An Economically-Principled Generative Model of AS Graph Connectivity

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ABSTRACT

We explore the problem of modeling Internet connectivity at the Autonomous System (AS) level and present an economically-principled dynamic model that reproduces key features of the AS graph structure. We view the graph as the outcome of optimizing decisions made by each AS given its business model. In our model, nodes (representing ASs) arrive over time and choose and change providers to maximize their utility. Our formulation of AS utility includes revenue from an AS's own generated demand for traffic, congestion and routing costs, as well as transfers to and from provider and customer ASs, respectively. Our model has the following features: it uses an empirically-motivated model of traffic demand (Chang, Jamin, Mao, Willinger, 2005) which considers the variation in demand with ASs' business models and the graph of business relationships; it allows for nodes to revise their connections over time, in a fashion similar to the well-known 'forest fire' model (Leskovec, Kleinberg, Faloutsos, 2005); a node's utility explicitly models many of the major economic and technological issues at play.

We validate our model-generated graphs against those of other generative models. Building on previous work that has shown that rule-based generative models like preferential attachment yield poorly-performing traffic routing graphs (Li, Alderson, Doyle, Willinger, 2006), we show that our graphs perform well as designed, engineered systems, while retaining measured statistical properties of the AS graph.

1. INTRODUCTION

The Internet is composed of tens of thousands of subnetworks (domains) called Autonomous Systems (ASs), each administered separately and following its own distinct objectives and constraints in controlling the traffic entering and leaving its network. In such a multilateral setting, understanding the factors affecting AS inter-connection policies and how these affect the overall, inter-domain network performance is a challenging problem.

While there is already a large literature devoted to understanding Internet connectivity at the AS level, many of these models suffer from common pitfalls. Many models are static and fail to capture the AS graph evolution and its reactivity to changes in business relationships, routing policies, and inter-domain demand. Other models have evolutionary dynamics based on rules or generalized stochastics (e.g. [5, 12]), such as preferential attachment and the copying model, which fail to give insight into the economic and technological issues that govern the growth and maintenance of the AS graph.

Moreover, in a thoughtful critique of such rule-based and randomized models, Li, Alderson, Doyle and Willinger [14] show that while many generative models reproduce features of the AS graph such as node-degree distributions obeying a power law, they fail to capture the high utility or good performance of realistic networks [14]. In a study of an AS's intra-domain graph, Li, Alderson, Willinger and Doyle [15] define performance as network throughput and show that "only a careful design process explicitly incorporating technological constraints, traffic demands, and link costs yields high-performance networks. In contrast, networks with the same degree distribution, resulting from even carefully-crafted random constructions result in poor-performing networks." In other words, it is very unlikely that randomized or rule-based generative models will yield graphs that have the highly-optimized structure of real-world networks.

1.1 Contributions

The present work explores the problem of modeling AS graph formation using economic principles. Notable features of our model include the incorporation of AS business models with an asymmetric gravity model of inter-domain traffic demand [7], an explicit representation of AS utility that incorporates benefits for traffic routed, congestion costs, and payment transfers between ASs, and a deterministic process for revising of links that can cascade through the network. While previous work on AS network growth has focused on at most one aspect of these characteristics [8, 9, 3, 11], we have not seen the conjunction of these ideas studied before. As far as we know, there is no previous work on AS graph modeling that incorporates a deterministic process for link revision that can cascade throughout the network.

We validate our model against the properties of other generative models. To do this, we define the social planner's problem which is parameterized by the business models of the graph and provide a method to compare earlier generative models with our model by optimizing the placement of business models on the network. We find that our model

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yields graphs that are better-performing as compared to other dynamic generative models. We also find that our model yields a structured placement of nodes endogenously, where this placement of nodes generally reflects ASs' business models and market power. Finally, we find that our generative model reproduces key statistical features of the actual AS graph. In summary, although the generative model uses simple *myopic* decisions, with some opportunity for revision, we find it leads to well-performing graphs.

1.2 The Model

We construct a graph where ASs are nodes and a link between two ASs reflects a business relationship such that traffic is directly routed between them, in both directions. The likelihood of a link between two ASs is largely dependent on their respective business models and current market position. For example, an AS that is predominantly a residential access provider will have most of its traffic demand as incoming to its network, whereas an AS that mainly provides web-hosting services will have much of its traffic demand as outward flow.

There are two main types of business relationships, otherwise called service-level agreements, between ASs: customer-provider and peer-to-peer relationships. In the former, customers pay providers for the connection, whereas no transfers are made between peers. In previous work, it was shown that peering links, partly because of their paucity in relation to customer-provider connections, are unlikely to be a causal factor in generating many of the statistical properties associated with the AS graph topology [8].

Our model uses an empirically-motivated and parameterized traffic demand model that determines traffic demands on the basis of AS relationships and the quality of the paths separating them. An AS can affect its own demand and that of others by its linking decisions. Implicit in our model is that ASs will provision their networks as required to meet demand, therefore we do not assume fixed link capacities.

Our utility function captures the fact that an AS's revenue stream comes from servicing both customers within its own network as well as transit traffic requests from its inter-domain customers. We model the operating costs of an AS as stemming from congestion associated with traffic being routed through its servers and sub-domains. In turn, the price an AS charges for connectivity by its inter-domain customers is a function of the running costs of an AS.

Another key feature of our model is that we incorporate link revision. Since previous work has shown that ASs are prone to short term volatility [17], we want to allow for the possibility that an AS may want to revise its links as its customers, demands and congestion change. Our link revision process is inspired by previous work of Leskovec, Kleinberg and Faloutsos [13] that proposes a "Forest Fire" based link revision for their generative model, which aims to capture the growth of social networks. In the forest fire revision process for our AS graph, changes propagate from customers to providers in a depth-first search manner where branches die out when a provider does not make a change.

1.3 Related Work

Many generative models of AS growth have been proposed based purely on randomized or rule-based behavior (e.g. [5, 12]). Our model is economically-principled and we show that it yields both important statistical properties as well as much better engineered systems.

We share this economic approach with previous models, that have centered on the notion that an AS benefits from the routing of its AS-originating and terminating traffic and loses utility from transiting traffic. However, many of these [3, 11] have been formulated as game-theoretic models with static equilibria (fixed points), which makes it difficult to understand the graph's evolution over time. Moreover, some of these assume that edges have intrinsic costs or capacities [2, 1, 4], and still others hinge on fixed demand models [3, 11].

Chang et al.'s [9] formulation of an AS's decision problem uses an empirically-motivated demand model previously introduced in [7], which we also employ. Our model differs from theirs in that our utility function is explicit about the economic tradeoffs at play (the utility functions are very different) and our model does not involve any randomization beyond the sampling of business models, which is tuned here to empirically-measured distributions. While Chang et al.'s model also allows for the revision of links, each AS revises its links when a periodic "timer" goes off. Their method of link revision does not cascade throughout the network as ASs react to their neighbors' link revisions.

2. PROPERTIES OF THE MODEL

We model the formation of the AS inter-domain routing graph. Nodes (representing AS domains) come into contact with other nodes and they lay down links to maximize their economic benefits. Our model only considers customer-provider links and so the decision to establish a link is always initiated by the customer, who pays the provider for the link. The joint action of customers choosing their providers defines a directed graph reflecting the customer-provider business relationships between ASs. While link payments are one-sided, traffic flows in both directions since the customer pays for transit traffic to and from its providers.

Inter-domain traffic demand is tied to ASs' respective customer bases. We capture this by a model given in [7], assigning each node a business model according to their distribution from empirical data and defining traffic demands based on these. Generally speaking, a customer AS's utility for connecting to a provider AS's domain is a function of how that connection will affect its own customers' traffic demand, the link's impact on its links and network congestion, and finally on the balance of payments made and received by it for routing traffic along all of its adjacent customer-provider connections.

We proceed to a detailed explanation of the properties of our model. For the sake of clarity, a detailed discussion of the dynamic process by which the AS graph is grown is deferred until Section 3. In what follows, let N denote the set of nodes with $n = |N|$.

2.1 Strategies

The action of a node $i \in N$ is a vector $s_i \in \{0,1\}^n$ indicating which nodes i has chosen as its providers. We let $s = s_1 \times \cdots \times s_n$ be the joint action of all nodes. Note that an AS node's actions are restricted to choosing its providers, not its customers.

2.2 The Graph

The joint action s defines a directed graph $G(s)$ as follows. The nodes of $G(s)$, hereafter G, are the nodes N. An edge

$e = (i, j)$ is established if and only if $s_i(j) = 1$ and designates that i is a customer of j, which is to say that i pays j for the link. Let $E_i(s_i) = \{(i, j)|s_i(j) = 1\}$ be the set of node i's provider links, with $E(s) = \cup_{i \in N} E_i(s_i)$. Moreover, let $E_i^u(s) = \{(k, l), (l, k) : (k, l) \cup (l, k) \in E(s)\}$ refer to *all* edges adjacent to i.

2.3 AS Business Models

An AS's business model reflects its utility for incoming and outgoing traffic, as well as its disutility for routing traffic through its domain. Formally, each node $i \in N$ has a business model parameterized with coefficients $(\alpha_i, \beta_i, \gamma_i) \in (0, 1]^3$ where α_i reflects AS i's demand for inbound traffic and β_i reflects its demand for outbound traffic. The parameter γ_i captures an AS's relative capacity to be an effective inter-domain access provider. A high value of γ_i suggests that i is an effective provider. We can think of the business model parameters $(\alpha_i, \beta_i, \gamma_i)$ as representing an AS's utility for providing residental access, web hosting, and business access services, respectively. This is exactly the business model representation studied by Chang et al. [7] The business model coefficients $(\alpha_i, \beta_i, \gamma_i)$ are chosen from the joint distribution $F(a, \Sigma)$, where a refers to the distribution of γ_i, which is currently drawn from a power law [7]. Since business model coefficients are highly correlated in real-life, we use a measured pairwise correlation matrix Σ to compute α_i and β_i [7].

2.4 Traffic Demands

Let $B(G)$ be the traffic demand matrix where entry b^{kl} represents the total demand for traffic from k to l. Let $\mathcal{S}$ designate the routing policy. Accordingly, given $G = (N, E)$, let $P_{kl}(\mathcal{S}, G)$ designate the set of edges $e \in E$ along which traffic from k to l is routed.

Traffic demands between ASs are given by a realistic demand model that captures many key features of traffic demand in the actual AS routing graph [7]. The traffic demand model of Chang et al. [7] is an asymmetric extension of the gravity model [18], where in the extension, bilateral demands in each direction depend on the business models of each pair of ASs, as well as those along the routing path between them. The gravity model assumes that traffic demand from AS k to AS l is expressed as $\frac{R_k \times A_l}{\Omega_{kl}}$, where R_k is a repulsive factor associated with traffic originating at k, A_l is an attractive factor associated with traffic destined for l, and Ω_{kl} is a bottleneck factor that opposes traffic from k to l. The attractive, repulsive, and bottleneck factors are tied to the business models of ASs. For example, an AS that is predominantly a residential access provider will have most of its traffic demand as incoming to its network, whereas an AS that mainly provides web-hosting services will have much of its traffic demand as outward flow.

We associate the rank vector $(r_i^\alpha, r_i^\beta, r_i^\gamma)$ associated with node i's $(\alpha_i, \beta_i, \gamma_i)$ business model coefficients in relation to those of all other nodes N. Let ω, ρ, χ be positive constants that reflect the sensitivity of traffic to variations in business model rank values. In particular, a value of $\chi = 0$ will mean that demand is not dependent on graph structure.

Demands are then expressed as follows:

$$b^{kl} = \frac{b_\beta^{kl} + \kappa_\alpha \cdot b_\alpha^{kl}}{(\overline{r}_{kl}^\gamma)^\chi}, \qquad (1)$$

where

$$b_\beta^{kl} = (r_k^\beta)^{-\omega} \cdot (r_l^\alpha)^{-\rho} + \kappa_\beta \cdot (r_k^\alpha)^{-\rho} \cdot (r_l^\beta)^{-\omega},$$
$$b_\alpha^{kl} = (r_k^\alpha)^{-\rho} \cdot (r_l^\alpha)^{-\rho},$$
$$\overline{r}_{kl}^\gamma = \max\{r_u^\gamma : (u, v) \cup (v, u) \in P_{kl}(\mathcal{S}, G), v \in N\}.$$

These expressions can be justified by the fact that inter-domain traffic is, for the most part, either communication between web servers and clients ("web" traffic) or communication between two clients (inter-residential traffic). The amount of web traffic between k and l is given by b_β^{kl}. Web traffic consists of client-to-server requests for web resources and server-to-client responses. The first term of b_β^{kl} is meant to capture server-to-client responses ("response" traffic) from k to l and the second is meant to capture client-to-server requests ("request" traffic) from k to l. Response traffic is typically significantly greater than request traffic, so the parameter K_β is meant to capture the asymmetric nature of web traffic. In the spirit of the gravity model, the total amount of web traffic between a pair of ASs should be dependent on their client population size and their web content population size. Likewise, the total amount of inter-residential traffic between a pair of ASs should be dependent on the ASs' client population size. The amount of inter-residential traffic between k and l is given by b_α^{kl}. The parameter K_α determines the relative weight of web traffic and inter-residential traffic. The bottleneck factor, $\overline{r}_{kl}^\gamma$, is the worst-case service quality of the path that routes the traffic between k and l. Given business models for all nodes N and the graph G, the demand matrix B is computed by Equation 1.

We also define x_e^{kl} as the flow of traffic originating from k and destined for l traveling along edge e and assume that no packets are dropped by ASs. Therefore, we have

$$x_{i,j}^{kl} = \begin{cases} b^{kl} & \text{if } (i, j) \in P_{kl}(\mathcal{S}, G) \\ 0 & \text{otherwise.} \end{cases} \qquad (2)$$

This means that all of the demand is successfully routed through the network.

2.5 The Routing Policy

The routing policy that we use is the "No Valley and Prefer Customer" Routing Algorithm [10]. In a "No Valley" path $(v_1, v_2, ..., v_n)$, if (v_i, v_{i+1}), denotes a provider-customer relationship, then (v_j, v_{j+1}) is a provider-customer relationship for all j such that $i < j < n$. In addition to this, all ASs prefer to route their traffic through their customers because an AS does not have to pay its customers to carry traffic. The chosen AS paths are the shortest paths among the "No Valley and Prefer Customer" paths. This routing algorithm takes $O(NE)$ time where N is the number of nodes in the graph and E is the number of edges in the graph and is closer to the way actual traffic is routed on AS graphs than shortest path routing [10].

2.6 Costs and Payments

We model the cost associated with traffic routed through node q as a *congestion cost*, $\tau_q \cdot \sum_{e \in E_q^u} \sum_{k,l \in N} x_e^{kl}$. Recall that E_q^u designates all edges adjacent to q. Note that this is a cost τ_q applied to all traffic flow through q, capturing q's cost for routing inbound and outbound traffic as well as for all transit traffic. We discuss how to set the cost function τ_q below.

ASs with lower transit routing costs in turn provide more affordable and more reliable service to customer domains, making them the preferred inter-domain access providers. The price charged to a customer AS, p, by the provider AS, q, is modeled as a function of the congestion associated with traffic being routed through q's servers and sub-domains as well as a mark-up for the bilateral traffic flow along the purchased link. Precisely, node q charges node p

$$t_{pq}(G) = \lambda_q \cdot \sum_{e \in E_q^u} \sum_{k,l \in N} x_e^{kl} + \mu_q \cdot \sum_{e \in (p,q) \cup (q,p)} \sum_{k,l \in N} x_e^{kl} \quad (3)$$

The first term reflects the congestion cost experienced by q that is passed onto customer p. The second term represents the mark-up on all traffic along the link (p,q) and can be thought of as a per-packet price of traffic flow. t_{pq} is a linear function that assumes that the mark-up on traffic from costs incurred is separable from price of flow on a link. That p should pay for traffic in both directions is how most customer-provider arrangements are made, reflecting the fact that p is paying q for access to the rest of the AS graph network.

We stress that λ_q and μ_q are a function of q's identity, both terms relating to q's effectiveness as an access provider, and that they are customer-anonymous, i.e. independent of p. In practice, p's traffic demand along the proposed link does matter in these per-unit charges. Our assumption holds particularly well for small customers linking to much larger providers and for the rare cases where large customers link to relatively small providers [17].

An AS q's costs for routing inter-domain traffic are very much tied to how its network is provisioned. Two important factors affecting an AS's transit costs are the length of inter-domain links and the inter-domain bandwidth capacity. Lower transit costs are associated with topologies with greater geographic coverage and that are optimized for larger traffic volumes. In our model, the effectiveness of q as an access provider is captured by the coefficient γ_q of its business model. With this in mind, we choose parameters τ_q, λ_q and μ_q to vary super-linearly in γ_q to reflect the large variability among ASs' prices (and presumably costs) for customer traffic [9]. Precisely, we have that

$$\tau_q = \tau \cdot e^{-\gamma_q}, \; \lambda_q = \lambda \cdot e^{-\gamma_q}, \; \mu_q = \mu \cdot e^{-\gamma_q}. \quad (4)$$

where $\tau, \lambda, \mu > 0$ are model parameters.

2.7 The Utility Function

Transfer payments between ASs aside, an AS's revenue comes from the customers in its own domain. We adopt a simple model whereby a node obtains 1 unit of utility for every traffic packet either originating from it or destined for it.

Given the set of nodes N, each with its associated business model, the graph $G(s) = (N, E(s))$ of inter-domain connections and its associated traffic demand matrix $b^{kl} \in B(G)$, and the transit costs and transfers made and received, the utility of an AS node i is as follows:

$$u_i(G) = \sum_{j \in N} b^{ij} + \sum_{j \in N} b^{ji} - \tau_i \cdot \sum_{e \in E_i^u} \sum_{k,l \in N} x_e^{kl} - \sum_{j:(i,j) \in E_i} t_{ij}(G) + \sum_{j:(j,i) \in E_j} t_{ji}(G)$$

3. THE DYNAMIC MODEL

In this section, we explain the dynamic process by which the AS inter-domain graph is formed.

3.1 Overview of the Dynamic Process

As nodes arrive in the network, they are given a business model that is chosen randomly from the joint distribution described in Section 2. The newly born node chooses to connect to the existing AS inter-domain graph in a way that maximizes its utility function. We assume that this strategic behavior is myopic in that nodes base their decisions on the immediate effect on their utility, and ignore the potential consequences that current decisions have on the future evolution of the network. This is a reasonable assumption if nodes have limited information about the network structure or about how their actions will affect others decisions, and this approach provides a starting point to consider far-sighted behavior in later work.[1]

The dynamic process unfolds as follows: Time proceeds in discrete periods. In period t, a single AS node is born with business attributes $< \alpha_i, \beta_i, \gamma_i >$. The newly born AS proceeds to place a *single* link to maximize its utility. In the same time period, the new node's provider then has the occasion to revise its links. It can either lay down a new link and/or delete a single link (assuming it has more than one provider) based on the action that maximizes its utility. If it decides to make a change, then each of its providers has the occasion to do the same. This process continues as described below.

3.2 Best Response of an AS

When a node i is born, it chooses to form a single link with an existing node j in the network, such that connecting to j maximizes i's utility. The following is the best response function of node i given that its decision space is to add a link:

$$BR_i^A(G) = \underset{i' \neq i, i' \notin E_i}{\operatorname{argmax}} \; u_i(G' = (N, E \cup (i, i')))$$

A node i can also add a link or delete a link or do both during the revision of nodes and will have a similar best response function.

3.3 Initial Conditions

Our random process starts with a single node being born. The second node is similarly born and (necessarily) links to the first node as its provider. The third node that is born has a choice of either linking to the first node or the second node as its provider. One might think that the nodes that arrive early are severely restricted in their choice of providers, but

[1]Indeed, in a survey of AS interconnection arrangements, Norton [17] suggests that AS relationships are sometimes prone to short-term volatility from unforeseen changes in traffic demand routing patterns stemming from new contract agreements.

given that we incorporate revision of links into our model, nodes have the opportunity to improve their situation as more nodes arrive.

3.4 Revision of Links

Once a node lays down a link to its provider, this provider is given the opportunity to revise its links and this process continues recursively until no providers make a change. This process propagates upstream from customers to providers. Customers may add a link to a new provider or delete a link to an existing provider, but a provider may not add or delete customer links. In order to make our revision process tractable and ensure that our revision process does not cycle, we perform a depth first search where branches die out once a provider decides not to make a change. The revision of links process in our model is much like the Forest Fire model of Leskovec et al. [13] although our revision process is deterministic rather than randomized.

At each node in the depth first search, a node first computes its best response function for adding a link given the current topology of the network and adds a link if doing so increases its current utility. It then computes its best response function for deleting a link given the current topology of the network and deletes a link if doing so decreases its utility.

Node i does not have a single best response function, but its best response during the link revision process should be thought of as computing the best response addition and subsequently computing the best response deletion:

$$BR_i^D(G) = \underset{i' \in E_i}{\operatorname{argmax}}\, u_i(G' = (N, E/(i,i')))$$

We note that we run the depth first search once and do not repeat the revision process until we reach an equilibrium. We note that in many cases it may be computationally expensive to do so, since each computation of the best response function involves multiple computations of all-pairs shortest paths and traffic demands. We also believe it may be unnecessary to run the link revision process until it reaches equilibrium at every time step since nodes will have the opportunity to revise links in future time intervals.

4. SIMULATION RESULTS

4.1 Statistical Properties

The graphs generated from our dynamic process satisfy some simple properties of the AS graph. We observe that our generated AS graphs satisfy power law degree distributions (as we would expect [14, 16]), shown by approximately linear behavior on a log-log scale. Though power law degree distributions are not unusual, they are still a key statistical property that AS graphs satisfy. Any valid AS model must generate graphs with power law degree distributions, however they should not be the only metric by which to judge AS graph generative models [14]. We observe that the customer degree distribution, the provider degree distribution and the overall degree distribution of our dynamically generated graphs satisfy a power law degree distribution. We show an example of the degree distributions in Figure 1[2].

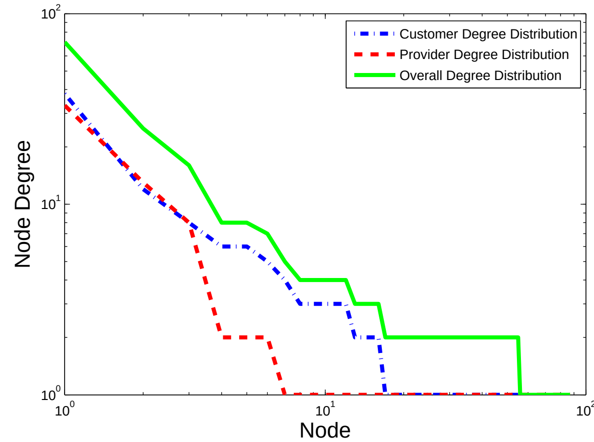

Figure 1: Power-law relationship of node vs. vertex degree

We observe that the exponent of the customer degree distribution is smaller (i.e. more negative) than the exponent of the overall degree distribution, which is in turn smaller than the exponent of the provider degree distribution, which is what is found in practice [9].

4.2 Defining the Performance of a Graph

Our main interest is in evaluating the relative performance of generated graphs against those of other generative models, including a highly optimized manual construction by Li et al. [14], based on the notion of Highly Optimized Tolerance (HOT) of Carson and Doyle [6]. We use a measure of social welfare to compare the relative performance. In this, we follow Li et al. [14] who notice that rule-based and purely stochastic generative models may reproduce certain statistical features of the graph, such as power-law degree distributions, but fail to capture important structural features related to the performance of the graph in question.

We define the performance of a graph as the social welfare function $W(G) = \sum_{i \in N} u_i(G)$. Notice that all the payments cancel out, so this objective function is just the total demand met by the network discounted by the congestion cost experienced by all nodes.

$$W(G) = \sum_{i \in N} \left(\sum_{l \in N} b^{il} + \sum_{k \in N} b^{ki} - \tau_i \cdot \sum_{e \in E_i^u} \sum_{k,l \in N} x_e^{kl} \right) \quad (5)$$

This is a reasonable model of social welfare for a network of utility-maximizing ASs.

4.3 Comparing Network Performance

We compare graphs generated by our generative model against a number of graph topologies: Erdős-Rényi random graphs, preferential attachment graphs [5], copying model graphs [12], randomized 2-geodetic graphs[3], and highly-optimized

[2]We observe that the exponents of the power laws are -0.8219, -0.3892, -0.7684 for the customer, provider, and total degree distribution, respectively. The model is set to parameters $\tau = 0.4, \lambda = 0.3, \mu = 0.2$, with traffic demand give by $a = -0.9, \omega = 1, \rho = 1, \chi = 2, \kappa_\alpha = 0.5, \kappa_\beta = 0.5$.

[3]All graphs have the probability of link addition p set to 0.3. The copying model also has $k = 2$. Our model graphs have $\tau = 0.6, \lambda = 0.2, \mu = 0.31$.

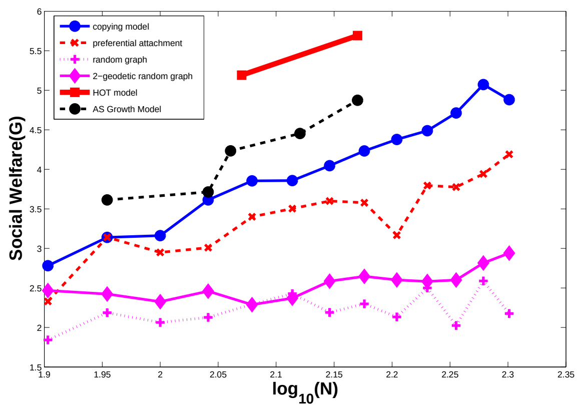

Figure 2: Social Welfare of Different Graph Models

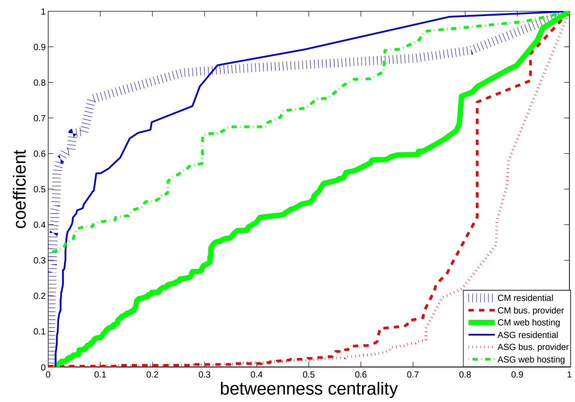

Figure 3: The average dominant business model coefficient against betweennness centrality.

tolerance (HOT) graph constructions [6]. The Erdős-Rényi random graphs have arguably the least structure and serve as a baseline for performance. The HOT graphs we evaluate are hand-crafted constructions (very similar to those presented in [14]) and exhibit a great deal of structure, which we would expect to be reflected in high performance.

We ensure that the graphs are of the same size and that the N nodes that comprise all graphs are identical, meaning that all graphs are made up of exactly the same set of business models. Given a set of business models and a graph, we match the business models to nodes in the graph to optimize the graph's performance objective function, in the spirit of presenting all comparison graphs in the best possible light. That is, given a collection of N nodes, each with an associated business model, and the undirected graph $G = (V, E)$, where $|V| = |N|$, we match each $i \in N$ to a $v \in V$, uniquely. The traffic demand model makes this a non-linear assignment problem which we solve by adaptive simulated annealing. We emphasize that node placements are achieved endogenously by our model.

Figure 2 shows the relative performance, measured according to the aggregate welfare of nodes, of different network topologies with business models optimally allocated (save for our AS graph generative model). The graphs are not normalized for the number of edges so more nodes and more edges will benefit performance. We find that our model-generated graphs fare well against all other graphs relative to hand-constructed HOT graphs, even against much larger copying model graphs (which have many more edges). This speaks to the power of economic constraints on AS strategic behavior to achieve good (social welfare-maximizing) results, even if AS actions are uncoordinated and myopic.

We also observe that our model reproduces, endogenously, something near the optimal, social welfare-maximizing placement of business models in the graph. We judge this by labeling all nodes according to their dominant (highest value) business model coefficient and measuring a node's location in the graph by its betweenness centrality. Given the graph $G = (V, E)$, the betweenness centrality $C_B(v)$ of a node $v \in V$ is $\sum_{s,t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}$, where σ_{st} refers to the number of shortest geodesic paths between s and t, and $\sigma_{st}(v)$, the number of shortest geodesic paths between s and t that pass through node v. Nodes lying on many shortest paths have higher betweenness centrality values than those that do not.

Figure 3 plots the average value of the dominant business model coefficient of nodes against their betweennness centrality. The graph compares results for copying model graphs, with the business models assigned to nodes to maximize social welfare, and those for our model-generated graphs, with the business models assigned to nodes as per the model[4] The plot suggests that our model endogenously achieves an optimizing placement of business models in the graph. For both our model-generated graphs and the optimally-assigned copying model graphs, as the dominant coefficient grows, business access providers move toward the center of the graph quickest of all, whereas residential access providers are much more likely to be located on the fringes of the graph.

5. CONCLUSIONS AND FUTURE WORK

We believe our model represents a significant departure from previous generative models by incorporating many of the tradeoffs affecting AS behavior rather than using simple stochastic models. A key feature of this model is the incorporation of revision of links since an AS's linking decisions are rarely permanent. We find that our model retains important statistical properties found in the actual AS graph. We also find that the performance of the graphs yielded by our model suggests well-engineered systems, in that they have high social welfare and an optimizing placement of business models. This is despite the relative simplicity of the dynamic, forest fire-like revision process that we consider.

We are presently validating our model against empirical data for a wide range of parameters. We are also exploring the use of different traffic demand models as well as incorporating peering links, i.e. links without transfers between parties.

Finally, we plan to study how the social welfare of graphs generated by our model are affected by different AS routing policies and to explore how new mechanisms for establishing interconnections may improve graph properties.

[4] The plot averages over 10 148-node graphs, with model parameters as before.

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