A Projective Framework for Radiometric Image Analysis

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Radiometric Image Analysis

$I \rightarrow \text{shape, illumination, reflectance}$
Reflectance
Reflectance

\[ f_r(\vec{\omega}_{in}, \vec{\omega}_{out}) \]

Bi-directional Reflectance Distribution Function
Symmetry I: Reciprocity

\[ f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}}) = f_r(\vec{\omega}_{\text{out}}, \vec{\omega}_{\text{in}}) \]
Symmetry II: Isotropy

\[ \vec{\omega}_{\text{out}} \quad \hat{n} \quad \vec{\omega}_{\text{in}} \]

[Westin et al. 92]
Shape from Symmetries: Prior Work

Co-located cameras and light sources
- Isotropy [Lu & Little 99]
- Reciprocity [Zickler et al. 02]

Ring of light sources
- Isotropy [Alldrin & Kriegman 07]

Arbitrary light sources
- Isotropy & reciprocity [Tan et al. 07]
Shape from Symmetries: This Work

Constraints satisfied
\[ g(I_n, I_{n'}) = 0 \]

Incorrect shape & lighting: constraints violated
\[ g(I_{\bar{n}}, I_{\bar{n}'}) \neq 0 \]
Problem Statement

Given a set of images and an initial shape as input, determine the correct shape using reflectance symmetries.

- When is correct shape guaranteed?
- How many input images are required, and under what source directions?
- What shape/lighting ambiguities exist when fewer images are available?
- What types of spatial reflectance variation can be tolerated?
Geometry of Reflectance Symmetries

GAUSSIAN SPHERE \((S^2)\)  

PROJECTIVE PLANE \((\mathbb{P}^2)\)  

\[ I(n) = f_r(n, v, s) \langle n, s \rangle \]

HOMOGENEOUS COORDINATES: \(n = \alpha n\) \(\forall \alpha \neq 0\)

“PRINCIPAL MERIDIAN”
Geometry of Reflectance Symmetries

Equi-reflectance quadruple \((n_1, n_2, n_3, n_4)\)

- Local light/view directions equivalent up to isotropy and reciprocity
- Equal reflectance values:

\[
\frac{I(n_1)}{\langle n_1, s \rangle} = \frac{I(n_2)}{\langle n_2, s \rangle} = \frac{I(n_3)}{\langle n_3, s \rangle} = \frac{I(n_4)}{\langle n_4, s \rangle}
\]

[Tan et al. 07]
Geometry of Reflectance Symmetries

\[ \frac{I(n_1)}{\langle n_1, s \rangle} = \frac{I(n_2)}{\langle n_2, s \rangle} = \frac{I(n_3)}{\langle n_3, s \rangle} = \frac{I(n_4)}{\langle n_4, s \rangle} \]

GAUSSIAN SPHERE \( (S^2) \)

PROJECTIVE PLANE \( (\mathbb{P}^2) \)
Case I: Uncalibrated photometric stereo

Uncalibrated Lambertian photometric stereo
[Hayakawa 94; Yuille & Snow 97]

\[ n = \frac{A^\top \tilde{n}}{||A^\top \tilde{n}||}, \quad s = \frac{A^{-1}\tilde{s}}{||A^{-1}\tilde{s}||} \]
Case I: Uncalibrated photometric stereo

Uncalibrated photometric stereo
[Hayakawa 94; Yuille & Snow 97]

Specular/diffuse separation
[Sato & Ikeuchi 94]

Uncalibrated Lambertian photometric stereo

\[
\begin{align*}
I(\bar{n}) & \neq I(\bar{n}) \\
\langle \bar{n}, s \rangle & \neq \langle \bar{n}, s \rangle \\
\langle n_1, s \rangle & \neq \langle n_3, s \rangle \\
\langle n_2, s \rangle & \neq \langle n_4, s \rangle
\end{align*}
\]
Case I: Uncalibrated photometric stereo

Uncalibrated Lambertian photometric stereo
[Hayakawa 94; Yuille & Snow 97]

Specular/diffuse separation
[Sato & Ikeuchi 94]
Case I: Uncalibrated photometric stereo

INPUT

CORRECTED

“GROUND TRUTH”
## Uniqueness Results

<table>
<thead>
<tr>
<th>Unknown Transform</th>
<th>Input</th>
<th>Remaining Ambiguity</th>
</tr>
</thead>
<tbody>
<tr>
<td>GL(3)</td>
<td>Two images; one known source</td>
<td>None (unique solution)</td>
</tr>
<tr>
<td>GL(3)</td>
<td>Two images; one known source azimuth</td>
<td>Depth scale</td>
</tr>
<tr>
<td>GL(3)</td>
<td>Two images</td>
<td>Rotation + depth scale</td>
</tr>
<tr>
<td>GBR</td>
<td>One image</td>
<td>Discrete choice (4 sol’ns)</td>
</tr>
<tr>
<td>GBR</td>
<td>Two images</td>
<td>None (unique solution)*</td>
</tr>
</tbody>
</table>

* [Tan et al. 07]
Intuition

- Projective transformations: $A^\top$, $A^{-1}$
- Destroy isotropic/reciprocal structure
- GBR: special case of translation & scale
Case II: Calibrated Photometric Stereo

Output:
Iso-depth contours
[Alldrin & Kriegman 07]

Benefits:
1. Only requires isotropic BRDF at each point
2. Allows arbitrary spatial variation
Intuition

"Iso-slope" contour
A suggestive example

**OUR METHOD**

**COMPARISON**
[Alldrin et al., 08]

**Theorem:** Generically, dense iso-slope and iso-depth contours uniquely determine the surface.
Conclusion

• Isotropy and reciprocity induce shape/lighting constraints

• These constraints can be represented succinctly on the projective plane

• This representation allows us to analyze two different formulations of photometric stereo

• This representation may benefit other image analysis problems as well
Future Work

- Additional reflectance symmetries

\[ \vec{n} \]
\[ \vec{\omega}_{out} \]
\[ \vec{\omega}_{in} \]

“Half-vector symmetry”
[Romeiro et al. 08]

- Photogeometric reconstruction

[Joshi & Kriegman, 2007]
[Zickler et al., 2003]