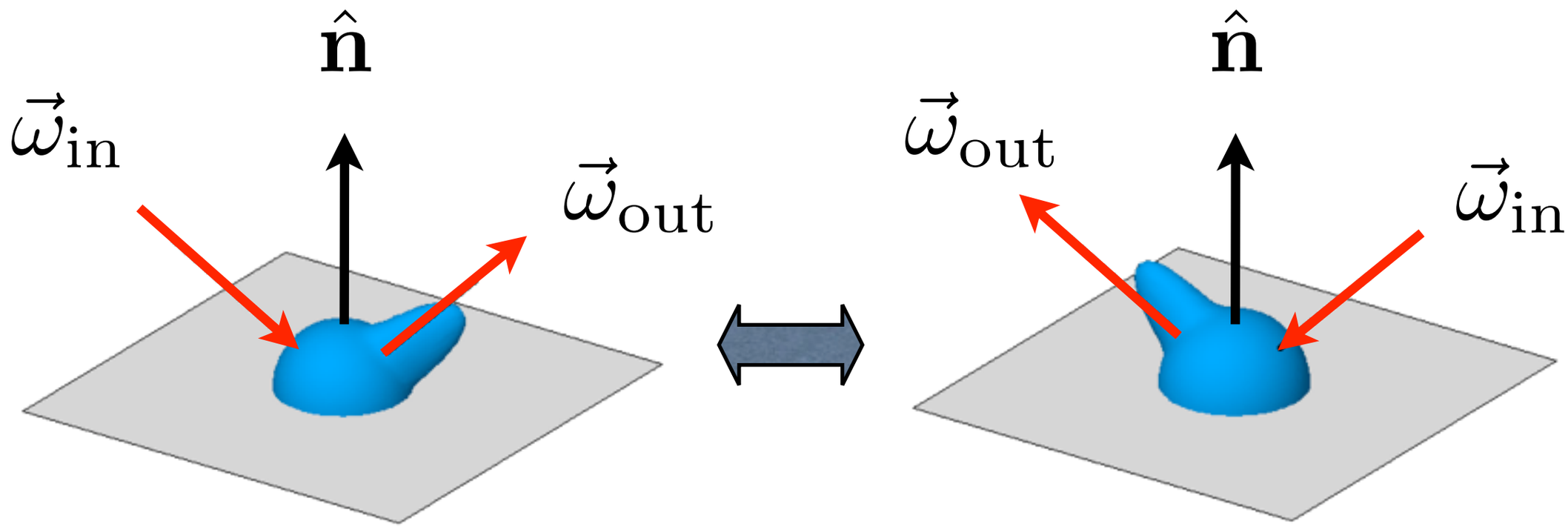
Reflectance



$$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})$$

Bi-directional Reflectance Distribution Function

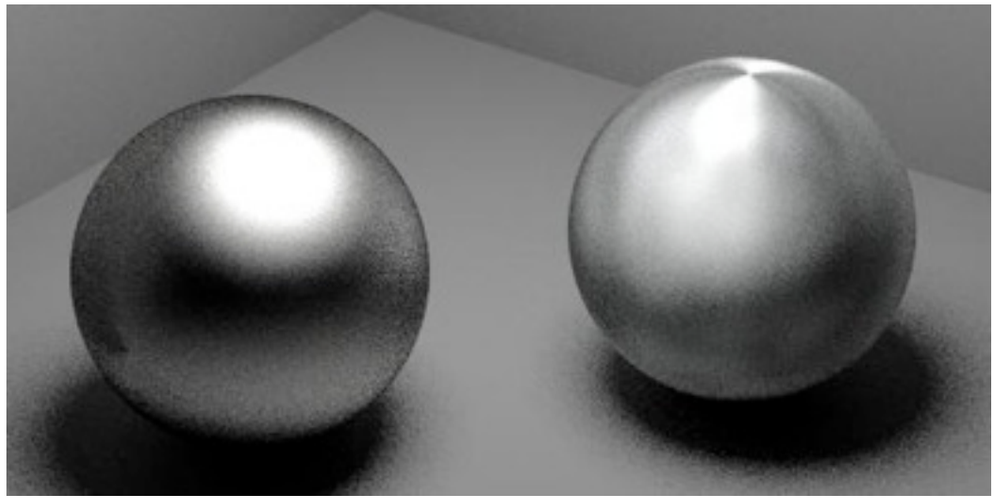
Symmetry I: Reciprocity



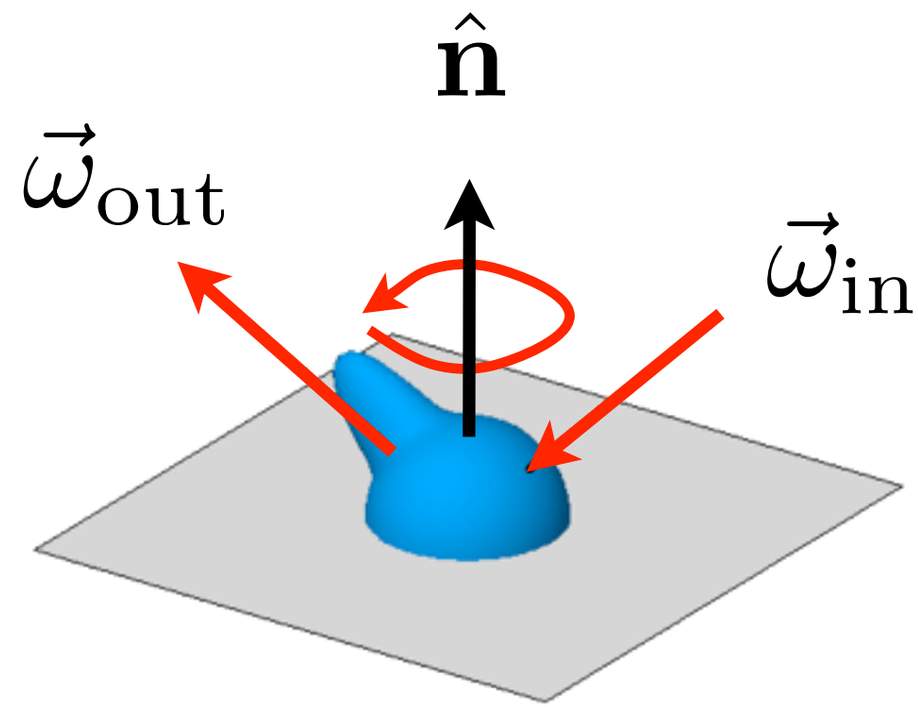
$$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}}) = f_r(\vec{\omega}_{\text{out}}, \vec{\omega}_{\text{in}})$$

Symmetry II: Isotropy

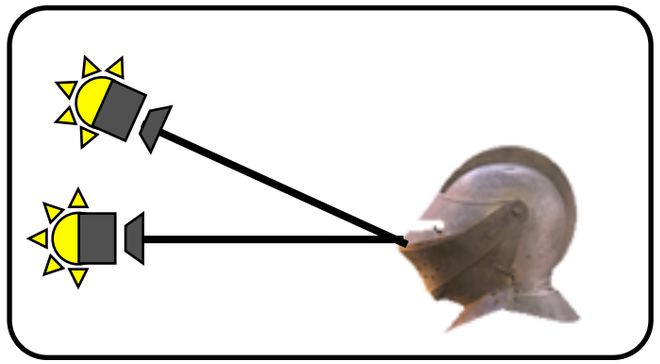
ISOTROPIC ANISOTROPIC



[Westin et al. 92]

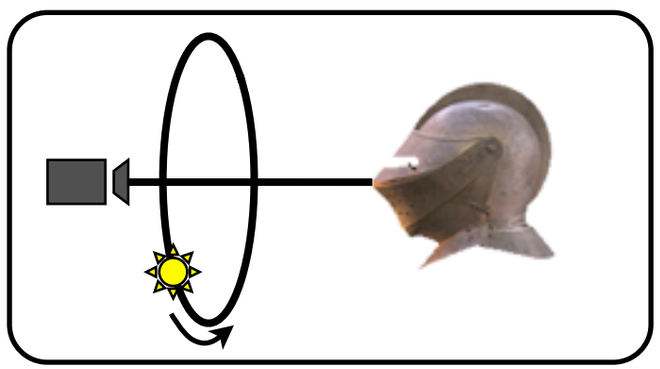


Shape from Symmetries: Prior Work



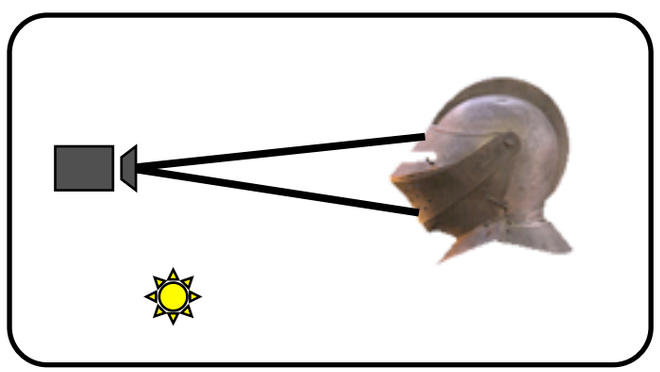
Co-located cameras and light sources

- Isotropy [Lu & Little 99]
- Reciprocity [Zickler et al. 02]



Ring of light sources

- Isotropy [Alldrin & Kriegman 07]



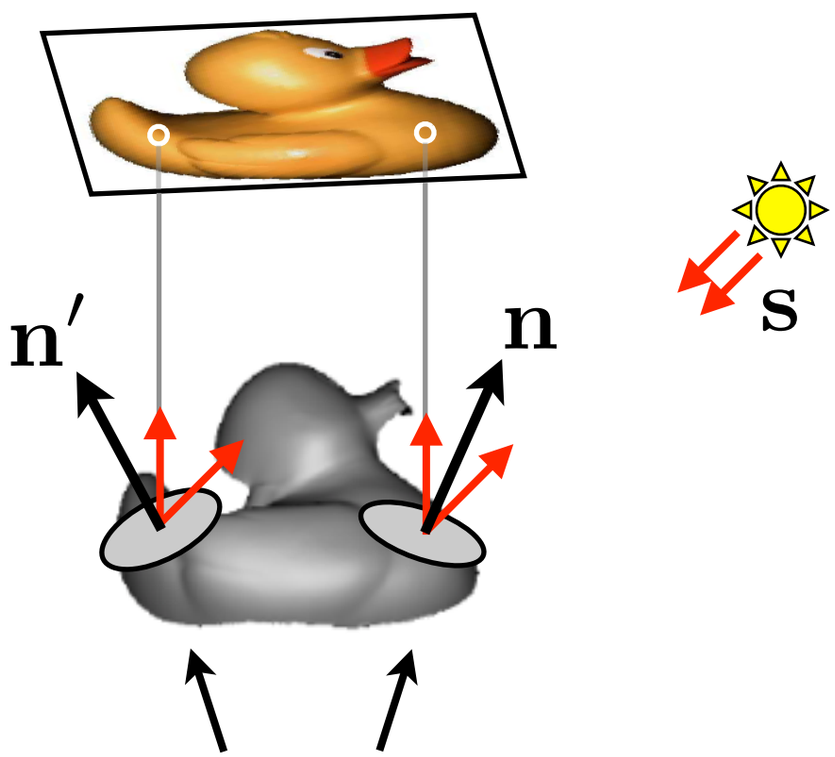
Arbitrary light sources

- Isotropy & reciprocity [Tan et al. 07]

Shape from Symmetries: This Work

constraints satisfied

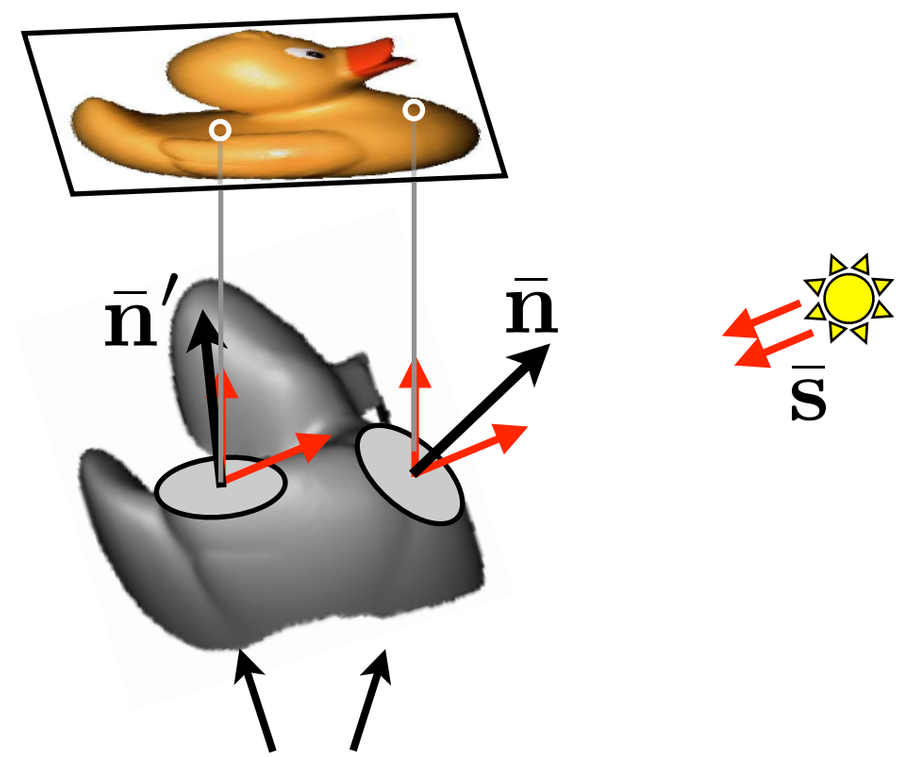
$$g(I_{\mathbf{n}}, I_{\mathbf{n}'}) = 0$$



"Equivalent" local light/view configurations

Incorrect shape & lighting: constraints violated

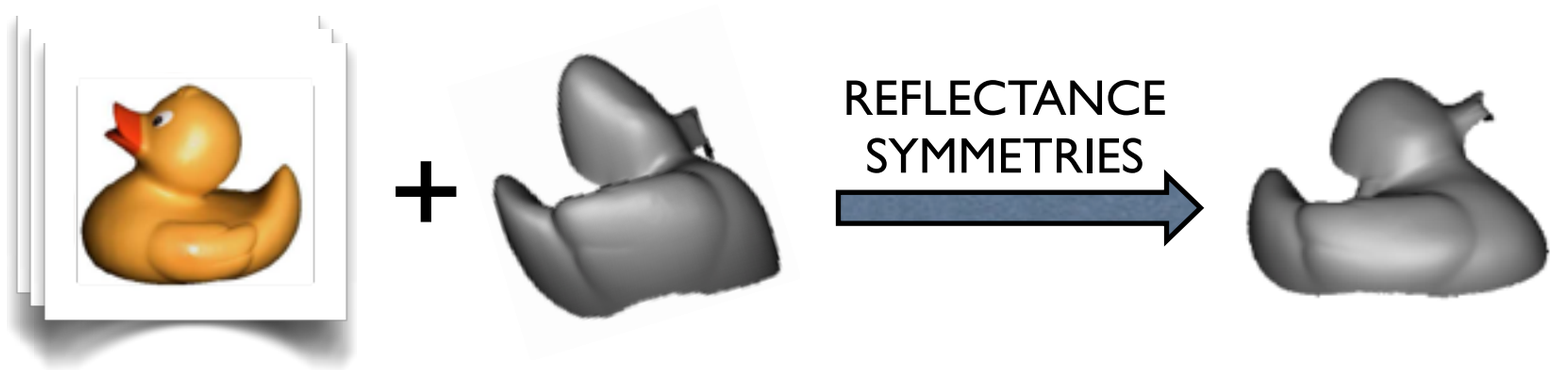
$$g(I_{\bar{\mathbf{n}}}, I_{\bar{\mathbf{n}}'}) \neq 0$$



"Equivalent" local light/view configurations

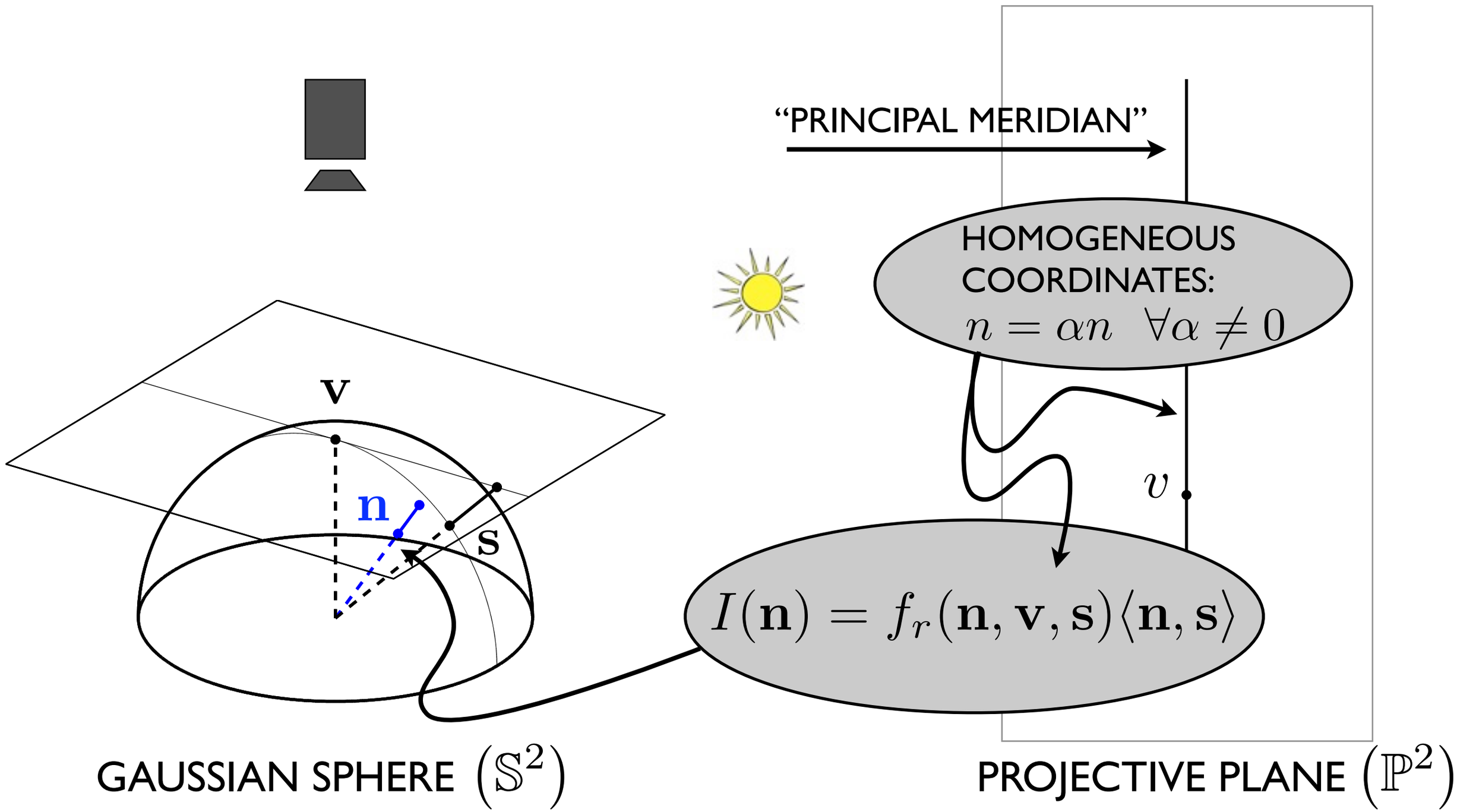
Problem Statement

Given a set of images and an initial shape as input, determine the correct shape using reflectance symmetries.



- When is correct shape guaranteed?
- How many input images are required, and under what source directions?
- What shape/lighting ambiguities exist when fewer images are available?
- What types of spatial reflectance variation can be tolerated?

Geometry of Reflectance Symmetries



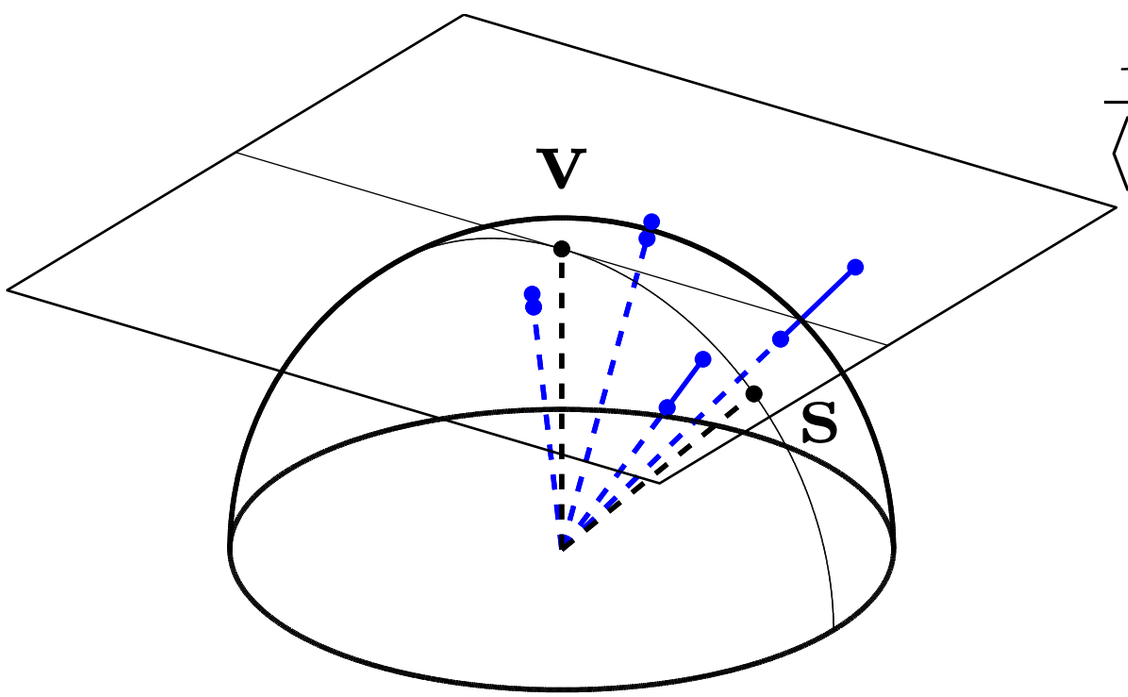
Geometry of Reflectance Symmetries

Equi-reflectance quadruple $(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, \mathbf{n}_4)$

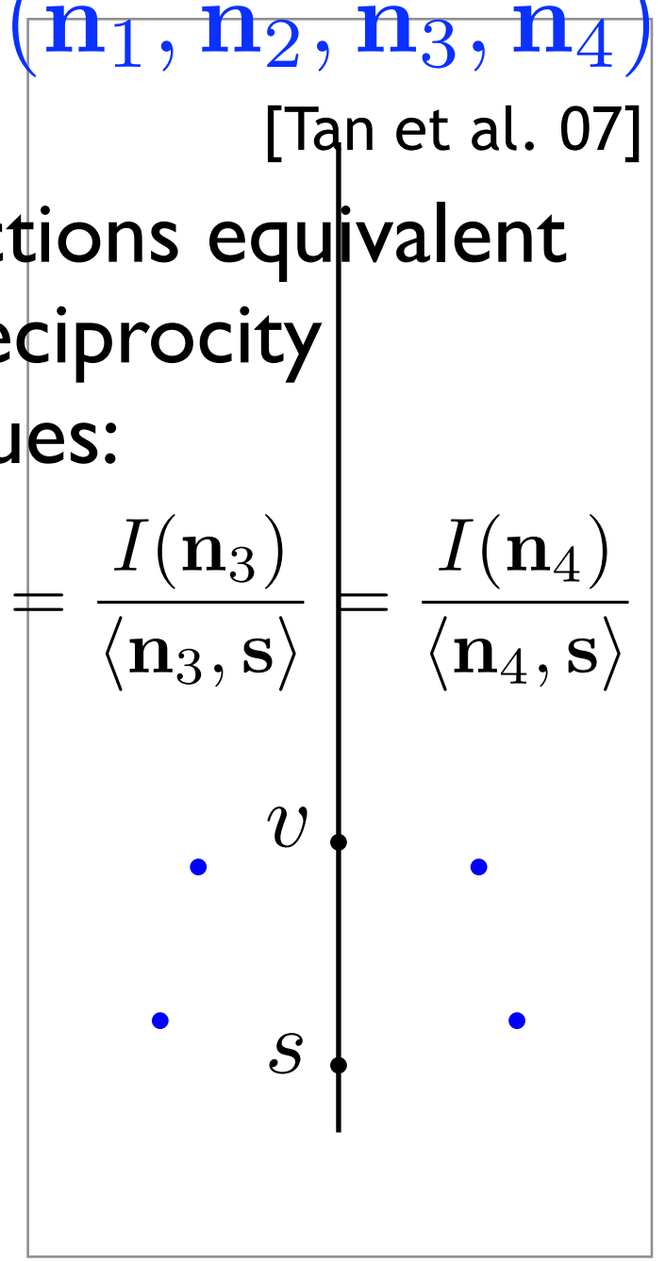
[Tan et al. 07]

- Local light/view directions equivalent up to isotropy and reciprocity
- Equal reflectance values:

$$\frac{I(\mathbf{n}_1)}{\langle \mathbf{n}_1, \mathbf{s} \rangle} = \frac{I(\mathbf{n}_2)}{\langle \mathbf{n}_2, \mathbf{s} \rangle} = \frac{I(\mathbf{n}_3)}{\langle \mathbf{n}_3, \mathbf{s} \rangle} = \frac{I(\mathbf{n}_4)}{\langle \mathbf{n}_4, \mathbf{s} \rangle}$$



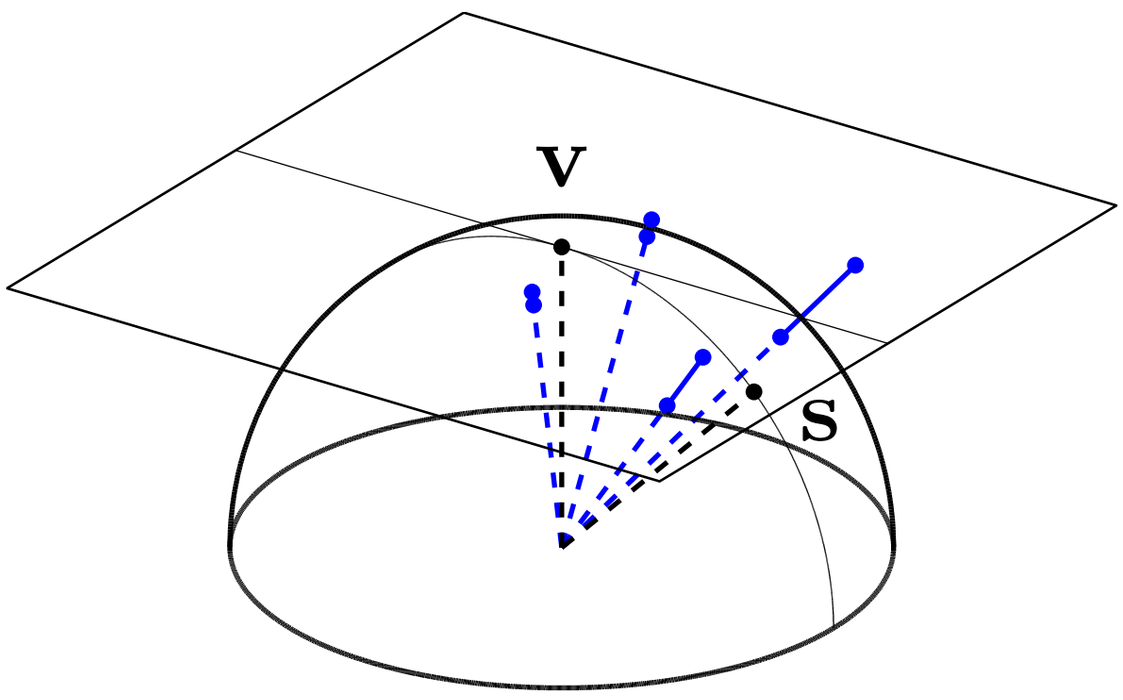
GAUSSIAN SPHERE $(\mathbb{S}^2)$



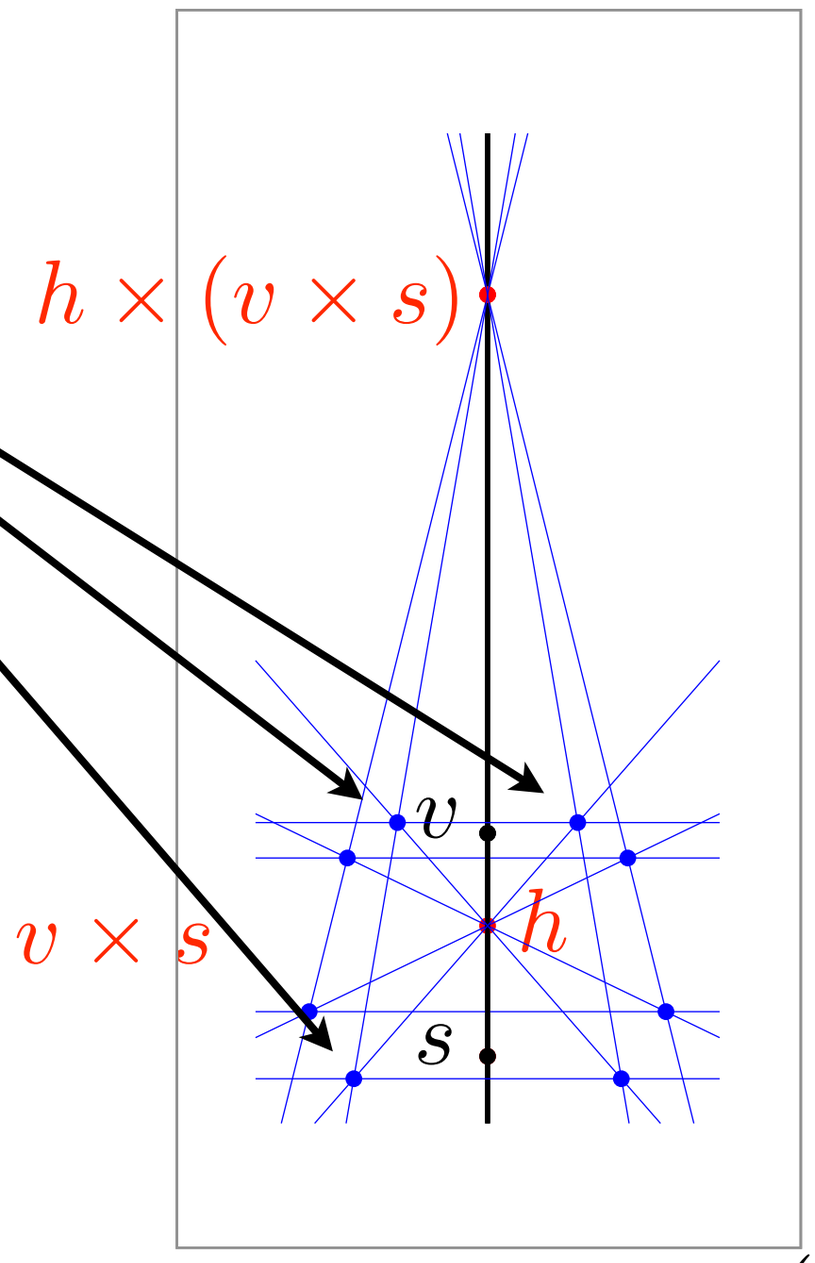
PROJECTIVE PLANE $(\mathbb{P}^2)$

Geometry of Reflectance Symmetries

$$\frac{I(\mathbf{n}_1)}{\langle \mathbf{n}_1, \mathbf{s} \rangle} = \frac{I(\mathbf{n}_2)}{\langle \mathbf{n}_2, \mathbf{s} \rangle} = \frac{I(\mathbf{n}_3)}{\langle \mathbf{n}_3, \mathbf{s} \rangle} = \frac{I(\mathbf{n}_4)}{\langle \mathbf{n}_4, \mathbf{s} \rangle}$$

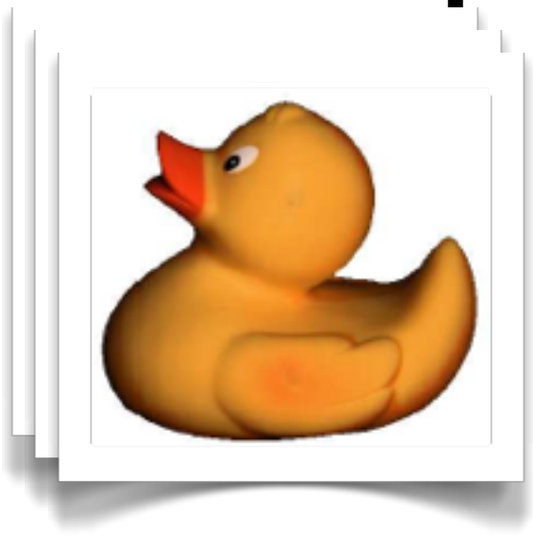


GAUSSIAN SPHERE $(\mathbb{S}^2)$

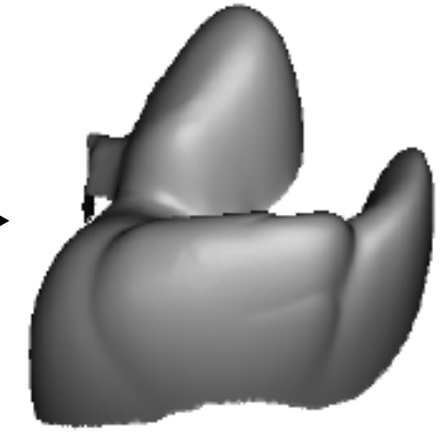


PROJECTIVE PLANE $(\mathbb{P}^2)$

Case I: Uncalibrated photometric stereo



Uncalibrated Lambertian photometric stereo
[Hayakawa 94;
Yuille & Snow 97]



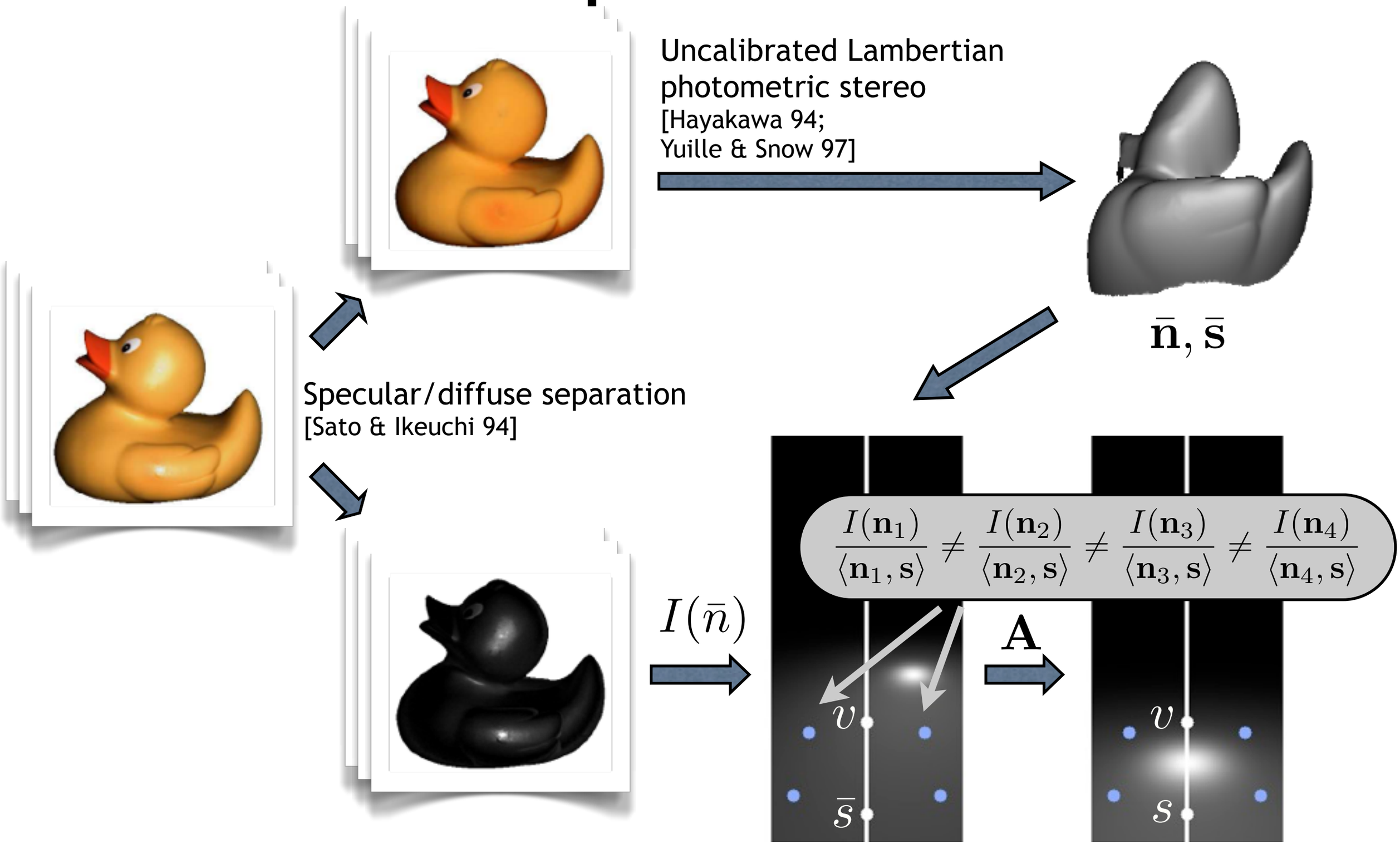
$\bar{\mathbf{n}}, \bar{\mathbf{s}}$

$$\mathbf{n} = \frac{\mathbf{A}^\top \bar{\mathbf{n}}}{||\mathbf{A}^\top \bar{\mathbf{n}}||} \quad \mathbf{s} = \frac{\mathbf{A}^{-1} \bar{\mathbf{s}}}{||\mathbf{A}^{-1} \bar{\mathbf{s}}||}$$

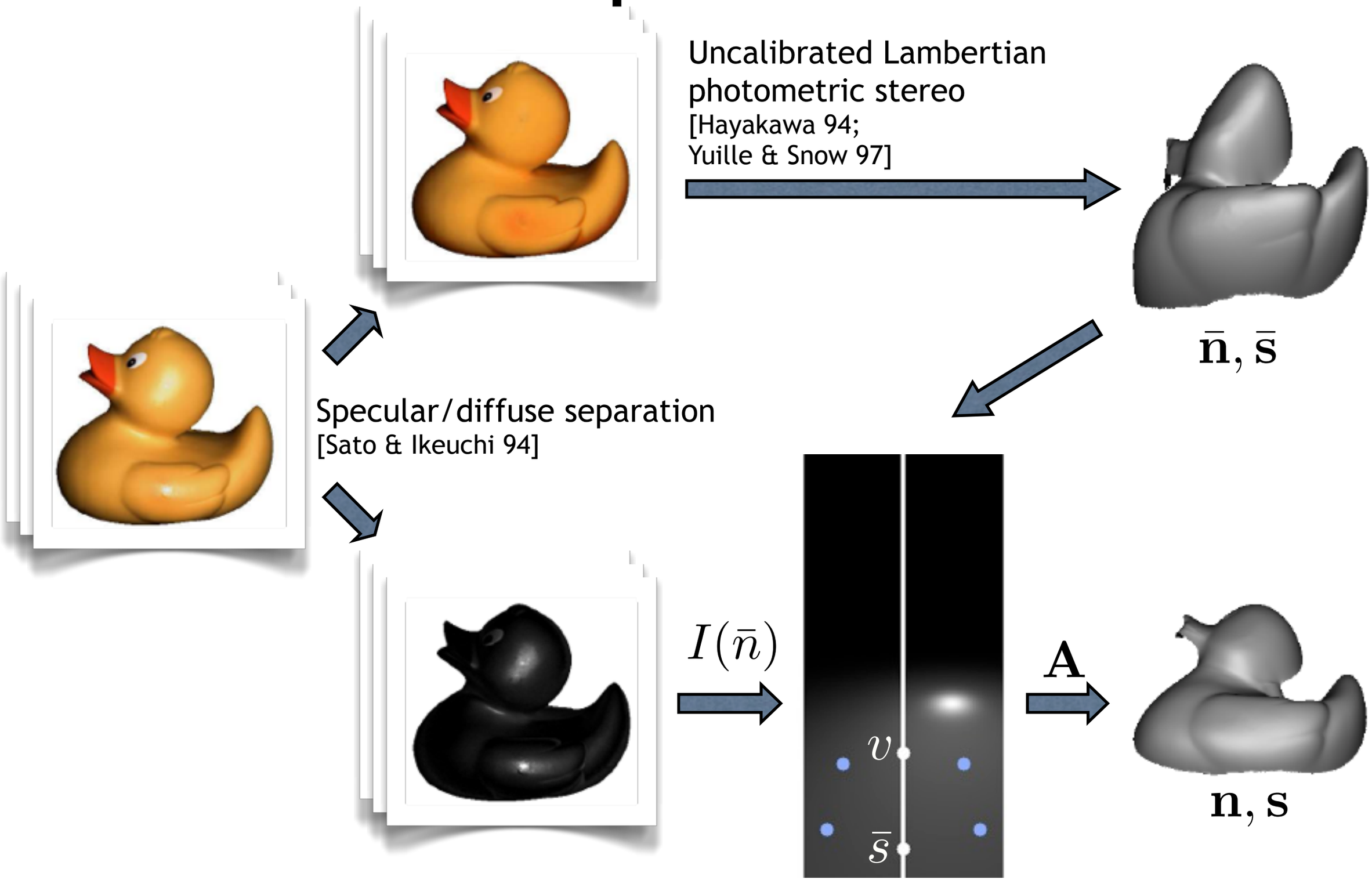
$\mathbf{A}$?

$\mathbf{n}, \mathbf{s}$

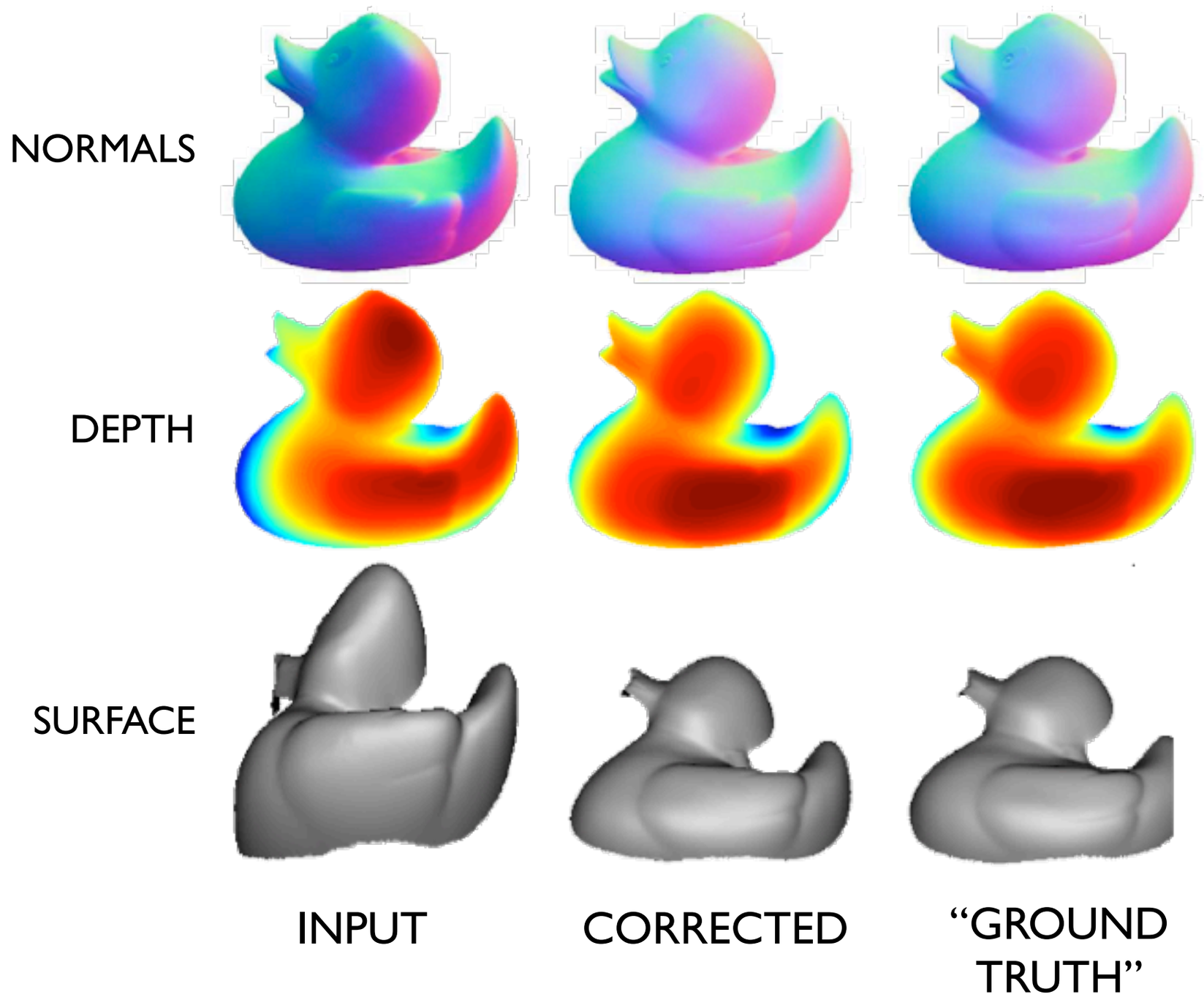
Case I: Uncalibrated photometric stereo



Case I: Uncalibrated photometric stereo



Case I:
Uncalibrated photometric stereo

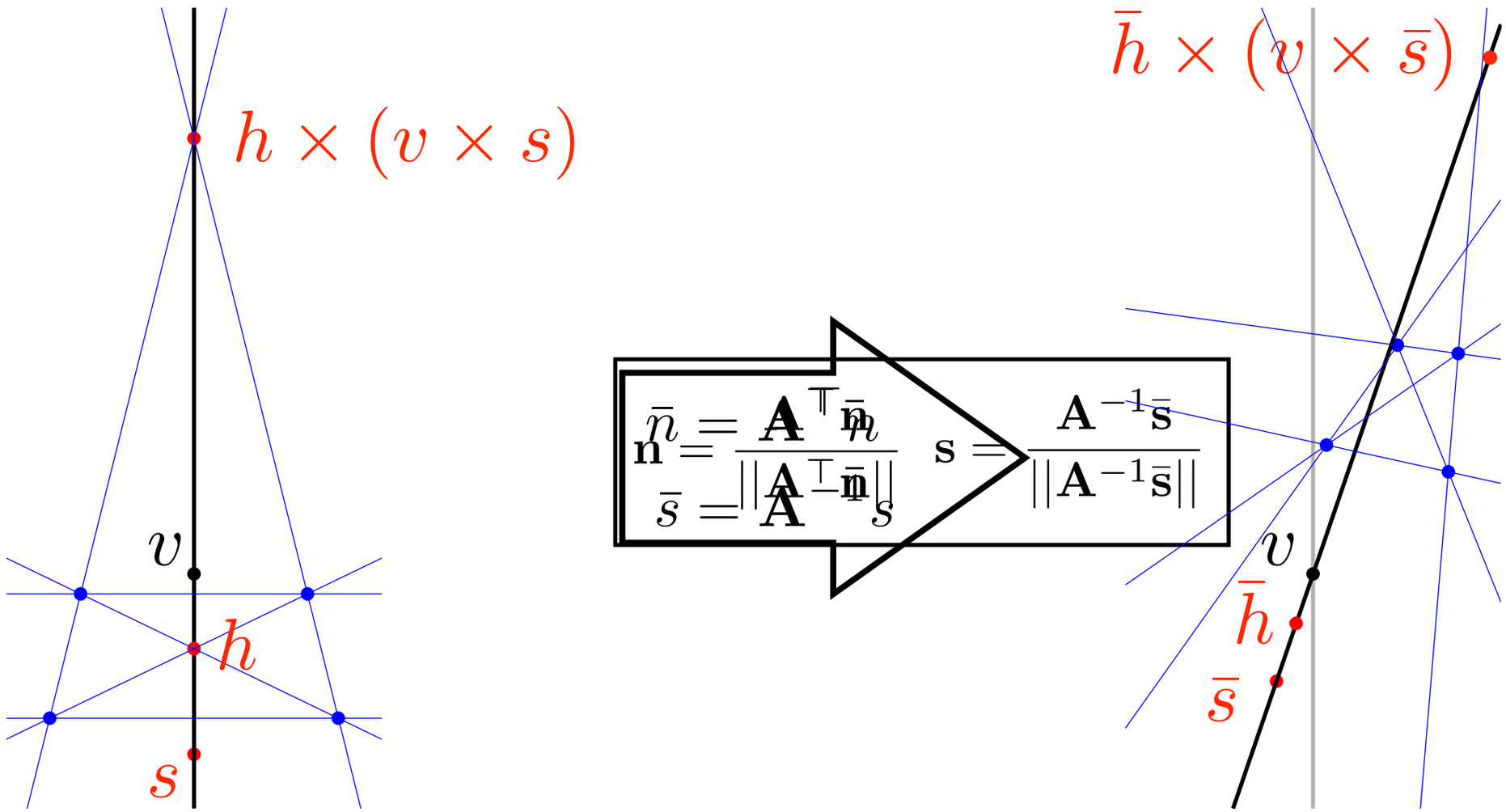


Uniqueness Results

Unknown Transform	Input	Remaining Ambiguity
GL(3)	Two images; one known source	None (unique solution)
GL(3)	Two images; one known source azimuth	Depth scale
GL(3)	Two images	Rotation + depth scale
GBR	One image	Discrete choice (4 sol'ns)
GBR	Two images	None (unique solution)*

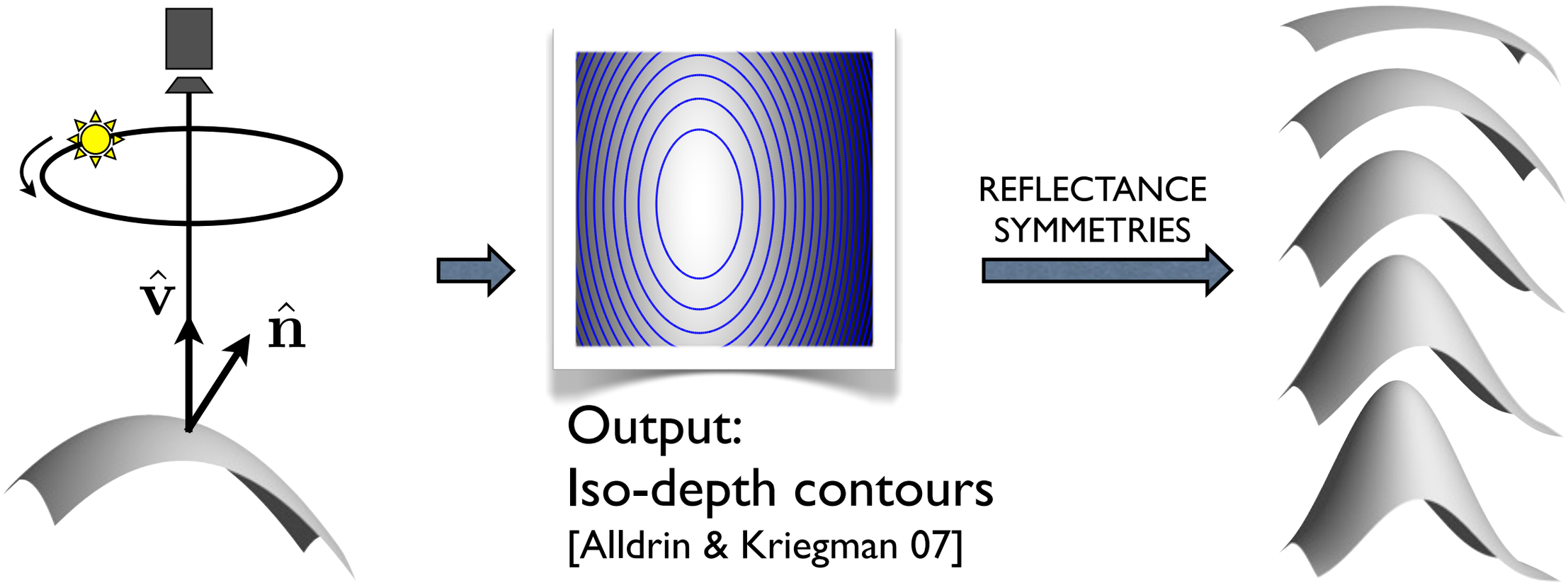
*[Tan et al. 07]

Intuition



- Projective transformations: $\mathbf{A}^\top$, $\mathbf{A}^{-1}$
- Destroy isotropic/reciprocal structure
- GBR: special case of translation & scale

Case II:
Calibrated Photometric Stereo

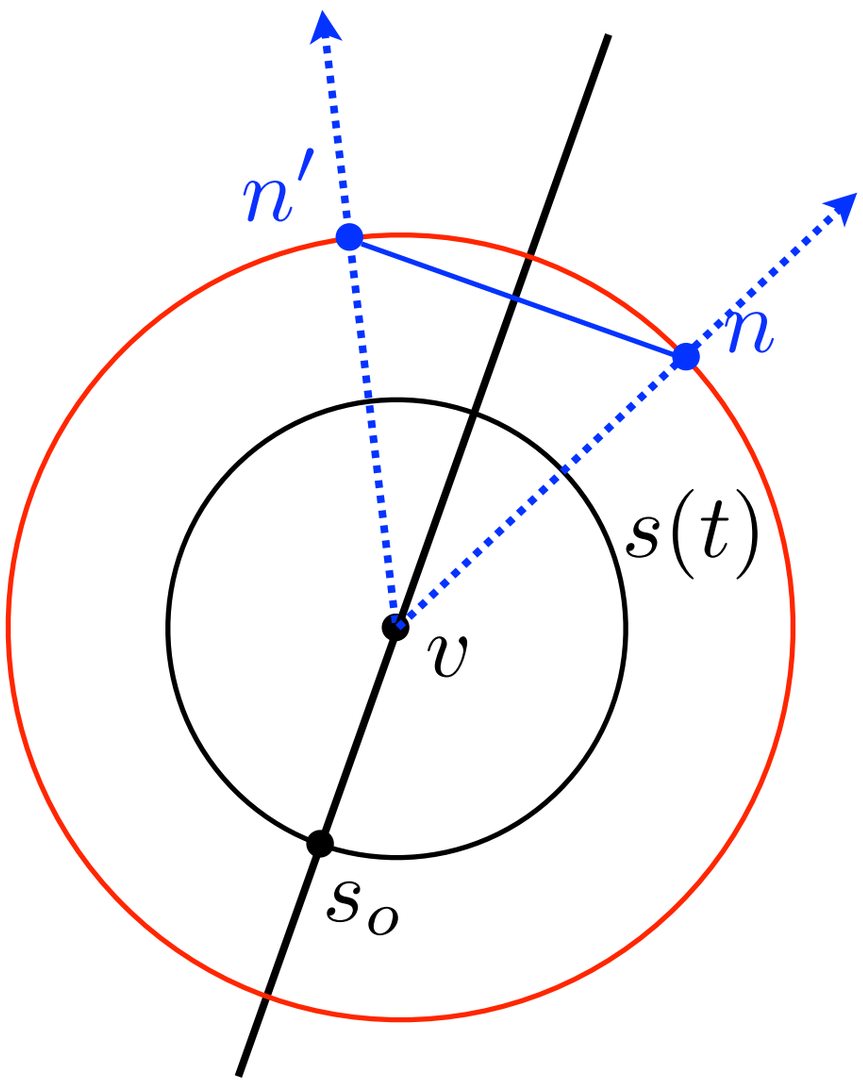


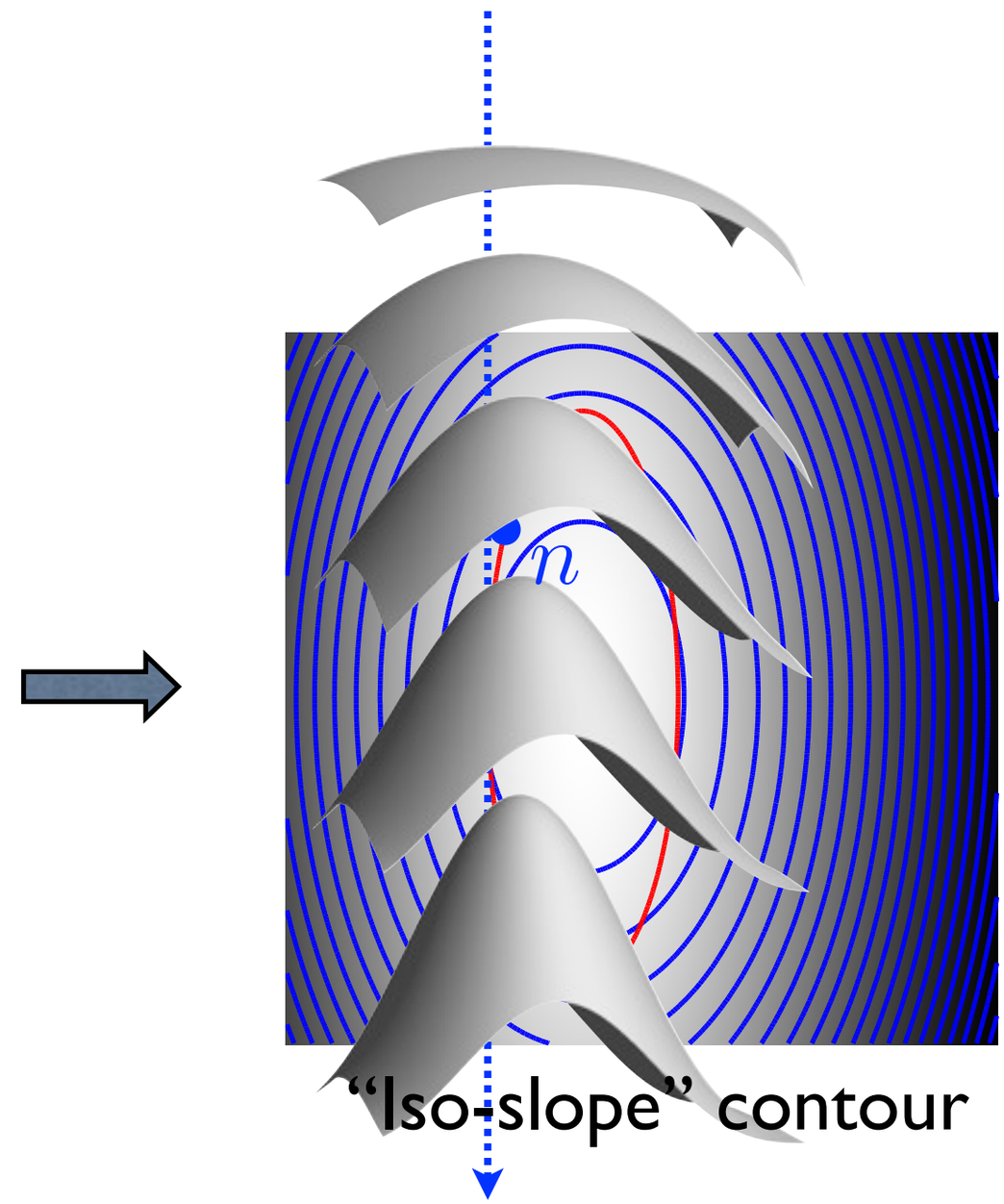
Output:
Iso-depth contours
[Alldrin & Kriegman 07]

Benefits:

1. Only requires isotropic BRDF at each point
2. Allows arbitrary spatial variation

Intuition





A suggestive example

OUR METHOD

COMPARISON
[Alldrin et al., 08]

ISO-DEPTH ISO-SLOPE

Theorem: Generically, dense iso-slope and iso-depth contours uniquely determine the surface.

Assumption:

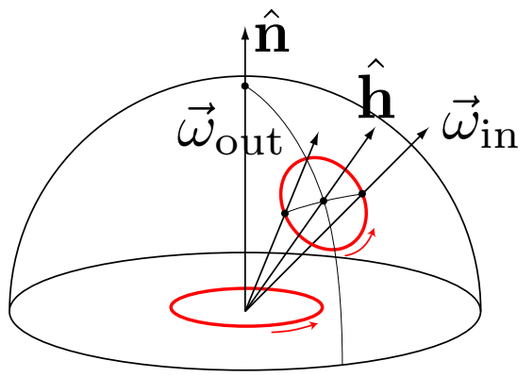
$$f(\theta_{in}, \phi_{in}, \theta_{out}, \phi_{out}) = f_1(\theta_{in}, \phi_{in}) + f_2(\theta_{out}, \phi_{out}) + f_3(\theta_{in}, \theta_{out}, |\phi_{in} - \phi_{out}|)$$

Conclusion

- Isotropy and reciprocity induce shape/lighting constraints
- These constraints can be represented succinctly on the projective plane
- This representation allows us to analyze two different formulations of photometric stereo
- This representation may benefit other image analysis problems as well

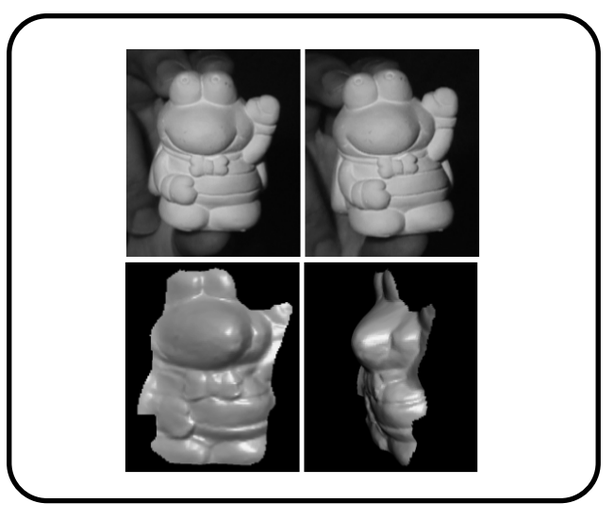
Future Work

- Additional reflectance symmetries

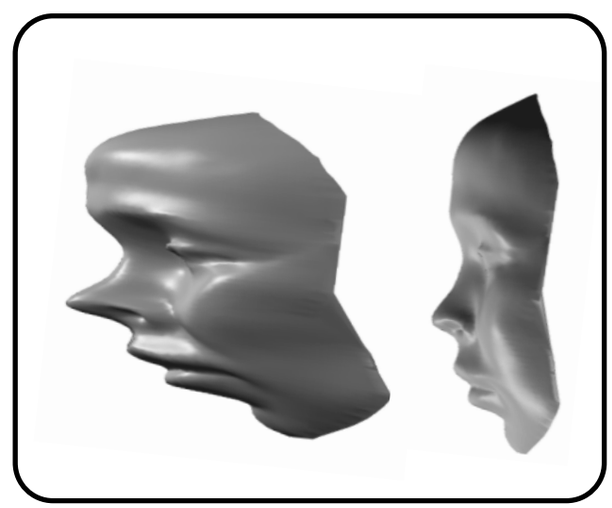


"Half-vector symmetry"
[Romeiro et al. 08]

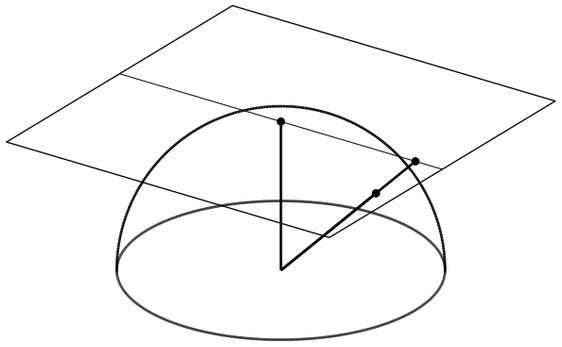
- Photogeometric reconstruction



[Joshi & Kriegman, 2007]



[Zickler et al., 2003]



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