

# Errata: First Printing of Mitzenmacher/Upfal Probability and Computing

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We would like to thank the many of you who have bought our book, and we would especially like to thank those of you who have taken the time to send us errors when you have found them. Below are errors, most of which should be corrected for the second printing (around March 2006).

Also, for those of you who are instructors, we would like to announce that we are working on a partial solution manual that will contain solutions for many of the exercises found in the book. Please contact the publisher (or us) if you would like a copy. Also, please feel free to contribute solutions if you would like!

- Page xvi, line -5: change “Matwani” to “Motwani”. (How’d that get by!)
- Page 9, line -6: change “events  $B \cap E_i$ ” to “events  $E_i$ ”.
- Page 16, line -1: in the equation, the last part should be  $(1 - p)^{n-2k}$ , not  $(1 - p)^{2k}$ .
- Page 17, line 7: in the equation for part c, it should be

$$\frac{1 + (1 - 2p)^n}{2}.$$

- Page 27, line 6: At the end of the paragraph, the following additional sentence would be helpful: One can similarly define the conditional expectation of a random variable  $Y$  conditioned on an event  $E$  as

$$\mathbf{E}[Y \mid E] = \sum_y y \mathbf{Pr}(Y = y \mid E).$$

- Page 73, line -2: “Exercise 4.20” should be “Exercise 4.21”.
- Page 79, Caption of Figure 4.2 should read: The butterfly network. In the wrapped butterfly levels 0 and 3 are collapsed into one level.
- Page 80, line -12: “bound (4.3)” should be “bound (4.1)”.
- Page 84, Exercise 4.6. It should be clear that in both problems we are looking only for upper bounds on these probabilities. Also, in the last line, we should be trying to bound:

$$\Pr((X > k) \cup (Y > \ell)).$$

That is, the intersection, not the union.

- Page 87, Exercise 4.19, part b: The exercise would be clearer if written as: “Use the fact that  $f(x) = e^{tx}$  is convex for any  $t \geq 0$  to obtain a Chernoff bound for the sum of  $n$  independent random variables with distribution  $Z$  as in part (a), based on a Chernoff bound for independent Poisson trials.”
- Page 93, line 1: “is not more” should be “is more”.
- Page 98: the variables  $b$  and  $n$  have gotten confused. Best to remove the  $b$ ’s entirely, with the following changes:
  - Page 98, line 15: change  $b$  to  $n$  TWO times.
  - Page 98, line 15: change  $m/b =$  to  $m/n =$ .
  - Page 98, line 15: change  $\lim_{n \rightarrow \infty}$  to  $\lim_{m \rightarrow \infty}$ .
  - Page 98, line 16: change  $X_n$  to  $X_m$  TWO times.
  - Page 98, line 17: change  $1/b$  to  $1/n$ .
  - Page 98, line 18 (equation): change  $X_n$  to  $X_m$ .
  - Page 98, line 18 (equation): change  $\lim_{n \rightarrow \infty}$  to  $\lim_{m \rightarrow \infty}$ .
- Page 99. equation in line 13 (just after “Combining, we have...”): The inequalities are going in the wrong direction.
- Page 103, line 5: The proof of the following theorem is a bit harder than originally thought; part of it is given in Exercises 5.13 and 5.14.

- Page 111, lines 9 and 10: the  $m/n$  should be  $(n/m)$ .
- Page 112, line 10: Just before the equation, change “hash value is” to “hash value when  $n \geq 2$  is”. Change the period at the end of the equation to the comma, and add after the equation: “where we have used the fact that  $(1 - p)^a \geq 1 - ap$  when  $0 < p < 1$  and  $a \geq 1$ .”
- Page 113, line 17; change “the probability that there are any empty bins” to “the probability that there are no empty bins”
- Page 113, line 19; change “the probability that there are any isolated vertices” to “the probability that there are no isolated vertices”
- Page 121, Exercise 5.13: The path suggested by the last parts of the Exercise, parts (c)-(f), are unfortunately not quite correct. (In particular, you might try showing part (c) is wrong!) The statement is correct, that is  $\Pr(Z \leq \mu) \leq 1/2$ . I just haven’t found an “elementary” proof. You could remove parts (c)-(f) and, in its place, show that  $\Pr(Z \leq \mu) \leq 1/2$  for all integers  $\mu$  from 1 to 10 by explicitly performing the calculation.
- Page 121, Exercise 5.14: Change part (b) to read: “(b) Using part (a) and Exercise 5.13, prove Theorem 5.10 for the case that  $\mathbf{E}[f(X_1^{(m)}, \dots, X_n^{(m)})]$  is monotonically increasing.”
- Page 122, line -12, Exercise 5.18: Change the  $O(e^{-n})$  to  $1 - o(1)$ .
- Page 127, Theorem 6.1: change  $\binom{n}{2}$  to  $\binom{n}{k}$  in the statement of the theorem.
- Page 129, line -9. It is worth noting that the number of vertices is never used in the proof, and hence we don’t even need to say that there are  $n$  vertices.
- Page 133, Line 17. It should be noted that technically we require that  $d = 2m/n \geq 1$ ; the case where  $d < 1$  is trivial and can be handled separately.
- Page 134, Theorem 6.7: you can change the statement by replacing “a nonnegative integer-” with “an integer-” and it is still true.

- Page 162, line -9: 2 lines before Lemma 7.3, the reference to equation (5.2) is incorrect. The reference should be to an unnumbered equation that appears on page 93, namely  $k! > \left(\frac{k}{e}\right)^k$ . The bounds referred to appear specifically in Lemmas 5.1 and 5.8.
- Page 183, Exercise 7.9: change “polynomial in  $p$ ” to “polynomial in  $n$ ”
- Page 185, Exercise 7.23: Change “0.99” in the opening paragraph to “0.9999” to match part c.
- Page 228, line 4:  $H(X) + H(Y)$  should be  $H(X_1) + H(X_2)$ .
- Page 231-232: The last part of the proof of Theorem 9.4 is wrong; we maximized where we should have minimized. The statement is of course correct and the proof is easily corrected. It is easiest to change the last equation of page 231 to

$$= \alpha - \frac{m - 2^\alpha}{m} (\alpha - \lfloor \log_2(m - 2^\alpha) \rfloor + 1).$$

Then the remainder of the proof is:

Suppose  $\lfloor \log_2(m - 2^\alpha) \rfloor = \beta$ , where  $0 \leq \beta \leq \alpha - 1$ . Then  $(m - 2^\alpha)/m$  is maximized when  $m$  is as large as possible, which corresponds to  $m = 2^\alpha + 2^{\beta+1} - 1$ . Hence

$$\begin{aligned} \mathbf{E}[Y] &\geq \alpha - \frac{2^{\beta+1} - 1}{2^\alpha + 2^{\beta+1} - 1} (\alpha - \beta + 1) \\ &\geq \alpha - \frac{1}{2^{\alpha-\beta-1}} (\alpha - \beta + 1) \\ &\geq \alpha - 1, \end{aligned}$$

completing the induction.

- Page 233, lines 6, 11, and -11: In all three cases, where it says  $\log_2(n + 1) - 1$ , it should say  $-\log_2(n + 1) - 2$ .
- Page 233, line -9: where it says  $\log_2(n + 1) + 1$ , it should say  $\log_2(n + 1) + 2$ .
- Page 273, line -15: ”spaces” should be ”spades”.

- Page 275, line 15: Change “Here the second line follows from the union bound. For the third line...” to “Here the third line follows from the union bound. For the fourth line...”
- Page 278, Equation at line 9: in the numerator of the rightmost expression change  $(n - 3k + 3)$  to  $n - (3k - 3)$
- Page 278, line -16: the right most-term should be  $\leq ke^{-t\dots}$ , instead of just  $e^{-t\dots}$ . That is, a factor of  $k$  was dropped.
- Page 278, line -13: change  $\ln \epsilon^{-1}$  to  $\ln(k/\epsilon)$ .
- Page 281, line 15: change “it make it” to “to make it”
- Page 283, line -7: the last inequality can be an equality.
- Page 284, Theorem 11.8 (line 14): Change  $n(c - \Delta)$  to  $nc$  in the numerator of the expression.
- Page 301, Definition 12.3: The definition is clearer if you change  $Z_0, Z_1, \dots, Z_n$  to  $Z_0, Z_1 \dots$  (i.e. the sequence just goes on).
- Page 304, line 3: The upper limit in the product should be  $t$ , not  $t - 1$ . Similarly, in lines 4 and 5, the upper limits in the products should be  $t - 1$  and not  $t - 2$ , and in line 4, the final expression should be

$$\mathbf{E}[e^{\alpha Y_t} \mid X_0, X_1, \dots, X_{t-1}].$$

- Page 304, lines 6 and 9: Change  $c_i$  to  $c_k$
- Page 310, Exercise 12.6: This is much harder than was intended; it’s hard to get through even with the original paper by one’s side! It is worthwhile to look up the original paper by Hoeffding to see the proof... (Journal of the American Statistical Association 58 (301): 13-30, March 1963.)
- Page 316, line 5: Change to “Without loss of generality, let  $z$  be an element...” Page 336, line -7: To be clear, the  $\Theta(\log \log n)$  results is meant only for *constant*  $d$ .